J. M. Nester, Rev. Mod. Phys. 48, 393 (1976).

[10] The superscript f should indicate that besides the vector fields $B_{\mu j}$ and $\tilde{B}_{\mu j}$ henceforth only a Christoffel-type connection $\Gamma_{\mu abj}$ derived from $f^{\mu\nu}$ contributes to the minimal coupling in (11).

[11] W. Heisenberg, *Introduction to the Unified Field Theory of Elementary Particles* (Wiley, London, 1966).

[12] W. Heisenberg, Naturwissenschaften 61,1 (1974).

[13] Bjorken and Drell, Ref. 6.

[14] V. E. Zakharov and A. B. Shabat, Zh. Eksp. Teor. Fiz. 61, 118 (1972) [Sov. Phys. JETP 34, 62 (1972)].

[15] A. C. Newell and L. G. Redekopp, Phys. Rev. Lett. 38, 377 (1977); H. C. Morris, J. Math. Phys. (N.Y.) 18, 285, 533 (1977).

[16] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[17] A. Salam and J. Strathdee, Phys. Lett. 61B, 375 (1976).

[18] A. Salam and J. Strathdee, International Center for Theoretical Physics, Trieste, Report No. IC/77/30, and "A Steeply-Rising Potential in Tensor Gauge Theory" (to be published).

[19] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).

[20] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964).

[21] S. Okubo, Phys. Rev. D 11, 3261 (1975).

[22] It should be noted, however, that the Dirac spinors of Ref. 3 represent leptons, which do not directly interact with hadrons.

[23] A. Salam and J. Strathdee, Phys. Rev. D 14, 2830 (1976).

[24] S. Weinberg, Rev. Mod. Phys. 46, 255 (1974), and Phys. Rev. D 13, 974 (1976).

[25] A. Salam and J. Strathdee, Phys. Lett. 66B, 143 (1977).

[26] R. Hagedorn, Nucl. Phys. B24, 93 (1970).

[27] S. W. Hawking, Phys. Rev. D 13, 191 (1976).

[28] E. E. Fairchild, Jr., "A Yang-Mills Formulation of Gravitational Dynamics", Phys. Rev. D (to be published).

[29] J. A. Wheeler, *Einstein's Vision* (Springer, Berlin, 1968); contained in part in *Battelle Recontres: 1967 Lectures in Mathematics and Physics*, edited by C. DeWitt and J. A. Wheeler (Benjamin, New York, 1968).

[30] C. J. Isham, Proc. Roy. Soc. London, Ser. A 351, 209 (1976).

[31] E. W. Mielke, "Knot Wormholes in Geometrodynamics?" (to be published).

[32] J. A. Wheeler, in *Proceedings of the Sixteenth Solvay Conference* (Editions de l'Université de Bruxelles, Brussels, Belgium, 1974), p. 279.

Coexistence of Three Nuclear Shapes in ^{151}Eu

Gordon L. Struble, Ivan C. Oelrich,[a] Joseph B. Carlson, Lloyd G. Mann, and Robert G. Lanier
University of California, Lawrence Livermore Laboratory, Nuclear Chemistry Division,
Livermore, California 94550

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Levels in ^{151}Eu were populated using the (d,t) reaction. Most of the cross section is concentrated between an excitation energy of 2.2 and 2.6 MeV while levels below 261 keV were not observed. We interpret this as evidence for the coexistence of three distinct shapes in ^{151}Eu.

The idea that a nucleus can exist in eigenstates with two different shapes is not new. The phenomenon was first seen[1] in ^{16}O and later was observed in many light nuclides.[2] The effect is attributed to the $T=0$ component of the nucleon-nucleon interaction, which favors a large spatial overlap of the single-particle orbitals. Thus, two highly symmetrical SU(3) configurations that possess very different shapes are often close in total binding energy.

The situation in heavy nuclei is more obscure. The transitions from spherical to deformed nuclei can be calculated quantitatively in Hartree-Fock-type models, and regions of stability can be qualitatively predicted from a knowledge of the single-particle shell structure as a function of deformation.[3] Thus, the filling of a strongly down-sloping (prolate) or the emptying of a strongly up-sloping (oblate) Nilsson orbital will drive the nucleus toward a larger deformation.

