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Dynamic Polarization Potential for Coulomb Excitation Effects on Heavy-Ion Scattering

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The very large deviations from Fresnel diffraction due to Coulomb excitation seen in heavy-ion scattering can be reproduced by a polarization potential which is derived without making the adiabatic approximation. This potential is predominantly imaginary.

Recent measurements[1-3] have confirmed that Coulomb excitation occurring during the scattering of two heavy ions can have dramatic effects upon the elastic cross sections when there is strong coupling to low-lying 2^+ states. The cross section deviates from the typical Fresnel shape by falling below the Rutherford cross section at small scattering angles which correspond to impact parameters much larger than those for grazing collisions. This is due to the long range of the Coulomb excitation interaction.

This effect can be reproduced by coupled-channel calculations which include Coulomb excitation.[1-3] However, these are time consuming and expensive, so there is considerable interest in having an effective potential to add to the usual optical-model potential which will reproduce this effect. We present here such a potential together with some applications.

The existence of such a polarization potential was suggested long ago.[4] However, previous attempts to construct it have relied upon an adiabatic approximation which results in a real potential; our approach shows that the polarization potential is predominantly imaginary.

We construct the potential to second order in the interaction V coupling the two ions. We will describe the derivation in detail elsewhere, but the essential approximation is to use plane waves in the intermediate states and to consider the effect of the potential acting on a plane wave. To lowest order, this is the approach used by Mott and Massey[5] for atomic collisions and it results in a local potential. We next make a sudden approximation in which it is assumed that the excitation energy of any important intermediate state is small compared to the bombarding energy. This yields a polarization potential for a 2^λ-pole excitation which has the form

$$U_{p,\lambda}(R) = -\frac{2\lambda+1}{16\pi}\frac{V_\lambda(R)}{E_{\mathrm{c.m.}}}\left[V_\lambda(R) + i2kF_\lambda(R)\right], \quad (1)$$

where

$$F_\lambda(R) = 2\pi^2 i^\lambda \int_0^\infty q dq j_\lambda(qR)\tilde{V}_\lambda(q), \tag{2}$$

and $\tilde{V}_\lambda(q)$ is the 2^λ-pole Fourier transform of $V_\lambda(R)$. Here $V_\lambda(R)$ is the radial dependence of the 2^λ-pole part of the interaction V, $E_{\text{c.m.}}$ is the center-of-mass energy, k is the wave number, and R is the separation of the centers of mass of the two ions. F_λ is real and we can show that $U_{p,\lambda}$ is predominantly imaginary.

The plane-wave approximation assumes no deflection or acceleration by the Coulomb field as the two ions approach. However, we are most interested in scattering where the deflection is not very large. Furthermore, the braking effect in the repulsive Coulomb field may be accounted for approximately by using the local energy and local wave number at radius R in place of $E_{\text{c.m.}}$ and k in Eq. (1). This gives a radially dependent correction factor

$$K(R) = (1 - Z_1 Z_2 e^2/RE_{\text{c.m.}})^{-1/2}. \tag{3}$$

[$K(R)$ diverges for $R = R_d$ where $R_d = Z_1Z_2e^2/E_{\text{c.m.}}$, so we arbitrarily set $K(R) = K(R_d/0.9)$ for $R < R_d/0.9$.] In addition, the derivation of the expression (1) involved neglecting the excitation energy ΔE of the intermediate state relative to the bombarding energy. However, the probability of Coulomb excitation is known[4] to depend strongly upon this excitation energy, because of the lack of high-frequency components in the long-range Coulomb interaction. This dependence may be included by introducing another correction factor,

$$g_\lambda(\eta, \xi) = f_\lambda(\eta, \xi)/f_\lambda(\eta, 0), \tag{4}$$

where η is the usual Coulomb parameter,[4] ξ is the usual adiabaticity parameter,[4]

$$\xi = \tfrac{1}{2}\eta \Delta E/E_{\text{c.m.}}, \tag{5}$$

and $f_\lambda(\eta, \xi)$ is the standard factor[4] that appears in expressions for the cross section for 2^λ-pole Coulomb excitation. The classical limit for f_λ is adequate for the large η values that occur for most heavy-ion scatterings; values of f_λ are tabulated in Ref. 4.

When we collect these results, using the Coulomb interaction for V in Eqs. (1) and (2), we obtain an explicit expression for the imaginary part of U_p. (The real part is found to be negligible.) Quadrupole excitation is by far the most important, and for this we find

$$\operatorname{Im} U_p(R) = \begin{cases} -\left[1 - \frac{2}{7}\left(\frac{R_c}{R}\right)^2 - \frac{1}{21}\left(\frac{R_c}{R}\right)^4\right] K(R)\frac{W_p}{R^5} & \text{for } R \geq R_c \\ -\frac{2}{3}K(R) W_p R^4/R_c^{\,9} & \text{for } R < R_c, \end{cases} \tag{6}$$

where R_c is the charge radius; typically, $R_c = 1.2(A_1^{1/3} + A_2^{1/3})$. The strength is given by

$$W_p = C(\mu Z_p^{\,2}/k)B(E2, 0 \to 2)g_2(\xi) \text{ MeV fm}^5, \tag{7}$$

where $C = (8\pi c^2/75)(e^2/\hbar c)^2$, or $C = 0.0166$ if the reduced mass μ is given in atomic mass units, the wave number k is in fm^{-1}, and the $B(E2)$ is in $e^2\cdot\text{fm}^4$ units. $Z_p e$ is the charge on the ion which is causing the excitation. (When more than one state may be excited, the various contributions are simply to be added.)

