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[14]Barger and Phillips (Ref. 12) estimate on the basis of a charm-production model, that the corrected rate for $\sigma(2\mu)/\sigma(1\mu)$ is 0.5×10^{-2}.

Classification of Yang-Mills Fields

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A classification of the Yang-Mills fields is presented using spinor methods. Each class of fields is associated with certain values of five invariants, of which four are complex and one is real. The whole classification is described in terms of two diagrams.

In this Letter I give a classification of the classical Yang-Mills fields along with their invariants using spinor methods.[1] Two diagrams are consequently presented that describe the classification. Not only the problem of classification of gauge fields is of interest per se, but it is of considerable importance in obtaining exact solutions of the Yang-Mills field equations. This fact is well-known in general-relativity theory.

The problem of classifying the Yang-Mills fields has recently been discussed using the method of infinitesimal holonomy group.[2] As has been pointed out[2] the classification obtained in this method, however, is not gauge invariant. Hence the physical meaning of such a classification is not clear since one class of fields can be transferred into another by a guage transformation. My method of classification is invariant under the product of the space-time and gauge groups. This group is taken here as $SL(2,C)\otimes SU(2)$.

The invariants and the classification[3] of the Yang-Mills fields were also discussed using the vector methods.[4] A total of nine real invariants were found that describe a complete set of independent invariants. However, the method proved to be useless for the classification problem. The eigenvalue-eigenvector calculation becomes so cumbersome that computer use was needed without achieving the desired classification. The problem of classification was thus left unsolved. It was pointed out, however, that three types of different fields can be isolated and associated with different values of the invariants. These are those fields for which (1) all invariants are different from each other; (2) all invariants are zero; or (3) the invariants satisfy a certain algebraic relation between themselves. It is well known, however, that when the invariants satisfy a certain relation between themselves, it is not necessary that one obtain only one kind of field. Both the electromagnetic and the gravitational fields are of such nature. For example, when all invariants of the gravitational fields vanish, one obtains three different types of fields rather than just one. In the Yang-Mills case the situation is even more complicated because of the double group structure. I show in the following that one has exactly six independent relations between the invariants (see Figs. 1 and 2) rather than the three relations that were pointed out using the vector method.[4] I also show that associated with these relations between the invariants there are exactly twelve independent and physically different types of Yang-Mills fields (see Figs. 1 and 2), rather than the only three fields found so far.[4] I thus have a complete and maximally detailed classification of the Yang-Mills

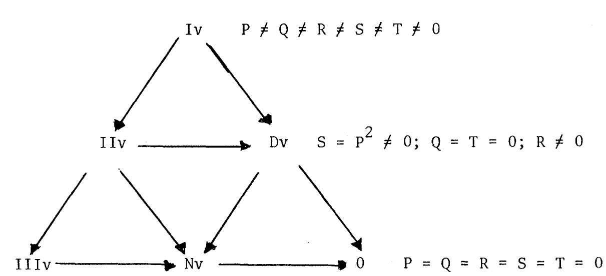

FIG. 1. Isovector diagram of classification. An arrow $A\to B$ indicates that type-B field is obtained from type-A field. The symbols in the diagram are as follows: $\mathrm{Iv}=\chi_{ABk}$; $\mathrm{IIv}=\alpha_{(A}\beta_{B)k}$; $\mathrm{Dv}=\alpha_{(A}\beta_{B)}\gamma_k$; $\mathrm{IIIv}=\alpha_{(A}\alpha_{B)k}$ (where α_{Bk} is defined by $\alpha_{Bk}\alpha_{Ck}=\alpha_B\alpha_C$); $\mathrm{Nv}=\alpha_{(A}\alpha_{B)}\gamma_k$; and 0 is the zero field (included for completeness). If one chooses the vector γ_k in case Dv to be real then in addition to satisfying the conditions indicated in the diagram it satisfies $R=P\bar{P}$.

