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Drift-Modified Tearing Instabilities Due to Trapped Electrons

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It is shown that the collisional detrapping of magnetically trapped electrons in tokamaks can excite drift tearing modes with high azimuthal mode numbers. For normal temperature gradients ($d\ln T_e/d\ln n > 0$), the modes are unstable in the collisional regime ($\nu_{\rm eff} > \omega_{*e}$), but stable in the collisionless regime ($\nu_{\rm eff} < \omega_{*e}$).

The parameters of present and future generations of tokamaks lie in the trapped-electron regime, in which the trapped-electron bounce frequency exceeds the effective collision frequency. Electrostatic microinstabilities of the drift-wave type (in particular, trapped-electron modes[1]), which are driven unstable by the expansion free energy associated with the density and temperature gradients, have been studied in some detail, because of their possible contribution to the anomalous cross-field transport processes. In this Letter, we show that the same free energy can also drive a tearing[2] or, more precisely, a drift-modified tearing instability.[3] In contrast to the finite-β modified trapped-electron instability,[4] these modes connect to long-wavelength magnetohydrodynamic (MHD) perturbations, rather than propagating sound waves, away from the mode rational surfaces. This new instability, which we call the dissipative trapped-electron drift tearing mode, will cause the formation of magnetic islands. In this way, the new instability could have a significant effect on cross-field transport, by mechanism different from those involving only *electrostatic* trapped-electron modes.

The wave equation for this mode is derived following fairly standard procedures.[4] For tokamaks with $\beta \equiv 8\pi p/B^2 \lesssim \epsilon/q^2 \ll 1$, the compressional Alfvén wave can be ignored, and the perturbed magnetic vector potential is given by $\vec{A} = A_\parallel \vec{B}/B$. Here, $\epsilon = r/R$, and $q = rB_T/RB_P$ is the safety factor. The other perturbation-field quantity is the electric potential φ. We then have $\vec{E} = -\nabla\varphi - \partial\vec{A}/c\partial t$ and $\vec{b} = \nabla\times\vec{A}$.

The perturbed electron distribution function is determined by the drift kinetic equation:

$$f_e = e\varphi f_{e0}/T_e + h_e, \tag{1}$$

$$(\omega - \omega_D - iC + iv_\parallel \nabla_\parallel)h_e = -\frac{ef_{e0}}{T_e}(\omega - \omega_*^T)\frac{\varphi - v_\parallel A_\parallel}{c}. \tag{2}$$

Here $\omega_*^T = \omega_{*e}[1 - \eta_e(\frac{3}{2} - \bar{v}^2)]$, $\bar{v} = v/v_{Te}$, $v_{Te} = (2T_e/m_e)^{1/2}$, $\eta_e = d\ln T_e/d\ln n$, ω_* is the electron diamagnetic drift, and ω_D is the combined electron ∇B and curvature drift. For the collision operator C, we assume a number-conserving, velocity-dependent Krook model; in this model, number conservation is satisfied by including in the equation for the untrapped electrons a Maxwellian source term equal to the number of electrons scattered out of the trapped region. For trapped electrons, Eq. (2) then gives

$$h_e^t = -\frac{ef_{e0}}{T_e}\frac{(\omega - \omega_*^T)A_\parallel}{ck_\parallel} + \hat{h}_e^t; \tag{3}$$

$$\hat{h}_e^t = -\frac{ef_{e0}}{T_e}\frac{\omega - \omega_*^T}{\omega - \langle\omega_D\rangle + i\nu_e(v)}\left(\varphi - \frac{\omega A_\parallel}{ck_\parallel}\right). \tag{4}$$

In Eq. (4), and in the following analysis, we neglect terms of order ω_D/ω; we keep ω_D only in denominators such as the one in Eq. (4), where it can give rise to resonance contributions, especially at small ν_e. For untrapped electrons, we obtain

$$h_e^u = -\frac{ef_{e0}}{T_e}\frac{\omega - \omega_*^T}{\omega - k_\parallel v_\parallel}\left(\varphi - \frac{v_\parallel A_\parallel}{c}\right) + \frac{if_{e0}}{\omega - k_\parallel v_\parallel}\frac{\int_t \nu_e(v)\hat{h}_e^t d^3v}{n_0[1-(2\epsilon)^{1/2}]}. \tag{5}$$

