

<sup>3</sup>In order to solve this equation by a Fourier analysis of both sides, it is required that the right-hand side not contain any contributions which are a solution to the homogeneous equation. This appears to be insured here since  $V(0,0) = 0$ . In the calculations of D. Bedeaux and P. Mazur, *Physica (Utrecht)* **73**, 431 (1974), this general requirement was not satisfied.

<sup>4</sup>This has the form of a model spectrum introduced in Ref. 1. Such a spectrum was obtained in linear response theory by adding a random force to the right-hand side of Eq. (1) and dropping  $\nabla \cdot \tilde{q}$ . Aside from acknowledged problems regarding the neglect of the corresponding dissipative flux that must characterize

such an open system, the strength of this local random force turned out to be geometry dependent.

<sup>5</sup>J. Clarke and T. Y. Hsiang, *Phys. Rev. B* **13**, 4790 (1976).

<sup>6</sup>See, for example, S. Eckstein, Y. Eckstein, J. Ketterson, and J. Vignos, in *Physical Acoustics*, edited by W. Mason and R. Thurston (Academic, New York, 1970), Vol. VI, p. 244.

<sup>7</sup>This result is implied by the calculations of P. J. Westervelt, *J. Am. Stat. Assoc.* **59**, 760 (1976). A direct derivation from the nonlinear hydrodynamics with fluctuations will be presented by the author at a later date.

## Macroscopic Parity Nonconservation Due to Neutral Currents?

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The parity-nonconserving electron-nucleon interaction induced by neutral currents should give rise to a (thermodynamically stable) electric-dipole-moment density directed along the characteristic rotation axis  $\hat{\omega}$  of superfluid  $^3\text{He-B}$ ; if observed, this would constitute the first example of parity nonconservation on a *macroscopic* scale. Although the magnitude of the effect depends critically on a poorly known chemical parameter, it quite probably lies within the experimentally detectable range.

The neutral currents postulated by modern theories of the weak interaction should induce an electron-nucleon interaction<sup>1,2</sup> which changes sign under spatial inversion ( $P$ ) but, in the simplest case, is invariant under time reversal ( $T$ ). Intensive searches for the effects of such an interaction in atomic physics have so far failed to produce a positive result.<sup>3</sup> In this Letter, I shall show that if this interaction indeed exists, then in the highly ordered condensed system of superfluid  $^3\text{He-B}$ <sup>4,5</sup> it should produce a *thermodynamically stable* effect which is *macroscopic*, that is, which increases linearly with the volume of the system.<sup>6</sup> Specifically, it will produce a permanent electric-dipole-moment density (a vector quantity) directed along the characteristic rotation axis<sup>5</sup>  $\hat{\omega}$  (an axial vector). Whether or not the magnitude of the effect is such that it could reasonably be looked for with present experimental techniques depends, as we shall see, rather crucially on a chemical parameter of the  $^3\text{He}$  dimer which is extremely difficult to estimate reliably; however, a very preliminary estimate makes the prospects quite hopeful.

The qualitative argument for the effect runs as follows. If a system (elementary particle, atom, or molecule) is characterized, in a stationary

state, by a single angular momentum  $\vec{J}$ , it is well known that  $T$  invariance forbids it to have an electric dipole moment  $\vec{d}$ , even if  $P$  is not conserved. If, however, the system is for some reason characterized by *two* independently conserved angular momenta, say an orbital angular momentum  $\vec{L}$  and a spin  $\vec{S}$ , then one can form the expression  $\vec{d} = c\vec{L} \times \vec{S}$  which, though  $P$  nonconserving, preserves  $T$  invariance. Now, in an ordinary atom or molecule  $\vec{L}$  and  $\vec{S}$  are of course not separately conserved but precess around their resultant  $\vec{J}$ ; moreover, even if by some means we could switch off the (hyper) fine interactions which cause this, it would still be impossible to detect a weak-interaction-induced permanent dipole moment of the above form, since it is so small that even in the largest electric fields its orientation energy is tiny compared to the thermal energy  $k_B T$ , and as a result the net moment of a gas of such particles would be zero. However, the Cooper pairs in superfluid  $^3\text{He}$ , which for present purposes are essentially giant diatomic molecules, have the unique property of spontaneously broken spin-orbit symmetry,<sup>7</sup> meaning that the relative orientation of spin and orbital coordinates is the same for all pairs, and remains constant in time. As a result, any elec-