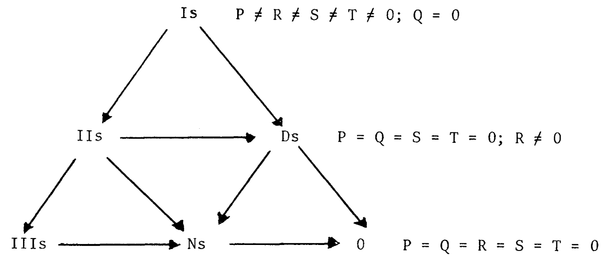

FIG. 2. Isospinor diagram of classification. An arrow $A \to B$ indicates that type-B field is obtained from type-A field. The symbols in the diagram are as follows: Is $= \alpha_{(AM}\beta_{B)N}$; IIs $= \alpha_{(AM}\beta_{B)}\delta_N$; Ds $= \alpha_{(A}\beta_{B)}\gamma_M\delta_N$; IIIs $= \alpha_{(AM}\alpha_{B)}\delta_N$ (where α_{AM} is defined by $\alpha_{AM}\alpha_{BM} = \alpha_A\alpha_B$ and $\alpha_{AM}\bar{\alpha}_{B'M} = \alpha_A\alpha_{B'}$); Ns $= \alpha_{(A}\alpha_{B)}\gamma_M\delta_N$; and 0 is the zero field (included for completeness).

fields along with their diagrams. For completeness, I also give the invariants of the combined Yang-Mills and other fields such as the gravitational field.

The spinor equivalent of a gauge-field strength $F_{\mu\nu k}$ is a complex function χ_{ABk} (for details see Ref. 1). Here the indices A and B are SL(2,C) spinor indices taking the values 0 and 1 whereas $k = 1$, 2, and 3 describe the isospin vector components in the SU(2) space. The spinor χ_{ABk} is symmetric: $\chi_{ABk} = \chi_{BAk}$; hence it has 3×3 complex components. This is equivalent to the eighteen real components of the field strengths $F_{\mu\nu k}$.

The same field can be described as a quantity having two SL(2,C) spinor indices and two SU(2) spinor indices, χ_{ABMN}, where M and N take the values 1 or 2. The quantity χ_{AB} can thus be described as a matrix whose rows and columns are fixed by the indices M and N, i.e., $(\chi_{AB})_{MN}$. The matrix χ_{AB} is then Hermitian and traceless. The relation between χ_{ABMN} and χ_{ABk} is given by

$$\chi_{ABMN} = \chi_{ABk}\sigma_{MN}{}^k/\sqrt{2}, \qquad \chi_{ABk} = \chi_{ABMN}\sigma_{NM}{}^k/\sqrt{2}, \tag{1}$$

or, in matrix notation,

$$\chi_{AB} = \chi_{ABk}\sigma^k/\sqrt{2}, \qquad \chi_{ABk} = \mathrm{Tr}(\chi_{AB}\sigma^k)/\sqrt{2}, \tag{2}$$

where σ^k are the usual Pauli matrices. It should be emphasized that χ_{ABMN} is not symmetric with respect to its indices M and N.

A general field χ_{ABMN} may or may not be decomposed into products of irreducible components of one or both types of spinor indices. Hence one can have fields with an isospin index having the form of a vector, χ_{ABk}, or having the form of products of SU(2) spinors, $\chi_{ABMN} = \alpha_{(AM}\beta_{B)N}$. Here brackets indicate symmetrization, i.e.,

$$\alpha_{(AM}\beta_{B)N} = \tfrac{1}{2}(\alpha_{AM}\beta_{BN} + \alpha_{BM}\beta_{AN}).$$

One can call the first type a *vector-type* field; the second, a *spinor-type* field. Each type can be further decomposed into irreducible products. The field χ_{ABk} can be decomposed into $\alpha_{(A}\beta_{B)k}$ or $\alpha_{(A}\beta_{B)}\gamma_k$. Of course, $\alpha_{(A}\beta_{B)k}$ can be decomposed into $\alpha_{(A}\beta_{B)}\gamma_k$, or $\alpha_{(A}\alpha_{B)k}$ or $\alpha_{(A}\alpha_{B)}\gamma_k$. Here α_{Bk} is defined by $\alpha_{Bk}\alpha_{Ck} = \alpha_B\alpha_C$. All of these field can naturally go over into the zero field (see Fig. 1). A spinor-type field, likewise, can be decomposed into $\alpha_{(AM}\beta_{B)}\delta_N$, $\alpha_{(A}\beta_{B)}\gamma_M\delta_N$, $\alpha_{(AM}\alpha_{B)}\delta_N$, $\alpha_{(A}\beta_{B)}\gamma_M\delta_N$, and the zero field (see Fig. 2). From the spinor χ_{ABk} one can construct one real and four complex invariants. These are given by