Here $\langle\omega_D\rangle \simeq \epsilon\omega_* \bar{v}^2 = \bar{\omega}_D \bar{v}^2$ is the bounce-averaged ∇B- and curvature-drift frequency of the trapped electrons, and $\nu_e(v) = \nu_{\rm eff}/\bar{v}^3$. In deriving Eqs. (3)–(5), we have employed the model collision operators $Ch_e{}^t = -\nu_e(v)\hat{h}_e{}^t$ and $Ch_e{}^u = f_{e0}\int_t \nu_e(v)\hat{h}_e{}^t d^3v / n_0[1-(2\epsilon)^{1/2}]$. In addition, we have made no distinction between φ and $A_\parallel$ and their bounce-averaged values; i.e., we have neglected the variation of φ and $A_\parallel$ along the field lines.

For the ions, we may neglect trapping and collisional effects. In the small-Larmor-radius limit, we obtain

$$n_i = \frac{n_0 e}{T_e}\left[\frac{\omega_*}{\omega} + \left(1 + \frac{\omega_*}{\omega\tau}\right)\rho_s{}^2 \frac{\partial^2}{\partial x^2}\right]\varphi. \tag{6}$$

Here $\tau = T_e/T_i$, $v_s = (T_e/m_i)^{1/2}$, $\rho_s = v_s/\Omega_i$, and we have ignored ion temperature gradients and ion curvature drifts. We have also assumed that $|\partial^2\varphi/\partial x^2| \gg k_y{}^2\varphi$ (slab geometry), as is appropriate for tearing modes for which the electrostatic part of the perturbation is confined to a narrow "singular layer."

From f_e and f_i, we obtain density and current perturbations to be substituted into the quasineutrality condition and Ampère's law, which then yield

$$\left(1 + \frac{\omega_*}{\tau\omega}\right)\rho_s{}^2 \frac{\partial^2\varphi}{\partial x^2} = F(x)\left(\varphi - \frac{\omega A_\parallel}{ck_\parallel}\right), \tag{7}$$

$$\frac{\partial^2 A_\parallel}{\partial x^2} = \frac{4\pi n_0 e^2\omega}{T_e c k_\parallel} F(x)\left(\varphi - \frac{\omega A_\parallel}{ck_\parallel}\right), \tag{8}$$

where

$$F(x) = [1-(2\epsilon)^{1/2} + \xi Z(\xi) - \xi Z(\xi_t)]\left[\left(1 - \frac{\omega_*}{\omega}\right)(1+B_1) - \frac{\eta_e\omega_*}{\omega}B_2\right] - \frac{\eta_e\omega_*}{\omega}\xi[Q(\xi) - Q(\xi_t)]. \tag{9}$$

Here, $\xi = \omega/|k_\parallel| v_{Te}$, $\xi_t = \xi/(2\epsilon)^{1/2}$, Z is the usual plasma dispersion function, $Q(\xi) = \xi + (\xi^2 - \frac{1}{2})Z(\xi)$, and

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = \frac{i(2\epsilon)^{1/2}}{1-(2\epsilon)^{1/2}} \frac{4}{\pi^{1/2}} \int_0^\infty \frac{\nu_{\rm eff}}{v} \frac{\exp(-v^2)d\bar{v}}{\omega - \bar{\omega}_D\bar{v}^2 + i\nu_{\rm eff}/\bar{v}^3} \begin{pmatrix} 1 \\ \bar{v}^2 - \frac{3}{2} \end{pmatrix}. \tag{10}$$

F is a function of x, the distance from the singular surface, and this function is given by setting $k_\parallel = k_\parallel' x$; we assume that this dependence on $k_\parallel$ provides the only significant x dependence of F. In deriving Eq. (8), we have noted that the parallel current is carried almost entirely by the untrapped electrons. The influence of the trapped-electron dynamics enters through the collisional trapping and detrapping effects, which are proportional to $\nu_{\rm eff}$.

For tearing modes, we may assume that the radial component of the magnetic-field perturbation (in our case, $A_\parallel$) is approximately constant within a narrow singular layer around $x=0$. With this approximation, the first step is to solve Eq. (7) for the electrostatic perturbation φ within the singular layer.