Experimental evidence for shape coexistence in the rare-earth nuclei is not as conclusive as in s-d-shell nuclei. A number of authors[4,5] have speculated about coexistence in the even-even nuclei around $N=90$, because the (p,t) and (t,p) $l=0$ strength is fragmented over several 0^+ states in a manner consistent with shape coexistence. Similar studies in odd-A nuclei also were interpreted as evidence for shape coexistence.[6,7] Using γ-ray spectroscopy, Kleinheinz *et al.*[8] reported coexistence in ^{151}Gd. They observed a rotational band with very small deformation based on the $i_{13/2}$ orbital and a strongly deformed band built on the $h_{11/2}$ orbital.

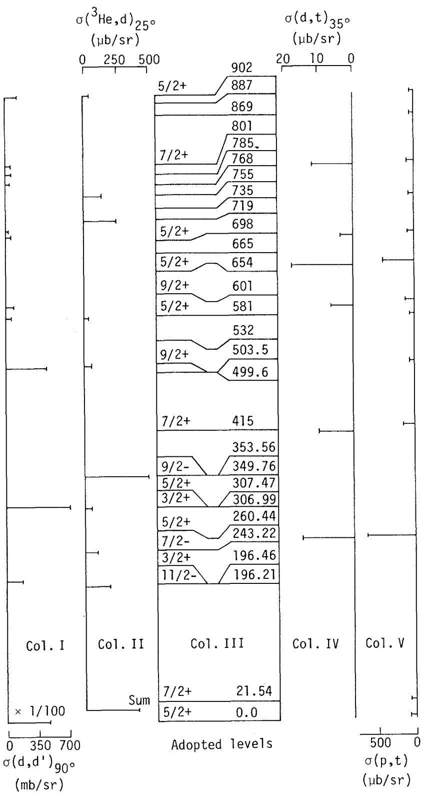

FIG. 1. Experimental data below 1 MeV for ^{151}Eu. The data displayed are the following: inelastic deuteron scattering (column I), the (^{3}He,d) reaction (Refs. 9 and 10) (column II), adopted levels from Nuclear Data Sheets (Ref. 13) (column III), our experimental data at 35° (column IV), and the (p,t) results of Ref. 7 (column V).

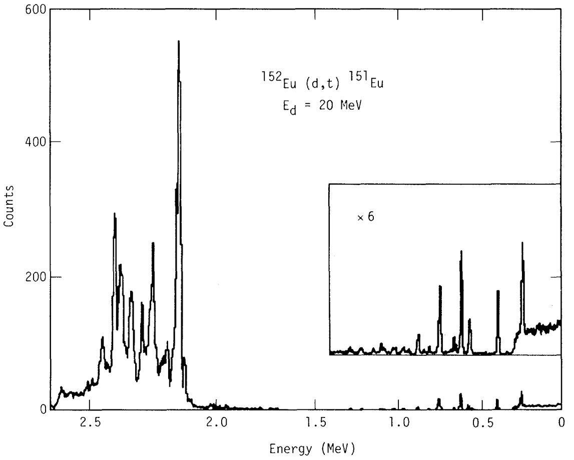

FIG. 2. Experimental spectrum of ^{151}Eu between 0 and 3 MeV at 35°. The inset magnifies the region between 0 and 1.5 MeV. The first peak is due to the 261-keV level, which is shown riding on the background of a light impurity.