tric dipole moment produced as above is a permanent one and the same for all pairs, and the orientation energy in an external field is proportional to  $N$  (cf. the ferromagnetic analogy discussed in Ref. 7, Sect. 3). In particular, it is easy to see that in the Balian-Werthamer (BW) phase conventionally identified with  ${}^3\text{He-B}$ , which is obtained<sup>5</sup> by starting with a  ${}^3P_0$  state ( $\vec{L} \equiv -\vec{S}$ ) and rotating the spin coordinates relative to the orbital ones<sup>9</sup> through an angle  $\theta$  around the axis  $\hat{\omega}$ , the average value of  $\vec{L} \times \vec{S}$  for a pair is just  $-\frac{4}{3}\sin\theta \hat{\omega}$ . Thus we may have a permanent electric-dipole-moment density along  $\hat{\omega}$ .

To calculate the magnitude of the effect we proceed as follows. In the case of the He atom, which has zero spin in its ground state, we need keep only those terms in the parity-nonconserving electron-nucleon interaction<sup>1,2</sup> which do not involve the electron spin. Moreover, for the  ${}^3\text{He}$  nucleus the total spin of the protons is zero to a good approximation. Hence for the present purposes the effective parity-nonconserving interaction for a system of  ${}^3\text{He}$  atoms may be written

$$H_{\text{pnc}} = -\frac{G_{\text{F}}C_{\text{An}}}{\sqrt{2}} \sum_{i,n} \vec{S}_n \cdot \left( \frac{\vec{p}_i}{m} - \frac{\vec{P}_n}{M} \right) \delta(\vec{r}_i - \vec{R}_n) + \text{H.c.} \quad (1)$$

Here,  $\vec{r}_i$ ,  $\vec{p}_i$ , and  $m$  are, respectively, the coordinate, momentum, and mass of the  $i$ th electron; and  $\vec{R}_n$ ,  $\vec{P}_n$ , and  $M$ , those of the  $n$ th nucleus.  $\vec{S}_n$  is the nuclear spin of the  $n$ th nucleus.  $G_{\text{F}}$  the conventional weak-interaction strength, and  $C_{\text{An}}$  the dimensionless axial-vector-coupling constant to the neutron.

Consider a single homonuclear diatomic molecule or dimer. We define its *electrohelicity*  $\epsilon(R)$  as follows. Imagine that the system is rotating with some total angular momentum  $\vec{K}$  but with the internuclear distance  $R$  held fixed for the moment. We then apply a weak perturbation of the form

$$V' = \beta \hat{n} \cdot \sum_{i,n} \left\{ \left( \frac{\vec{p}_i}{m} - \frac{\vec{P}_n}{M} \right) \delta(\vec{r}_i - \vec{R}_n) + \text{H.c.} \right\} \equiv \beta \hat{n} \cdot \vec{V}, \quad (2)$$

where  $\hat{n}$  is a unit vector (which may have arbitrary behavior under space reflection and time reversal). The perturbation on the rotating state will in general induce an electric dipole moment  $\langle \vec{d} \rangle$  in the system, and since the unperturbed Hamiltonian  $H_0$  is  $P$  and  $T$  invariant this can only be proportional to  $\vec{K} \times \hat{n}$ . We then define  $\epsilon(R)$  by

the equation

$$\langle \vec{d} \rangle(R) = \epsilon(R) \beta \vec{K} \times \hat{n}. \quad (3)$$

The perturbation (1) can be expressed in the form (2) if we write  $\vec{S}_1 \equiv \frac{1}{2}\vec{S} + (\vec{S}_1 - \vec{S}_2)$ , etc., where  $\vec{S} \equiv \vec{S}_1 + \vec{S}_2$  is the total nuclear spin of the molecule, and if we assume that the terms in  $\vec{S}_1 - \vec{S}_2$  make no contribution. (This is almost certainly right for the Cooper pairs in  ${}^3\text{He}$ , since for a triplet state  $\vec{S}_1 - \vec{S}_2$  is zero identically.) We then find that the electric dipole moment induced by neutral currents in a single dimer is

$$\langle \vec{d} \rangle(R) = -(G_{\text{F}}/2\sqrt{2}) \epsilon(R) \vec{K} \times \vec{S}. \quad (4)$$

