experimentally[10] and theoretically[11] for MnO, and a similar behavior is expected for UO_2.[12] Such fluctuation-induced tricritical behavior can be studied by the numerical simulation methods described here, provided that an anisotropy term is added to the Hamiltonian.

[1]P. Bak, D. Mukamel, and S. Krinsky, Phys. Rev. Lett. 36, 52 (1976); D. Mukamel and S. Krinsky, Phys. Rev. B 13, 5065, 5078 (1976); P. Bak and D. Mukamel, Phys. Rev. B 13, 5086 (1976).

[2]S. A. Brazovsky and I. E. Dzyaloshinsky, Pis'ma Zh. Eksp. Teor. Fiz. 21, 360 (1975) [JETP Lett. 21, 164 (1975)]; S. A. Brazovsky, I. E. Dzyaloshinsky, and B. G. Kukharenko, Zh. Eksp. Teor. Fiz. 70, 2257 (1976) [Sov. Phys. JETP 43, 1178 (1976)].

[3]See K. G. Wilson and J. Kogut, Phys. Rep. 12C, 75 (1974).

[4]B. C. Frazer, G. Shirane, D. E. Cox, and C. E. Olsen, Phys. Rev. 140, A1448 (1965).

[5]P. Lethuiller, J. Pierre, G. Fillion, and B. Barbara, Phys. Status Solidi 15, 613 (1973).

[6]S. J. Allen, Phys. Rev. 167, 492 (1968).

[7]K. Binder, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and M. S. Green (Adademic, New York, 1976), Vol. 5B, p. 1.

[8]O. G. Mouritsen and S. J. Knak Jensen, to be published.

[9]An alternative structure of UO_2, where the order parameter is a linear combination of those considered here, has been suggested by J. Faber and G. H. Lander, Phys. Rev. B 14, 1151 (1976). The Ginzburg-Landau-Wilson Hamiltonian, however, remains the same and the theory in Ref. 1 applies to this structure as well.

[10]D. Bloch, D. Hermann-Ronzaud, C. Vettier, W. B. Yelon, and R. Allen, Phys. Rev. Lett. 35, 963 (1975).

[11]P. Bak, S. Krinsky, and D. Mukamel, Phys. Rev. Lett. 36, 829 (1976).

[12]J. Sólyom and G. S. Grest, unpublished.

Theory of the Residual Resistivity Anomaly in Potassium

Marilyn F. Bishop and A. W. Overhauser
Department of Physics, Purdue University, West Lafayette, Indiana 47907
(Received 12 May 1977)

The induced-torque experiments of Holroyd and Datars show evidence for an anomalous anisotropy in the residual resistance of potassium of about five to one. We show that this is consistent with the hypothesis that the conduction electrons are in a static charge-density–wave state. The importance of torque and de Haas–van Alphen experiments on the same specimen is emphasized.

The experiments of Holroyd and Datars[1] have verified the existence of the giant torque anomaly in potassium that was originally observed by Schaefer and Marcus.[2] The results show a variation of the induced torque with magnetic-field direction in spherical samples of potassium, an effect that is sizable even in the limit of zero magnetic field and becomes enormous for high fields. The low-field results imply that the residual resistivity of potassium is highly anisotropic, in contradiction to the simple theory of metals. The purpose of this Letter is to show that this anisotropy can be explained if the conduction electrons in potassium are in a static charge-density–wave state.

In an induced-torque experiment, a magnetic field is rotated with respect to a suspended sample, usually spherical in shape. The time variation of the field induces currents, which, by interacting with the field, exert a torque on the sample. The magnitude of this torque is measured as a function of both the direction and magnitude of the magnetic field. For a simple metal, i.e., a metal with a spherical Fermi surface, the induced torque, for a spherical sample, should be independent of the direction of the magnetic field. De Haas–van Alphen measurements on potassium seem to indicate that its Fermi surface is spherical to within 0.1%.[3] In contradiction, the gigantic anisotropies in the induced-torque experiments suggest that potassium is not so simple and that further study is needed.

In order to facilitate the examination of this problem, we present in Fig. 1 the data for sample K-10 of Holroyd and Datars.[1] The induced torque is plotted versus direction of magnetic field for axes of rotation along the growth axis [Fig. 1(a)] and perpendicular to the growth axis [Fig. 1(b)], for field values from 1 to 23 kG. The field was rotated at a speed of 22°/min, and the sample was kept at a temperature of 1.5 K. In order that the sample acquire a precise shape, the potassium was grown in a Kel-F mold with a spherical cavity of diameter 1.11 cm, machined to within 10^{-3} in.[4] When the magnetic field was rotated in a plane perpendicular to the growth axis, as in Fig.