The expressions (6) and (7) were tested by seeing how well they would reproduce the results of coupled-channel calculations with Coulomb excitation alone. The results were excellent and will be described in detail elsewhere. We show here (Fig. 1) the results of using this polarization potential, together with a conventional Woods-Saxon nuclear optical potential U_0 to fit the data[1] for $^{18}\text{O} + {}^{184}\text{W}$ at 90 MeV. The Woods-Saxon–potential depth was fixed at $V = 40$ MeV, but the other three parameters, W, r_0, and a, were adjusted for an optimum fit. The $B(E2)$ for ^{184}W was taken to be $(3.76\times10^4)e^2\cdot\text{fm}^4$ and the resulting W_p was increased 7% to account for other excitations (e.g., of the ^{18}O ion). This gave $W_p = 88.5$ GeV fm^5. The fit shown in Fig. 1 was obtained with $W = 9.06$ MeV, $r_0 = 1.313$ fm, $a = 0.457$

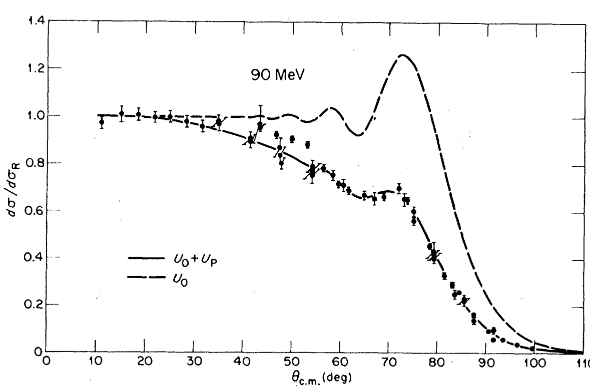

FIG. 1. Fits to the measured (Ref. 1) cross sections for $^{18}\text{O} + {}^{184}\text{W}$ with use of polarization potential U_p as described in the text.

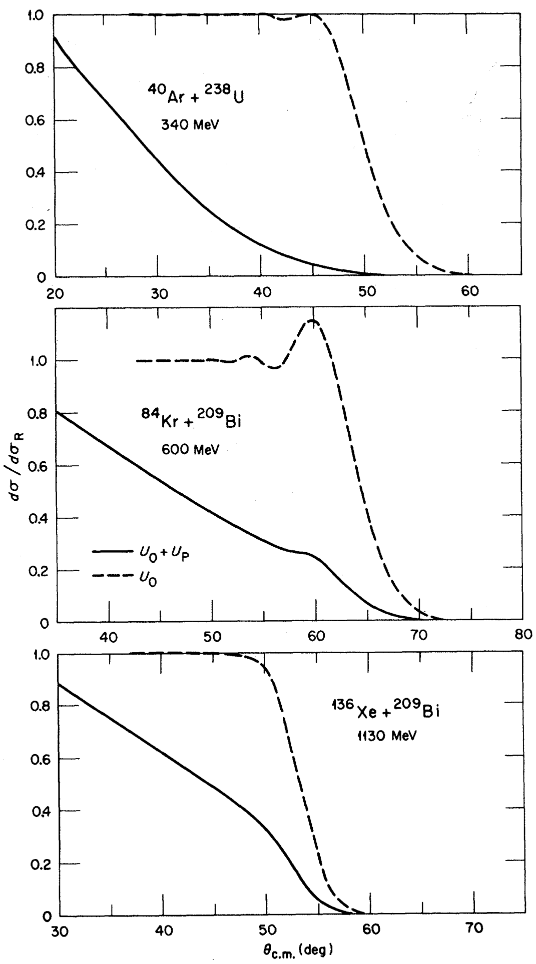

FIG. 2. Ilustrative calculations of the of $E2$ Coulomb excitation on the elastic scattering of very heavy ions. The polarization potential U_p was used with strengths $W_p = 1100$ GeV fm^5 (^{40}Ar + ^{238}U) and 500 GeV fm^5 (^{84}Kr + ^{209}Bi and ^{136}Xe + ^{209}Bi).

fm. Also shown is the scattering induced by the Woods-Saxon potential U_0 alone. The hope is that a U_0 obtained in this way is then a good starting point for a potential to be used in a full coupled-channel analysis of the elastic and inelastic data. This program has not been completed, but preliminary results[3] are encouraging. Alternatively, $U_0 + U_p$ may be used to represent some of the effects of Coulomb excitation in distorted-wave Born-approximation calculations of transfer reactions.

The effects of Coulomb excitation on the elastic scattering may be seen stronger for very-heavy-ion collisions. To indicate this, we present results using our polarization potential for three systems that have been studied experimentally.[6] The measurements were unable to resolve inelastic scattering to the low excited states, so we cannot compare directly with these data. However, for illustrative purposes, we simply took the optical potentials U_0, which have been adjusted[6] to fit these data, and added to them our potential (6) with reasonable estimates of the strengths W_p. The results are shown in Fig. 2. It is seen that the nuclear potential U_0 for ^{40}Ar + ^{238}U has little opportunity to act at all because of the strong damping due to Coulomb excitation.

Franey[2,7] has studied the use of a simple phenomenological imaginary potential of the form iW_p/R^n to reproduce the effects of Coulomb excitation. He found an optimum value of $n \approx 6.3$. Although our potential has a basic dependence of $1/R^5$, the effect of the local energy correction (3) is to steepen it so that in the important region of R it is very close to $1/R^{6.3}$. Of course, our strength W_p is determined by Eq. (7) so that our potential has predictive power, whereas the phenomenological one has to be adjusted to fit the data.

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