$$\begin{aligned} P &= \chi_{AB}\chi_k{}^{AB}, \\ Q &= \chi_{Ai}{}^B\chi_{Bj}{}^C\chi_{Ck}{}^A\epsilon_{ijk}, \\ R &= \chi_{ABk}\chi_l{}^{AB}\bar{\chi}_{C'D'l}\bar{\chi}_k{}^{C'D'}, \\ S &= \chi_{ABk}\chi_l{}^{AB}\chi_{CDl}\chi_k{}^{CD}, \\ T &= \chi_{ABk}\chi_l{}^{AB}\chi_{CDm}\chi_n{}^{CD}\chi_{EFp}\chi_q{}^{EF}\epsilon_{kmp}\epsilon_{lnq}. \end{aligned} \tag{3}$$

Here P, Q, S, and T are complex whereas R is real.

The above relations can also be written in terms of the matrices χ_{AB}. Using the fact that

$$\mathrm{Tr}(\sigma^k\sigma^l) = 2\delta^{kl}, \qquad \mathrm{Tr}(\sigma^i\sigma^j\sigma^k) = 2i\epsilon_{ijk},$$

one obtains

$$\begin{aligned} P &= \mathrm{Tr}(\chi_{AB}\chi^{AB}), \\ Q &= -i2^{1/2}\,\mathrm{Tr}(\chi_A{}^B\chi_B{}^C\chi_C{}^A), \\ R &= \mathrm{Tr}(\chi_{AB}\chi^{\dagger C'D'})\,\mathrm{Tr}(\chi^{AB}\chi_{C'D'}{}^{\dagger}), \\ S &= \mathrm{Tr}(\chi_{AB}\chi^{CD})\,\mathrm{Tr}(\chi^{AB}\chi_{CD}), \\ T &= -2\,\mathrm{Tr}(\chi_{AB}\chi_{CD}\chi_{EF})\,\mathrm{Tr}(\chi^{AB}\chi^{CD}\chi^{EF}). \end{aligned} \tag{4}$$

These five invariants are the only ones that the Yang-Mills fields have. Their calculated values for different fields are listed in Figs. 1 and 2 and in the sequel. The construction of these invariants is completely analogous to the invariants one obtains for the electromagnetic and the gravitational fields. When described in terms of a spinor, the Maxwell field φ_{AB} yields the only complex invariant[3]

$$K = \varphi_{AB}\varphi^{AB} = \tfrac{1}{4}(f_{\mu\nu}f^{\mu\nu} + if_{\mu\nu}{}^*f^{\mu\nu}). \tag{5}$$

Here $f_{\mu\nu}$ is the Maxwell tensor and ${}^*f^{\mu\nu}$ is its dual,

$$*f^{\mu\nu}=\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}f_{\rho\sigma}.$$

The gravitational field yields, on the other hand, the two complex invariants

$$I=\psi_{ABCD}\psi^{ABCD},\quad J=\psi_{CD}{}^{AB}\psi_{EF}{}^{CD}\psi_{AB}{}^{EF}. \tag{6}$$

Here ψ_{ABCD} is the Weyl spinor.

More invariants of the coupled gravitational-electromagnetic and gravitational–Yang-Mills fields can be constructed. In the first case one has

$$\begin{aligned} L&=\varphi_{AB}\psi^{ABCD}\varphi_{CD},\\ M&=\varphi_{AB}\psi_{CD}{}^{AB}\psi_{EF}{}^{CD}\varphi^{EF}. \end{aligned} \tag{7}$$

In the second case one has

$$\begin{aligned} U&=\chi_{ABk}\psi^{ABCD}\chi_{CDk},\\ V&=\chi_{ABk}\psi_{CD}{}^{AB}\psi_{EF}{}^{CD}\chi_k{}^{EF},\ \text{etc.} \end{aligned} \tag{8}$$

Of course, one can also find the invariants of the coupled three fields but I will not go through that here.