I will now *assume* that for a system in which Cooper pairs form, the correct total electric dipole moment is obtained by summing expression (4) over all the pairs and averaging over their radial separation, i.e.,

$$\langle \vec{d} \rangle = -\frac{G_{\text{F}}C_{\text{An}}}{2\sqrt{2}} \langle \frac{1}{2} \sum_{i,j} \epsilon(R_{ij}) (\vec{L}_{ij} \times \vec{S}_{ij}) \rangle, \quad (5)$$

where  $i$  and  $j$  now denote atoms (not electrons) and the notation for the total orbital angular momentum of the pair  $(i, j)$  is changed from  $\vec{K}$  to  $\vec{L}_{ij}$  to conform with the convention in low-temperature physics.<sup>5,7</sup> If the relative wave function of the pairs is known, the two-particle expectation value in (5) may be calculated by a straightforward generalization of the standard techniques (Ref. 5, Sects. VI.D and VII.D). For the BW phase ( ${}^3\text{He-B}$ ) let us introduce the total probability density  $|F(R)|^2$  to find the pair a distance  $R$  apart; this is identical to the quantity  $|\vec{F}(\vec{r})|^2$  defined in Ref. 5 [cf. Eq. (7.48)], and in second-quantized language is  $\frac{1}{2} \sum_{\alpha\beta} |\langle \psi_{\alpha}^{+}(0) \psi_{\beta}^{+}(\vec{R}) \rangle|^2$ . In the BW phase it is isotropic. In terms of this quantity the electric dipole moment per unit volume,  $\vec{D}$ , is given by

$$\vec{D} = + (G_{\text{F}}C_{\text{An}} \sqrt{2} 3(\sin\theta) Q \hat{\omega}; \quad (6)$$

$$Q = \int d^3R \epsilon(R) |F(R)|^2.$$

In general  $F$  and hence  $Q$  are pressure and temperature dependent; in what follows I shall consider the limit of low temperatures and of pressures of the order of the melting pressure.

Both the factors occurring in  $Q$  are difficult to estimate reliably.<sup>9</sup> The electrohelicity  $\epsilon(R)$  may be written formally as a static response function,

$$\epsilon(R) = \frac{1}{2M_{\text{K}}} \sum_n \frac{\langle 0 | d_x | n \rangle \langle n | V_y | 0 \rangle}{E_0 - E_n} - (x \leftrightarrow y) + \text{c.c.}, \quad (7)$$

where 0 denotes the state of the molecule (dimer) in question, which for definiteness is taken to have a well-defined projection  $M_K$  of  $\vec{K}$  on the  $z$  axis. Here  $\vec{V}$  is defined in Eq. (2) above and  $\vec{d}$  is the electric-dipole-moment operator of the molecule. For an electronic  $\Pi$  state (or, of course, for an atom in a  $p$  state)  $\epsilon(R)$  in expression (7) is of the order of magnitude  $e/\hbar a_0$ . For a  $^1\Sigma$  state such as the ground state of the  $^3\text{He}$  dimer, how-

$$\epsilon(R) = -(2\hbar^2/MR^2) \langle \{ L_\xi H_0^{-1} d_\eta H_0^{-1} V_\zeta + d_\eta H_0^{-1} V_\xi H_0^{-1} L_\xi + \frac{1}{2} (L_\xi d_\eta H_0^{-2} V_\zeta + d_\eta H_0^{-2} V_\xi L_\xi) - \frac{1}{2} d_\eta [[L_\xi, H_0^{-1}], H_0^{-1}] V_\xi \} \rangle - (\eta \leftrightarrow \xi), \quad (8)$$

where the energy  $H_0$  is measured from the ground state (which is excluded as a possible intermediate state) and  $\vec{L}$  is the electronic angular momentum operator. To calculate the expectation value in (8), even for the simplest models, appears to be a somewhat nontrivial exercise in chemical physics; however, it is evident after a little inspection that it vanishes for two separated neutral atoms with no *net* polarizations (or angular momenta) irrespective of correlation effects. In other words, it is a true "induction" property and might be expected to fall off exponentially with distance. It is convenient therefore to parameterize  $\epsilon(R)$  for the  $^3\text{He}$  dimer in the approximate form

$$\epsilon(R) = (e/\hbar a_0)(m/M)\epsilon_0 \exp[-\lambda(R - R_0)/a_0] \quad (R > R_0), \quad (9)$$