Shape coexistence has been postulated[6,7] for ^{151}Eu in particular. The experimental evidence relating to the ground state is consistent with its description as a spherical $d_{5/2}$ quasiparticle. The $g_{7/2}$ and $h_{11/2}$ quasiparticle states also were strongly populated in the (^{3}He,d) reaction.[9,10] Four of the five quadrupole-phonon states based on the $d_{5/2}$ ground state were seen in Coulomb excitation[11] and inelastic scattering.[12] However, in (p, t) reaction studies,[6,7] not only were the spherical states weakly populated, but a previously unobserved set of deformed states, starting with a $\frac{5}{2}^+$ level at 261 keV, was strongly populated. The authors postulated that this level is the band head of the $\frac{5}{2}$[413] intrinsic Nilsson state.[3] They also identified the $\frac{7}{2}^+$ member of this band and the $\frac{5}{2}^+$ and $\frac{7}{2}^+$ members of a β-vibrational band built on this orbital. All of these experimental data, presented in Fig. 1, are consistent with the notion that inelastic scattering and the (^{3}He,d) reaction act selectively to populate spherical states while the (p,t) reaction acts to preferentially populate deformed states.

In recent in-beam γ-ray studies,[14] evidence was found not only for this deformed band, but for a decoupled band built on the $h_{11/2}$ orbital. To explain these results, Dracoulis and Leigh[15] performed a Coriolis-mixing calculation and attempted to describe all of the structure of ^{151}Eu in terms of one small deformation ($\epsilon = 0.145$). By introducing arbitrary shifts in the positions of the single-particle orbitals, they obtained a consistent description of all the experimental data. Thus, the question of coexistence in ^{151}Eu is in doubt.

In the present experiment, we studied the levels in ^{151}Eu using a (d,t) reaction. We used a 20-MeV deuteron beam from the three-stage tandem Van de Graaff facility at the Los Alamos Scientific Laboratory and analyzed the tritons with a quadrupole–triple-dipole spectrometer and a 1-m-long, helical proportional detector.[16] The measurements were taken in 10° steps between 15° and 65°. The ^{152}Eu target was made by irradiating isotopically separated ^{151}Eu with thermal neutrons, and then by chemically purifying, separating, and collecting the ^{152}Eu on a thin carbon foil. The fabrication of this target will be discussed in more detail in another publication.[17]

For the 35° run, the experimental spectrum between 0 and 3 MeV is shown in Fig. 2. The striking feature is the concentration of cross section between 2.2 and 2.6 MeV. Because the ground state of ^{152}Eu is very deformed,[18] it is easy to postulate that these states are closely related to the ^{152}Eu ground state and possess a large deformation.

As shown in Fig. 1, no state below 261 keV was populated in our experiment, and only upper limits could be assigned for those cross sections. The cross section for the ground state is less than 30 nb at 25°. All the deformed states observed in the (p,t) reaction were populated, although the so-called β-vibrational state at 654 keV has a larger cross section than the $\frac{5}{2}[413]$ band head.

We have calculated cross sections for ^{151}Eu by assuming the same deformation as in ^{152}Eu ($\epsilon = 0.26$) and, except for the $\frac{1}{2}[400]$ and $\frac{3}{2}[402]$ orbitals, by using the theoretical energies and pairing parameters of Immele and Struble.[19] These $N=4$ orbitals appear much lower in the experimental spectrum[20] of ^{152}Eu than would be predicted on the basis of the single-particle energies tabulated in Ref. 19. The distorted-wave Born-approximation (DWBA) code DWUCK,[21] along with optical-model parameters from Oelert *et al.*,[22] was used to evaluate the single-particle cross sections. Assuming pure deformed configurations, we calculated the spectroscopic factors for each state of the various bands and used these along with the DWBA results to obtain the predicted cross sections. To account for configuration mixing, we distributed the cross section symmetrically about the position of a state using an energy-dependent Lorentz factor.[23]

A theoretical spectrum for 35° is shown in Fig. 3. The experimental and theoretical spectra are similar except that in the theoretical spectrum all of the low-energy strength is in the $\frac{5}{2}[413]$ band. Above 2 MeV, greater than 90% of the strength is due to removal of a neutron from $N=4$ orbitals.

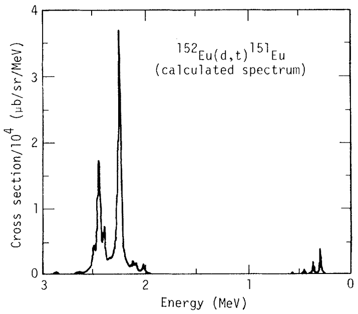

FIG. 3. Theoretical calculation for the (d,t) spectrum at 35° (see text).