The invariants P, Q, R, S, and T can also be expressed in terms of the matrices[4]

$$K_{ij}=F_{\mu\nu i}F_j{}^{\mu\nu},\quad J_{ij}=F_{\mu\nu i}*F_j{}^{\mu\nu}. \tag{9}$$

Here $F_{\mu\nu k}$ is the Yang-Mills field strengths and $*F_k{}^{\mu\nu}$ is its dual,

$$*F^{\mu\nu}=\tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}, \tag{10}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is equal to $+1$ or -1, depending upon whether $\mu\nu\rho\sigma$ is an even or an odd permutation of 0123, and zero otherwise. Then one obtains

$$\begin{aligned} P&=\tfrac{1}{4}(\mathrm{Tr}K+i\,\mathrm{Tr}J),\quad Q=\tfrac{3}{2}(t+it'),\\ R&=\tfrac{1}{16}(\mathrm{Tr}K^2+\mathrm{Tr}J^2),\\ S&=\tfrac{1}{16}(\mathrm{Tr}K^2-\mathrm{Tr}J^2)+\tfrac{1}{8}i\,\mathrm{Tr}(JK),\\ T&=\tfrac{1}{8}(\det K+i\det J), \end{aligned} \tag{11}$$

where

$$t=\tfrac{1}{6}\epsilon_{ijk}F_{\mu i}{}^{\nu}F_{\nu j}{}^{\rho}F_{\rho k}{}^{\mu}, \tag{12a}$$

$$t'=-\tfrac{1}{6}\epsilon_{ijk}*F_{\mu i}{}^{\nu}*F_{\nu j}{}^{\rho}*F_{\rho k}{}^{\mu}. \tag{12b}$$

It has been shown that the invariants $\mathrm{Tr}J$, $\mathrm{Tr}J^2$, $\det J$, $\mathrm{Tr}K$, $\mathrm{Tr}K^2$, $\det K$, $\mathrm{Tr}(JK)$, t, and t' can also be taken as independent invariants.[4] However, it appears that it is more natural to work with the invariants P, Q, R, S, and T. For example the condition $S=P^2$ is equivalent to the conditions $\mathrm{Tr}(JK)=(\mathrm{Tr}J)(\mathrm{Tr}K)$ and $\mathrm{Tr}K^2-(\mathrm{Tr}K)^2=\mathrm{Tr}J^2-(\mathrm{Tr}J)^2$. One also notices that *the number of classes and subclasses of different fields far surpasses that thought to exist before.*[4] In the sequel we find the value of each of these invariants for all possible different fields.

As a simple illustration of the above consider the field of a monopole that has both "electric" and "magnetic" charges.[5] Such a field is given by

$$F_{0jk}=-\frac{e}{g}\frac{x^jx^k}{r^4},\quad F_{ijk}=\frac{1}{g}\epsilon_{ijm}\frac{x^mx^k}{r^4}. \tag{13}$$

The spinor χ_{ABk} is then given by

$$\chi_{ABk}=l_{(A}n_{B)}\gamma_k,\quad \gamma_k=-\frac{e+i}{g}\frac{x^k}{r^3}, \tag{14}$$

where l_A and n_A are two arbitrary $\mathrm{SL}(2,C)$ spinors satisfying $l_An^A=1$. Hence this field is of type Dv (see Fig. 1). Its invariants (see below) are given by

$$\begin{aligned} P&=\frac{(1-e^2)-2ie}{2g^2r^4}=\tfrac{1}{4}(F_{\mu\nu k}F_k{}^{\mu\nu}+iF_{\mu\nu k}*F_k{}^{\mu\nu}),\\ Q&=0,\quad R=P\overline{P},\quad S=P^2,\quad T=0. \end{aligned} \tag{15}$$

In conclusion I list below the values of the invariants P, Q, R, S, and T for each particular case: Class Iv,

$$P\neq Q\neq R\neq S\neq T\neq 0; \tag{16}$$

class IIv,

$$\begin{aligned} P&=-\tfrac{1}{2}A_kA_k,\quad Q=0,\quad R=\tfrac{1}{4}(A_k\overline{A}_k)^2,\\ S&=P^2,\quad T=0, \end{aligned} \tag{17}$$

where $A_k=\alpha_A\beta_k{}^A$; class Dv,

$$\begin{aligned} P&=\tfrac{1}{2}B^2\gamma_k\gamma_k,\quad Q=0,\\ R&=\tfrac{1}{4}(B\overline{B})^2(\gamma_k\overline{\gamma}_k)^2,\quad S=P^2,\quad T=0, \end{aligned} \tag{18}$$

where $B=\alpha_A\beta^A$; if γ_k can be chosen to be real, then one has in this case $R=P\overline{P}$.