where  $a_0$  is the Bohr radius and  $\epsilon_0$  the reduced electrohelicity at the hard-core radius  $R_0$  ( $\sim 4.7 \times a_0$ ). We may try to obtain a *very tentative* order of magnitude for  $\epsilon_0$  (and  $\lambda$ ) as follows: We replace the energy denominators in the first four terms of (8) by a typical excitation energy, say 1 hartree, and for consistency neglect the commutator term. We then take the  $^1\Sigma_g^+$  electronic wave function to be an antisymmetrized product of molecular orbitals  $(1\sigma_g)^2(1\sigma_u)^2$  as done, e.g., by Kestner,<sup>12</sup> and evaluate the expectation values in (8), neglecting any exchange terms and also a spurious term proportional to  $R^2$  which is introduced by the approximation. In this way we obtain

$$\epsilon(R) = -12 \left( \frac{m}{M} \right) \left( \frac{ea_0^4}{\hbar R} \right) \sum_{i=u,g} \left[ \Psi_i \frac{\partial \Psi_i}{\partial \xi} \right]_{\vec{R}/2}, \quad (10)$$

where the sum is over the  $1\sigma_g$  and  $1\sigma_u$  molecular orbitals evaluated at the position of one nucleus [ $\vec{R}/2 \equiv (0, 0, R/2)$ ]. We now insert in (10) the composition of the molecular orbitals as computed

ever, a finite value of  $\epsilon$  is obtained only<sup>10</sup> by virtue of the coupling of the electronic motion to the molecular rotation. Treating the nuclear kinetic energy as a perturbation in the usual way,<sup>11</sup> one obtains after some calculation<sup>9</sup> the following expression, valid to order  $m/M$ , for  $\epsilon(R)$  in terms of the electronic matrix elements with respect to the coordinate system  $(\xi, \eta, \zeta)$  fixed in the molecule (with  $\zeta$  the internuclear axis):

numerically in, e.g., Ref. 12: For the distances of interest the dominant contribution to  $\partial \Psi_i / \partial \xi$  comes from the admixture of the  $2p$  atomic orbitals and we can safely confine ourselves to this. The four data points available from Ref. 12 do not actually fit formula (9) very well, but I believe that a reasonable set of values for the purpose of calculating the order of magnitude of  $Q$  is  $\epsilon_0 \cong 3 \times 10^{-4}$ ,  $\lambda \sim 0.3-0.5$ .

The other unknown in Eq. (6) is the Cooper-pair wave function  $F(R)$  for distances of the order of  $R_0$ . It would clearly be quite unrealistic to insert here the simple BCS-like form commonly used in the literature,<sup>5</sup> since this completely neglects the hard-core repulsion. I shall instead use the phenomenological *Ansatz*<sup>13</sup>

$$F(R) = A \sin[k(R - R_0)]/R, \quad (11)$$

where  $k$  is of the order of the Fermi wave vector  $k_F$ , and fit the constant  $A$  by inserting Eq. (11) in the expression for the (experimentally known) dipole energy [ $g_D = \frac{1}{2} \gamma^2 \hbar^2 \int R^{-3} |F(R)|^2 d^3R$ : cf. Ref. 5, Eqs. (10.5-10.8)]. This gives  $Q = f(e/\hbar a_0)(m/M)\epsilon_0 a_0^3 (g_D/\gamma^2 \hbar^2)$ , where  $f$  is a numerical constant which for  $\lambda \sim 0.3-0.5$  and  $k \geq k_F$  is practically independent of  $k$  and equal to about  $(250/3)\lambda^{-1}$ .

To put the final result in an experimentally useful form, it is convenient to define a "unit" ( $U$ ) of spontaneous electric-dipole-moment density as that amount which, for a volume of  $1 \text{ cm}^3$  acted on by a field of  $1 \text{ V/cm}$ , gives an orientation energy of  $10^{-7} \text{ eV}$  (which is roughly the thermal energy  $k_B T$  at  $1 \text{ mK}$ ). Substituting the above expression for  $Q$  in Eq. (6) and putting<sup>5</sup>  $\sin \theta = (\frac{15}{16})^{1/2}$ ,  $g_D \sim 10^{-3} \text{ ergs/cm}^3$ , and<sup>2</sup>  $G_F/\hbar a_0^2 \cong 1.6 \times 10^{-16}$ ,  $C_{An} \sim 1$ , finally yields  $\vec{D} = D_0 \hat{\omega}$  with  $D_0 \sim 2.6 \times 10^{-4} (f\epsilon_0) U$ ; if the above estimates for  $\epsilon_0$  and  $f$  are used, this gives  $D_0 \sim 2 \times 10^{-5} U$ . However, it should be emphasized that the above estimate for  $f\epsilon_0$  could easily be in error by as much as an order of