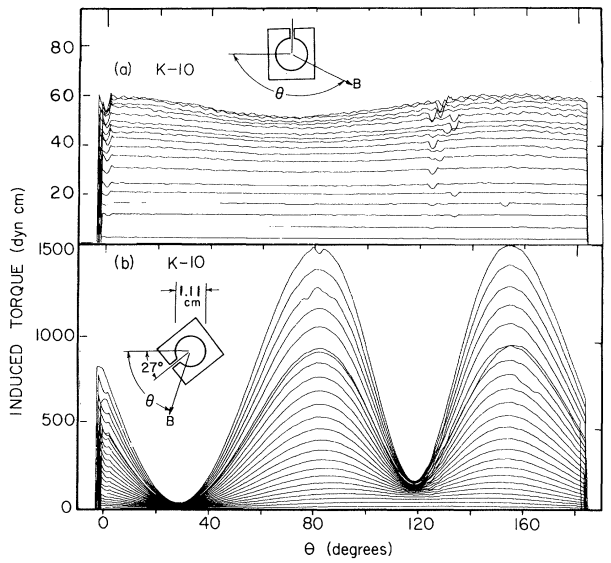

FIG. 1. Induced-torque vs magnetic-field direction for field values between 1 and 23 kG for sample K-10 of Holroyd and Datars, which was prepared in a mold. The sample was a sphere of diameter 1.11 cm with the field (a) rotated about the growth axis; (b) rotated in a plane containing the growth axis. The lowest curve in (a) is for 500 G; the second is for 1 kG. Their ratio leads to an x-z average $\omega_c\tau \sim 1.5$ at 1 kG.

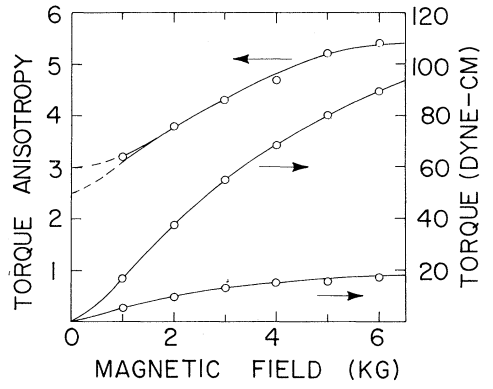

FIG. 2. Low-magnetic-field values of induced-torque shown in Fig. 1, with data points plotted as circles. The lower curve is the minimum in induced torque taken at an angle $\theta = 28°$ in Fig. 1, while the middle curve is taken at $\theta = 118°$. The upper curve is the ratio of these limiting values and is the low-field torque anisotropy. Solid curves are drawn through the experimental points.

1(a), the magnetic-field dependence was essentially isotropic, which is the result expected from a metal with a spherical Fermi surface. In contrast, when the growth axis was in the plane of rotation, as in Fig. 1(b), an anomalous four-peaked anisotropy of 45 to 1 appeared in the torque pattern for high fields. At low fields, this pattern became twofold, the maximum occurring when the field was perpendicular to the growth axis, and the minimum when it was parallel. The observed dependences clearly isolate the growth axis as a unique direction in this specimen.

In order to consider the low-field results in more detail, we plot with circles in Fig. 2 the data[4] at the minimum, 28° of Fig. 1(b) (lower curve), and at 118° (middle curve) in the torque pattern versus magnetic field. The upper circles are the ratios for each field value. The solid curves are drawn through the data, while the dashed extensions bracket all reasonable extrapolations of the ratio to zero field. Thus, the zero-field limit of the torque anisotropy is between 2.5 and 3 to 1. In this limit, the residual resistivity tensor $\overleftrightarrow{\rho}$ is diagonal; and with the directional dependence for this sample, we have $\rho_{xx} = \rho_{yy} = \rho_0$ and $\rho_{zz} = \gamma\rho_0$, where z is along the growth axis. The ratio R of the torque with magnetic field in the x direction to that with the field in the z direction for a spherical sample may thus be written as[5]

$$R = \frac{(\gamma+1)^2 + (\omega_c\tau)^2}{2(\gamma+1) + (\omega_c\tau)^2}. \quad (1)$$

With R at $H = 0$ between 2.5 and 3, γ must be between 4 and 5. This is consistent with the value $\gamma = 4$ that was originally estimated from the data of Schaefer and Marcus.[6] The $\omega_c\tau$ in Eq. (1) is that for the x or y direction. The data of Figs. 1 and 2 imply $\omega_c\tau \sim 2.5$ at 1 kG. The corresponding value for relaxation along the z axis would be ~ 0.5.