In the case of the drift tearing mode, there are *two* singular layers.[5] In the *outer* singular layer, we may assume that $|x| \gg \omega/|k_\parallel'| v_{Te}$. Denoting the solution in this outer singular layer by φ_O, Eq. (7) may be written

$$\frac{\partial^2\varphi_O}{\partial x^2} = \Gamma\left(\varphi_O - \frac{\omega A_\parallel}{ck_\parallel' x}\right), \tag{11}$$

where $\Gamma = F(\infty)/[\rho_s{}^2(1+\omega_*/\tau\omega)]$, with $F(\infty) = [1-(2\epsilon)^{1/2}][(1-\omega_*/\omega)(1+B_1) - \eta_e(\omega_*/\omega)B_2]$. The solution of Eq. (11) that properly connects to the external MHD solution, i.e., that has $\varphi_O \to \omega A_\parallel / ck_\parallel' x$ as $x\to\infty$, is given by

$$\varphi_O = \frac{\omega A_\parallel \Gamma^{1/2}}{ck_\parallel'}\int_0^\infty \frac{\sin(k\Gamma^{1/2}x)}{1+k^2}dk. \tag{12}$$

As $x\to 0$, this solution has the asymptotic form $\varphi_O \sim -(\omega A_\parallel\Gamma/ck_\parallel')x\ln|\Gamma^{1/2}x|$. The outer singular layer has a width given by the typical scale of the solution φ_O, namely $x \sim x_O \sim |\Gamma|^{-1/2}$. In order of magnitude, we have $|\varphi| \sim |\omega A_\parallel/ck_\parallel' x_O|$ throughout this region; physically, this is the region within which the parallel electric field is significant.

There is also, however, an *inner* singular layer, in which $|x| \sim \omega/|k_\parallel'| v_{Te}$. In this region, we have $|\varphi| \ll |\omega A_\parallel/ck_\parallel' x|$, so that we may drop the term in φ on the right-hand side of Eq. (7). The resulting equation will have a unique solution, which we denote by φ_I, that properly connects to the solution in the outer singular layer, i.e., that has $\varphi_I \sim -(\omega A_\parallel\Gamma/ck_\parallel')x\ln|\Gamma^{1/2}x|$ as $x\to\infty$. However, the detailed form of this solution is not needed. The inner singular layer has a width given by $x \sim x_I \sim \omega/|k_\parallel'| v_{Te}$; physically the electrons can be readily accelerated by the parallel electric field only if $\omega \gtrsim k_\parallel v_{Te}$, so that most of the perturbed current lies within this inner region.

The dispersion relation can now be obtained in the usual way, by equating $[\partial A_\parallel/\partial x]/A_\parallel$ to the quantity Δ' obtained from the external MHD solutions. Here, the brackets denote the jump across the singular layer. The quantity Δ' depends on the detailed form of the radial current profile[6]; for our present purposes, we may simply regard it as a known quantity. The jump $[\partial A_\parallel/\partial x]$ is obtained by integrating Eq. (8) over the entire singular layer. However, it is clear from the x dependence of the right-hand side of Eq. (8) that the dominant contribution to the integral comes from the inner singular layer. As we have noted above, we have $|\varphi| \ll |\omega A_\parallel / c k_\parallel' x|$ in this region, so that the term in φ may be dropped from the right-hand side of Eq. (8). Thus, we obtain

$$\Delta' = \frac{[\partial A_\parallel/\partial x]}{A_\parallel} = -\frac{4\pi n e^2 \omega^2}{T_e c^2 k_\parallel'^2}\int_{-\infty}^{\infty}\frac{F(x)}{x^2}dx. \quad (13)$$

We substitute for $F(x)$ from Eq. (9). The integral on the right-hand side in Eq. (13) can easily be evaluated analytically, by contour integration over x, leading to the following dispersion relation:

$$(\omega - \omega_*)(1 + B_1) - \eta_e\omega_*(\tfrac{1}{2} + B_2) = \frac{i\gamma_T}{1 - 2\epsilon}, \quad (14)$$

where $\gamma_T = \Delta' |k_\parallel'| v_A^2 \rho_s^2/\pi^{1/2} v_{Te}$ is the growth rate of the usual "collisionless" tearing mode.[7]