magnitude in either direction. Clearly, a much better calculation of the electrohelicity is urgently needed, as is a better estimate of  $F(R)$  for  $R \sim R_0$ .<sup>14</sup>

The predicted parity-nonconserving spontaneous dipole-moment density is at first sight laughably small ( $\sim 10^{-12} e \cdot \text{cm}^{-2}$ ). However, it will provide a *linear* coupling  $-D_0 \hat{\omega} \cdot \vec{E}$  between an electric field  $\vec{E}$  and the vector  $\hat{\omega}$  which characterizes the macroscopic orientation of  ${}^3\text{He-B}$ , and this should be in principle detectable. Two obvious possibilities are (a) to observe the equilibrium dependence of  $\hat{\omega}$  on the *polarity* of a strong electric field in a situation where the much stronger parity-conserving orientation effects<sup>15</sup> are largely balanced off against one another, or (b) a resonance experiment, for instance using crossed rf electric and magnetic fields.<sup>9</sup> Preliminary estimates indicate that if  $D_0$  has the above order of magnitude, such experiments may just lie within the reach of currently available techniques. If successful, they not only might provide the first firm experimental evidence for electron-hadron coupling due to neutral currents, but would constitute the first ever indisputable manifestation of macroscopic parity nonconservation due to the weak interaction.

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<sup>1</sup>A. N. Moskalev, R. M. Ryndin, and I. B. Khriplovich,

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<sup>2</sup>C. Bouchiat, J. Phys. G **3**, 183 (1977).

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<sup>4</sup>J. C. Wheatley, Rev. Mod. Phys. **47**, 415 (1975) (experimental review).

<sup>5</sup>A. J. Leggett, Rev. Mod. Phys. **47**, 331 (1975) (theoretical review).

<sup>6</sup>A macroscopic (but thermodynamically unstable) parity-nonconserving effect in superconductors has been predicted by A. I. Vainshtein and I. B. Khriplovich, Zh. Eksp. Teor. Fiz. **68**, 3 (1975) [Sov. Phys. JETP **41**, 1 (1975)]. (Cf. also Ref. 1.)

<sup>7</sup>A. J. Leggett, J. Phys. C **6**, 3187 (1973).

<sup>8</sup>We make  $\theta$  positive by convention; i.e., the rotation is always clockwise in a right-handed coordinate system.

<sup>9</sup>I hope to discuss the details of this work, and in particular the calculation of the electrohelicity, at more leisure elsewhere.

<sup>10</sup>It is easy to convince oneself from symmetry considerations that the explicit occurrence of the nuclear coordinates in  $\vec{d}$  and of their momenta in  $\vec{V}$  does not contribute to  $\epsilon$  to order  $m/M$ .

<sup>11</sup>Some caution is necessary here: It is essential to keep not only the "Λ-doubling" term proportional to  $-\vec{K} \cdot \vec{L}$  but also the rotational-energy term proportional to  $K^2$ ; one finds that the leading contributions from these two terms cancel to give Eq. (8).

<sup>12</sup>N. R. Kestner, J. Chem. Phys. **48**, 252 (1968).

<sup>13</sup>The use of a radial function appropriate to *s*-wave rather than *p*-wave symmetry seems unlikely to introduce substantial error, since  $kR_0 \gtrsim 2.3$ .

<sup>14</sup>For various reasons which there is no space to discuss here, I believe that the above estimate of  $F$  and hence of  $D_0$  is very likely to err on the pessimistic side.

<sup>15</sup>Theoretically predicted orientation effects include a quadratic (parity-conserving) electric-field effect [see, e.g., D. A. Dahl, J. Low Temp. Phys. **27**, 139 (1977)]. An experimental determination of the existence, magnitude, and *sign* of this effect seems a prerequisite for the experiment envisaged.