Lass has suggested that the high-field torque patterns observed by Schaefer and Marcus could be explained with an isotropic resistivity if the sample shapes had been nonspherical by about 10–15%.[7] However, this hypothesis could not possibly explain the data of Holroyd and Datars presented here in Figs. 1 and 2. The low-field anisotropy could be reproduced by Lass's model if the sample were elliptical (i.e., pancake shaped) with the growth-axis dimension reduced to 45% of its reported value, as if the mold had been slightly less than half filled.[8] However, for a magnetic field of 1 kG, the value of the torque at the minimum in the angular pattern calculated using this model could never exceed 3.2 dyn cm, regardless of relaxation time. Consequently, the value of 5.4 dyn cm observed by Holroyd and Datars is much too large to be consistent with this explana-

tion.

Finally, we remark that an attempt to extrapolate the anisotropy curve of Fig. 2 into unity for $H=0$ would require a 500% magnetoresistance along the z axis at 1 kG. The observed magnetoresistance in K is typically 2% for 1 kG.

Faced with the dilemma of an anisotropic residual resistivity in potassium, we suppose that the conduction electrons are in a static charge-density–wave (CDW) state. This hypothesis has produced successful explanations of other anomalies in potassium.[9] A CDW is characterized by a total self-consistent one-electron potential of the form $G\cos(\vec{Q}\cdot\vec{r})$. This potential mixes plane-wave states $\vec{k}$ with $\vec{k}\pm\vec{Q}$, producing a periodic variation in electron density and energy gaps of magnitude G at $\vec{k}=\pm\frac{1}{2}\vec{Q}$. The wave vector $\vec{Q}$ of the CDW spans the Fermi surface so that $Q\sim 2k_F$, where k_F is the Fermi wave vector. Mixing of states $\vec{k}$ with $\vec{k}\pm\vec{Q}$ permits impurity-induced Umklapp scattering along $\vec{Q}$.[6] Therefore, if an electric field is applied along $\vec{Q}$, the resistivity will be larger than if it is applied perpendicular to $\vec{Q}$. For this reason we take $\vec{Q}$ to be in the z direction, the growth axis of Holroyd and Datars' sample K-10.

In the temperature region of interest, the resistivity of potassium is dominated by impurity scattering. Electrons can scatter from an impurity potential V_c, which for mathematical convenience we take to have the Gaussian form

$$V_c=A\exp[-(r/r_s)^2]. \tag{2}$$

Here r_s is the Wigner-Seitz radius, a value that approximately mimics results of pseudopotential calculations.[10] In addition, scattering can take place from a strain field[11] with potential

$$V_s=B\sum_{\vec{L}\neq 0}\{\exp[-(\vec{r}-\vec{L}-\vec{u}_{\vec{L}})^2/r_s^2]-\exp[-(\vec{r}-\vec{L})^2/r_s^2]\}, \tag{3}$$

where $\vec{u}_{\vec{L}}\sim\vec{L}/L^3$ and is the displacement of a neighbor at site $\vec{L}$ from its position in the absence of an impurity.[12] The resistivity along each principal axis was calculated by assuming a uniform displacement δ of the Fermi sphere. δ was determined by balancing impurity-induced momentum transfer with that caused by the applied field.

We calculated the resistivity anisotropy for each of these two potentials as a function of an assumed normalized gap G/E_F of the CDW, where E_F is the Fermi energy; the results are plotted in Fig. 3. In addition, because the total influence of a defect is actually described by the combined effect of these two potentials, we plot the maximum and minimum values of the anisotropy resulting from any linear combination of the two terms. These are labeled interference (maximum) and interference (minimum) in Fig. 3. Since the results are ratios of resistivities, parameters such as the absolute magnitude of potentials and the concentration of impurities drop out of the calculations.

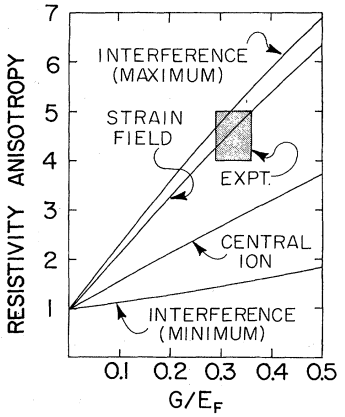

FIG. 3. Predictions of the resistivity anisotropy vs charge-density–wave gap G/E_F for the scattering-potential models described in the text. The shaded rectangle represents the range of values determined by experiments.

Experimental values are shown by the shaded rectangle in Fig. 3. The range of values of the CDW gap G/E_F (0.29 to 0.36) has been determined by other experiments.[9] It is clear that the calculated anisotropy can be large enough to account for the experimental results if scattering from the strain field is comparable to or greater than that from the central ion. This is certainly reasonable when the valence of an impurity (e.g., Na) is the same as that of the lattice ion or if the impurity introduces a significant strain.