To estimate the importance of the trapped-electron terms in Eq. (14) we must compare the magnitudes of ω_* and γ_T. We find that $\gamma_T/\omega_* = (2\Delta' r_n/\pi^{1/2}\beta_e)(\rho_s/L_s)(m_e/m_i)^{1/2}$ where $\beta_e = 8\pi n T_e/B^2$, and L_s is the shear length. Typical tokamaks have $\rho_s/L_s \sim 2\times 10^{-4}$ and $\beta_e \sim 10^{-2}$; for these parameters, we find $\gamma_T/\omega_* \sim 5\times 10^{-4}(\Delta' r_n)$. Since[6] $\Delta' r_n \lesssim 10^2$, we have $\gamma_T/\omega_* \lesssim 5\times 10^{-2}$. It follows that the term on the right-hand side in Eq. (14) can be treated as a small correction, so that the dispersion relation becomes

$$\omega \simeq \omega_*\left(1 + \frac{\eta_e}{2}\frac{1 + 2B_2}{1 + B_1}\right) + \frac{i\gamma_T}{(1 + B_1)(1 - 2\epsilon)}. \quad (15)$$

in the limit $\omega \ll \nu_{\rm eff}$, we find

$$B_1 \simeq \frac{(2\epsilon)^{1/2}}{1 - (2\epsilon)^{1/2}}\left(1 + \frac{4i\omega}{\pi^{1/2}\nu_{\rm eff}}\right)$$
$$B_2 \simeq \frac{(2\epsilon)^{1/2}}{1 - (2\epsilon)^{1/2}}\frac{6i\omega}{\pi^{1/2}\nu_{\rm eff}}, \quad (16)$$

giving

$$\omega \simeq \omega_*\{1 + 0.5\eta_e[1 - (2\epsilon)^{1/2}]\},$$
$$\gamma = \frac{2(2\epsilon)^{1/2}}{\pi^{1/2}}\frac{\eta_e\omega_*\omega}{\nu_{\rm eff}}[2 + (2\epsilon)^{1/2}] + \frac{\gamma_T}{1 + (2\epsilon)^{1/2}}. \quad (17)$$

Thus, for $\eta_e > 0$, the trapped-electron effects are destabilizing in the more collisional regime. On the other hand, for $\omega \gg \nu_{\rm eff}$, we find

$$B_1 \simeq \frac{(2\epsilon)^{1/2}}{1 - (2\epsilon)^{1/2}}\frac{4i\nu_{\rm eff}}{3\pi^{1/2}\omega}\ln\left|\frac{\omega}{\nu_{\rm eff}}\right|, \quad B_2 \simeq -\tfrac{3}{2}B_1, \quad (18)$$

giving

$$\omega \simeq \omega_*(1 + 0.5\eta_e),$$
$$\gamma \simeq -\frac{8(2\epsilon)^{1/2}}{3\pi^{1/2}[1 - (2\epsilon)^{1/2}]}\frac{\eta_e\omega_*\nu_{\rm eff}}{\omega} \times \ln\left|\frac{\omega}{\nu_{\rm eff}}\right| + \frac{\gamma_T}{(1 - 2\epsilon)}. \quad (19)$$

Thus, for $\eta_e > 0$, the trapped-electron effects are stabilizing in the less collisional regime. We might also note that, in contrast to the electrostatic trapped-electron modes, the ∇B-drift resonances do not play a major role here; at most, they cause a small modification of the real part of the frequency.

The destabilizing contribution in the more collisional regime arises from the term B_2, while B_1 contributes a small stabilizing term. In the less collisional regime, both B_1 and B_2 are stabilizing. More precisely, we find that ImB_2 has a broad peak at $\omega/\nu_{\rm eff} \simeq 0.1$ and vanishes at $\omega/\nu_{\rm eff} \simeq 0.8$. Thus, the largest growth rate occurs at $\omega/\nu_{\rm eff} \simeq 0.1$, and modes with $\omega/\nu_{\rm eff} > 0.8$ are stable. The assumption that $\omega > \nu_e$ then limits the unstable spectrum to $0.8 \gtrsim \omega/\nu_{\rm eff} > \epsilon$. Moreover, the asymptotic experssions given in Eqs. (16) and (17) are approximately valid for $\omega/\nu_{\rm eff} < 0.1$. It may be noted that the destabilizing mechanism introduced by the trapped electrons does not depend on Δ'; thus, the trapped-electron tearing modes can be unstable even if $\Delta' < 0$. Qualitatively similar destabilization mechanisms are also found to exist in the collisional (Pfirsch-Schlüter) regime.[5,8]