We compare experimental and model data for angular distributions in Fig. 4. The agreement for the $N=4$ states is good. However, the experimental distributions for the deformed states below 1 MeV peak at 35° instead of the predicted 25°.

This experiment and our calculation suggest that there are three distinct deformations in ^{151}Eu. States related to the ground state are spherical. States related to the $\frac{5}{2}^+$ state at 261 keV are modestly deformed, and states between 2.2 and 2.6 MeV are strongly deformed and are essentially one-neutron holes coupled to the ground state of ^{152}Eu. The properties of the states above 2 MeV agree well with those predicted by the calculation. These states are expected to be deformed since both the $\frac{11}{2}[505]$ and the $\frac{1}{2}[400]$ or $\frac{3}{2}[402]$ neutron orbitals are strongly up-sloping and are only half full in such three-quasiparticle states. The fractionation of (d,t) strength among low-energy, $K=\frac{5}{2}$ bands and the poor fit of the angular distributions are consistent with a dramatic change in the core between ^{152}Eu and the low-energy states in ^{151}Eu. Indeed, the β vibrations may have a better overlap with the strongly deformed ^{152}Eu while the angular distribution of the $\frac{5}{2}[413]$ state at 261 keV will not be pure $l=5$, because transitions with smaller l are permitted through the core. Additional evidence for the small deformation of the 261-keV state is provided by the mo-

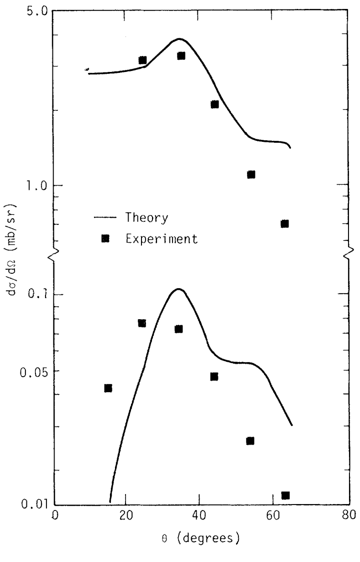

FIG. 4. Triton experimental and theoretical angular distributions. The upper curve shows the variation with angle of the sum of $d\sigma/d\Omega$ for all levels between 2.2 and 2.6 MeV. The lower curve exhibits similar data for levels between 0.261 and 1.0 MeV.

ment of inertia[7] associated with its band. It is half the magnitude of the comparable band in ^{153}Eu.

States between 0 and 261 keV were not observed in our experiments. Dracoulis and Leigh[15] suggest that the levels near the ground state are deformed and that the ground state is the $\frac{5}{2}$ member of a band built on the $\frac{3}{2}[411]$ Nilsson orbital while the 22-keV state is primarily the $\frac{7}{2}$ member of a $\frac{7}{2}[404]$ band. To reproduce the observed level sequence, Dracoulis and Leigh performed a Coriolis-mixing calculation that gave a ground-state wave function that contained a small component (~ 2%) of the $\frac{5}{2}[413]$ orbital but a much larger component (~ 30%) of this orbital in the 22-keV level. If we combine these admixtures with our predicted cross sections for a pure $\frac{5}{2}[413]$ configuration at 25° (32 μb/sr, $\frac{5}{2}^+$; 15 μb/sr, $\frac{7}{2}^+$), we predict ~ 600 nb/sr and ~ 4400 nb/sr for the 0- and 22-keV states, respectively. These predictions are one to two orders of magnitude larger than our conservatively estimated, experimental lower limits and suggest that the absence of (d,t) excitation to the 0- and 22-keV states is due primarily to differences in the nuclear cores. We further note that the moderately deformed $\frac{5}{2}^+$ state at 261 keV has a cross section reduced below the theoretical predictions by only a factor of 3–4. Thus, we speculate that the additional (d,t) hindrance to levels below 261 keV occurs because these states are nearly spherical.