The high-field torque peaks can be explained only when the multiply connected topology of the Fermi surface is recognized. A CDW structure provides this complication,[6] but a quantitative theory is not yet developed.

We conclude that the CDW model for potassium, for reasonable values of the parameters, provides a resolution of the puzzle of the anisotropic

residual resistivity that is consistent with experiment. We must note, however, that it is by no means necessary that any single crystal be characterized by a single $\vec{Q}$ direction throughout. It is possible for a $\vec{Q}$-domain structure to exist. This $\vec{Q}$-domain structure, in fact, provides an explanation for the sample dependence of the anisotropies seen in induced-torque experiments.[1,2] Since the torque anisotropies in sample K-10 of Holroyd and Datars are larger by an order of magnitude than those in other samples, we conjecture that it was probably the only single-$\vec{Q}$ sample that has ever been studied.

A difficulty for the CDW model in potassium stems from the results of the de Haas–van Alphen (dHvA) experiments.[3] However, it is not clear what effect a multiple $\vec{Q}$-domain structure would have on the results of these experiments.[13] Unfortunately, dHvA experiments have never been done on samples that are known to exhibit the large torque anisotropies shown in Figs. 1 and 2. Therefore, with induced-torque measurements as a tool for characterizing samples, it would be extremely interesting to perform dHvA experiments on a sample that exhibits the large torque anisotropies discussed here.

A CDW structure will lead to weak diffraction satellites. Phase excitations[9] may reduce their intensity in K by several orders of magnitude. A search for these satellites at 10^{-7} of the (110) intensity level should be attempted.

This work was supported by the National Science Foundation under Grant No. 41884.

[1]F. W. Holroyd and W. R. Datars, Can. J. Phys. 53, 2517 (1975).
[2]J. A. Schaefer and J. A. Marcus, Phys. Rev. Lett. 27, 935 (1971).
[3]D. Shoenberg and P. J. Stiles, Proc. Roy. Soc., Ser. A 281, 62 (1964); M. J. G. Lee and L. M. Falicov, Proc. Roy. Soc. Ser. A 304, 319 (1968).
[4]F. W. Holroyd and W. R. Datars, private communication.
[5]P. B. Visscher and L. M. Falicov, Phys. Rev. B 2, 1518 (1970).
[6]A. W. Overhauser, Phys. Rev. Lett. 27, 938 (1971).
[7]J. S. Lass, Phys. Rev. B 13, 2247 (1976).
[8]However, because the Kel-F mold was transparent, both Holroyd and Datars observed that it was completely filled. They also remarked that sample K-10 had a diameter of 1.11 cm (not 1.71), and that the ordinate scale of (their) Fig. 5(a) was incorrect. Magnet rotation rate for this specimen was 22°/min (not 35°/min).
[9]For a review, see A. W. Overhauser, Phys. Rev. B 3, 3173 (1971).
[10]T. M. Rice and L. J. Sham, Phys. Rev. B 1, 4546 (1970).
[11]A. W. Overhauser and R. L. Gorman, Phys. Rev. 102, 676 (1956).
[12]J. D. Eshelby, J. Appl. Phys. 25, 255 (1954).
[13]We note here that unrecognized $\vec{Q}$-domain structure of the spin-density-wave state of chromium led to initial reports that the dHvA effect had cubic symmetry.

Large-Lattice-Relaxation Model for Persistent Photoconductivity in Compound Semiconductors

D. V. Lang and R. A. Logan
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 6 April 1977)

A new model, based on an extremely strong coupling between the electronic and vibrational systems of certain defect centers, is proposed to explain the phenomenon of persistent photoconductivity observed in some compound semiconductors. The model is supported by data on donor-related defects in n-type $Al_xGa_{1-x}As$ which exhibit the features characteristic of this effect: a very large Stokes shift (thermal depth, ~ 0.1 eV; optical depth, ~ 1.2 eV); and a very small ($< 10^{-30}$ cm^2), thermally activated, electron-capture cross section at temperatures below 77 K.

Many compound semiconductors exhibit persistent impurity photoconductivity at temperatures below about 77 K. This very striking, and puzzling, effect is characterized by a large photoconductivity which persists for hours, or even days, after the optical excitation is removed. It can only be quenched by heating the sample above some characteristic temperature. Such effects have been observed in $Al_xGa_{1-x}As$,[1] $GaAs_{1-x}P_x$,[2] $Cd_{1-x}Zn_xTe$,[3] CdTe,[4-6] GaSb,[7] and CdS.[8] The long persistence time implies that the capture cross section for the photoexcited carriers is less than 10^{-30} cm^2. This is *six orders of magnitude* smaller than the smallest capture cross section of any well-characterized attractive, repulsive, or neutral center in Si, Ge, or III–V semiconductors.[9-11]