Finally, we must confirm the validity of our assumptions regarding the inner and outer singular layers, and of the "constant-$A_\parallel$" approximation. In the more collisional regime where the instabilities occur, the width of the outer singular layer is given by $x_O \sim \rho_s[(1 + \omega_*/\tau\omega)/F(\infty)]^{1/2} \sim \rho_s\{1 + 2(1 + T_i/T_e)/\eta_e[1 - (2\epsilon)^{1/2}]\}^{1/2}$. For $T_e \gtrsim T_i$ and $\eta_e \sim 1$, this is somewhat larger than ρ_i, so that the small-Larmor-radius approximation is justified for the ions. The width of the inner singular layer is given by $x_I \sim \omega/|k_\parallel'| v_{Te} \sim \{1 + 0.5\eta_e[1 - (2\epsilon)^{1/2}]\}(L_s/r_n)(m_e/m_i)^{1/2}\rho_s$. Thus, for $\eta_e \simeq 1$, the condition $x_O > x_I$ becomes $r_n/L_s > (m_e/m_i)^{1/2}$, a condition that is usually satisfied in tokamaks.

[However, for typical parameters ($\eta_e \simeq 1$, $L_s/r_n \simeq 15$, $\epsilon \sim \frac{1}{4}$, and $T_e/T_i \simeq 2$) we find that $x_I \lesssim \rho_i$, so that the small-Larmor-radius approximation may not be valid for the inner singular layer. On the other hand, we did not find it necessary to calculate the detailed solution for φ in the inner singular layer, so that our general conclusion would be unaltered if this region were handled with a finite-Larmor-radius treatment.] The "constant-$A_\parallel$" approximation requires that $x_0 > |\omega(1+\omega_*/\tau\omega)^{1/2}/k_\parallel' v_A|$. Substituting for ω from Eq. (18), this condition becomes

$$\beta_e < \frac{4}{[1-(2\epsilon)^{1/2}]\eta_e\{1+0.5\eta_e[1-(2\epsilon)^{1/2}]\}}\left(\frac{r_n}{L_s}\right)^2. \qquad (20)$$

This condition does not present a very severe limitation on the validity of our analysis; for $\eta_e \simeq 1$, $\epsilon \simeq \frac{1}{4}$, and $L_s/r_n \sim 15$, it only limits us to β_e less than about 4%.

One of the more interesting conclusions to be drawn from our analysis is that trapped-electron drift tearing modes can arise with high azimuthal mode numbers. This is in contrast to magnetically driven tearing modes, which are usually stable (i.e., $\Delta' < 0$) for aximuthal mode numbers $m > 3$. For high m, the external MHD solutions must decay away from the singular surface at r_s as $\exp(-m|r-r_s|/r_s)$. Accordingly, for these modes, we have $\Delta' \simeq -2m/r_s$. The trapped-electron drift tearing modes for these high m values will be unstable provided the trapped-electron contribution to the growth rate exceeds the (stabilizing) contribution γ_T. For $\omega/\nu_{\rm eff} \lesssim 0.1$, we can use the expression given in Eq. (17) for the trapped-electron contribution to γ and, in this case, the instability condition is independent of m: For $\eta_e \simeq 1$ and $\epsilon \simeq \frac{1}{4}$, it can be written $\beta_e > 0.3(r_n^2/qrL_s)\nu_{e*}$, where $\nu_{e*} = \nu_{\rm eff}(qR/\epsilon^{1/2}v_{Te})$. For typical device parameters ($L_s/r_n \sim 15$) this condition becomes $\beta_e > 1.3\times 10^{-3}\nu_{e*}$, which is generally satisfied in tokamaks with relatively collisionless plasma parameters. For $0.1 \lesssim \omega/\nu_{\rm eff} \lesssim 0.8$, the trapped-electron term is still destabilizing, although its contribution to γ is smaller than that given in Eq. (17). However, if $\beta_e \gg 0.3(r_n^2/qRL_s)\nu_{e*}$, as is usually the case, the stabilizing contribution γ_T is relatively very small compared with the trapped-electron contribution to γ, with the result that instability will still occur over much of the range $0.1 \lesssim \omega/\nu_{\rm eff} \lesssim 0.8$.

We conclude that trapped-electron drift tearing modes should be unstable for quite high m values, up to a limit given roughly by $\omega \lesssim 0.8\nu_{\rm eff}$; for our typical parameters, this becomes $k_y\rho_s \lesssim \nu_{e*}$.

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