Because deformations of nuclei in this transition region depend crucially on the occupation of the $\frac{11}{2}[505]$ orbital, it would be interesting to determine whether the low-energy neutron-excited states in ^{152}Eu have the same deformation as the ground state. A sensitive probe of this would be a (p,t) reaction on deformed ^{154}Eu. If the ground state of this target occurs as an excited state in ^{152}Eu and if the target configuration does not have the same deformation in the two nuclides, then the very strong (p,t) strength of this configuration should be spread among a number of states.

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[a] Present address: Department of Physics, The Technical University of Munich, Garching, Germany.

[1] G. E. Brown and A. M. Green, Nucl. Phys. 75, 401 (1966), and 85, 87 (1966).

[2] S. N. Tewari and G. L. Struble, Phys. Rev. C 1, 1156 (1970).

[3] S. G. Nilsson, C. F. Tsang, A. Sobiczewski, Z. Szymanski, S. Wycech, C. Gustafson, I. Lamm, P. Möller, and B. Nilsson, Nucl. Phys. A131, 1 (1969).

[4] S. Hinds, J. H. Bjerregaard, O. Hansen, and O. Nathan, Phys. Lett. 14, 48 (1965); J. R. Maxwell, G. M. Reynolds, and N. M. Hintz, Phys. Rev. 151, 1000 (1966).

[5] R. Chapman, W. McLatchie, and J. E. Kitching, Nucl. Phys. A186, 603 (1972).

[6] D. G. Burke, G. Løvhøiden, and J. C. Waddington, Phys. Lett. 43B, 470 (1973).

[7] H. Taketani, H. L. Sharma, and N. M. Hintz, Phys. Rev. C 12, 108 (1975).

[8] P. Kleinheinz, R. K. Sheline, M. R. Maier, R. M. Diamond, and F. S. Stephens, Phys. Rev. Lett. 32, 68 (1974).

[9] O. Straume, G. Løvhøiden, and D. Burke, Nucl. Phys. A266, 390 (1976).

[10] H. L. Sharma, H. Taketani, and N. M. Hintz, Willaims Laboratory of Nuclear Physics, University of Minnesota, Annual Report, 1974 (unpublished).

[11] J. E. Thun and T. R. Miller, Nucl. Phys. A193, 337 (1972).

[12] E. M. Bernstein, G. R. Boss, G. Hardie, and R. E. Shamu, Phys. Lett. 33B, 465 (1970).

[13] B. Harmatz, Nucl. Data Sheets 19, 33 (1976).

[14] J. R. Leigh, G. D. Dracoulis, M. G. Slocombe, and J. O. Newton, Australian National University, Canberra, Report No. ANU-P/655, 1976 (unpublished); H. Taketani, M. Adochi, T. Hattori, and T. Matsuzaki, Phys. Lett. 63B, 154 (1976).

[15] G. D. Dracoulis and J. R. Leigh, J. Phys. G 2, L87 (1976).

[16] S. D. Orbesen, J. D. Sherman, and E. R. Flynn, Los Alamos Scientific Laboratory Report No. LA-6271-MS, 1976 (unpublished).

[17] R. J. Dupzyk, C. M. Henderson, G. L. Struble, R. G. Lanier, W. M. Buckley, and L. G. Mann, to be published.

[18] I. D. Proctor, D. W. Heikkinen, G. L. Struble, and R. G. Lanier, Bull. Am. Phys. Soc. 21, 984 (1976).

[19] J. D. Immele and G. L. Struble, Phys. Rev. C 15, 1103 (1977).

[20] T. von Egidy, private communication.

[21] P. D. Kunz, private communication.

[22] W. Oelert, J. V. Maher, D. A. Sink, and M. Spisak, Phys. Rev. C 12, 1495 (1975).

[23] B. B. Back, J. Bang, S. Bjørnhølm, J. Hattula, P. Kleinheinz, and J. R. Lien, Nucl. Phys. A222, 377 (1974).