For classes IIIv, Nv, and 0 all invariants vanish. Class Is,

$$\begin{aligned} P&=-ab,\quad Q=0,\\ R&=\tfrac{1}{4}\{4a\overline{a}b\overline{b}+2ab\det\overline{C}\\ &\qquad+2\overline{a}\overline{b}\det C+[\mathrm{Tr}(C^t\overline{C})]^2\}, \end{aligned} \tag{19}$$

$$\begin{aligned} S&=\tfrac{1}{4}\{4a^2b^2+4ab\det C+(\mathrm{Tr}C^2)^2\},\\ T&=-\tfrac{1}{4}\{8a^3b^3-6a^2b^2\,\mathrm{Tr}C^2+\tfrac{3}{2}ab(\mathrm{Tr}C^2)^2-\mathrm{Tr}C^6\}. \end{aligned}$$

Here on has $\alpha_{MN}=\alpha_{AM}\alpha_N{}^A$, $\beta_{MN}=\beta_{AM}\beta_N{}^A$, and $C_{MN}=\alpha_{AM}\beta_N{}^A$, with

$$\alpha=a\begin{pmatrix}0&1\\-1&0\end{pmatrix},\quad \beta=b\begin{pmatrix}0&1\\-1&0\end{pmatrix}.$$

Class IIs,

$$P=Q=0, \quad R=\tfrac{1}{4}(\alpha_N\bar{\alpha}_N)^2(\delta_M\bar{\delta}_M)^2, \quad S=T=0, \tag{20}$$

where $\alpha_N=\alpha_{AN}\beta^A$; class Ds,

$$P=Q=0,$$
$$R=\tfrac{1}{4}(\alpha_A\beta^A\bar{\alpha}_{B'}\bar{\beta}^{B'}\gamma_M\bar{\gamma}_M\delta_N\bar{\delta}_N)^2, \quad S=T=0; \tag{21}$$

class IIIs,

$$P=Q=R=S=T=0.$$

For classes Ns and 0 all invariants vanish.

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Observation of a Resonance in e^+e^- Annihilation Just above Charm Threshold

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We observe a resonance in the total cross section for hadron production in e^+e^- annihilation at a mass of 3772 ± 6 MeV/c^2 having a total width of 28 ± 5 MeV/c^2 and a partial width to electron pairs of 370 ± 90 eV/c^2.

Previously, detailed studies of the total cross section for hadron production (σ_T) by e^+e^- annihilation have concentrated on center-of-mass energies ($E_{\text{c.m.}}$) above 3.9 GeV.[1-3] In this Letter we report high-statistics measurements of σ_T between the $\psi(3684)$ ($\equiv \psi'$) and 3.9 GeV. We observe a resonance near 3.77 GeV, just above the threshold for the production of charmed particles.

The data were collected with the Stanford Linear Accelerator Center–Lawrence Berkeley Laboratory magnetic detector at SPEAR.[4-6] In order to maintain consistency with previous measurements, the event-selection criteria and experimental corrections are substantially the same as those used in Ref. 1. Hadronic events are selected as events with two or more detected charged tracks which form a vertex within a cylindrically shaped fiducial volume 22 cm long and 4 cm in radius, centered about the interaction region. If only two oppositely charged particles are detected, they are required to be acoplanar with the incident beams by at least 20°, each to have a momentum greater than 300 MeV/c, and to have at least one particle not identified as an electron. Cosmic rays are rejected by time-of-flight measurements. Backgrounds from beam-gas interactions (~2%) are subtracted using events detected beyond the ends of the fiducial cylinder. A small correction (<1%) is also made for contamination from two-photon processes.[7,8] The luminosity is determined from measurements of large-angle e^+e^- scattering in the magnetic detector.[4,7]

To correct for the efficiency of the apparatus to detect hadronic events (ϵ), we used the same smooth function of energy which was used in Ref. 1. It is based on an unfolding procedure in which the produced-charged-particle multiplicity distribution is deduced from the observed distribution and on Monte Carlo calculations which determine the detector response to each produced multiplicity.[7,9] The use of a smooth function for ϵ is justified only if the observed mean multiplicity