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Nonlinear Evolution of Collisionless and Semicollisional Tearing Modes

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The evolution of a single tearing mode is investigated. The "collisionless" and "collisional" tearing modes nonlinearly evolve into the "semicollisional" regime where the dynamics of the "singular layer" are dominated by electron diffusion along the perturbed magnetic surfaces. The "semicollisional" mode grows algebraically in the nonlinear phase as in the "collisional" calculation of Rutherford.

Tearing instabilities are believed to have an important role in both the overall stability and energy confinement of tokamak discharges. The $m = 2$ tearing mode (where m is the poloidal mode number) is experimentally found[1] to precede the "disruptive instability", although the role of this mode in the disruption is still unknown. Higher-order tearing modes, though smaller in amplitude, may break up the magnetic surfaces in tokamaks, resulting in enhanced particle and energy transport.[2,3] It is important, therefore, to develop an understanding of the nonlinear evolution of these instabilities. Previous nonlinear theories have been largely based on the collisional magnetohydrodynamics (MHD) equations. We have recently shown,[4] however, that the usual linear stability analysis which results from these equations[5] is not adequate to describe present high-temperature discharges. Evidently, the nonlinear treatment of these instabilities[6,7] must be modified accordingly.

For simplicity, we consider a model in which a current slab J_{z0} of width a in the x direction and uniform in the y-z plane flows along an externally produced B_{z0} field. A self-consistent field $B_{y0}(x)$ is produced which is given by $B_{y0}(x) \simeq B_{z0}x/l_s$ near $x = 0$, with $l_s = B_{z0}(\partial B_{y0}/\partial x)^{-1}$ the shear length of the field. Density and temperature gradients are neglected. The magnetic energy in the field B_{y0} drives the tearing instability and, for a wave number k in the y direction, produces the magnetic islands shown in Fig. 1. The magnetic perturbations are represented by $\tilde{\vec{B}} = \nabla \times \tilde{A}_z \hat{e}_z$, where $\tilde{A}_z$ is the vector potential. The magnetic energy released is dissipated by an induced electric field $\tilde{E}_z = -c^{-1}\partial\tilde{A}_z/\partial t$ which accelerates electrons in a narrow region $|x| < \Delta \ll a$, where $\vec{k}\cdot\vec{B}_0 \approx 0$ ("singular layer"). The current J_z is filamented along the y direction by this induced field so

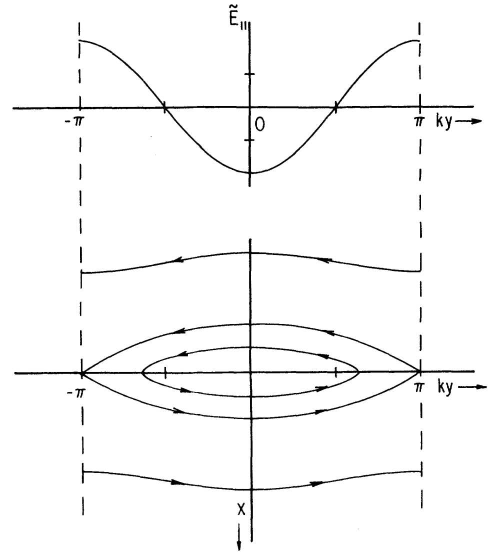

FIG. 1. The variation of $\tilde{E}_{\parallel}$ with y and the magnetic-field configuration around the tearing layer are shown. We assume $B_z \gg |B_y|, |B_x|$.

as to produce self-consistently the magnetic-field perturbations shown.

We first briefly review the linear theory[4,5] of this instability. Outside the region of particle acceleration, the ideal MHD equations are valid and $\partial/\partial x \sim a^{-1} \sim k$. In the region of particle acceleration, $\partial/\partial x \gg a^{-1}$ and $\tilde{A}_z$ satisfies the equation

$$\partial^2 \tilde{A}_z/\partial x^2 = -4\pi \tilde{J}_z/c, \qquad (1)$$

where $\tilde{J}_z$ is the electron response to $\tilde{E}_z$. $\tilde{J}_z$ is localized to the region $x<\Delta$ so $\partial\tilde{A}_z/\partial x$ suffers a discontinuity across the layer given roughly by

$$[\partial\tilde{A}_z/\partial x]_{-\Delta}^{\Delta} \equiv \Delta'\tilde{A}_z(0) = -(4\pi/c)\int dx\, \tilde{J}_z(x) \qquad (2a)$$

$$\simeq -4\pi\tilde{J}_z(0)\Delta/c. \qquad (2b)$$

The discontinuity in $\partial\tilde{A}_z/\partial x$, represented by Δ', must match a corresponding discontinuity in the outer solutions and hence Δ' is independent of the dynamics of the layer. $\Delta' \sim a^{-1}$ has been calculated previously in both slab[5] and cylindrical[8] geometries.

In the "collisionless" and "semicollisional" tearing instabilities Δ is limited by electron thermal motion along $\vec{B}_0$.[4] The electron thermal motion coupled with finite $k_\parallel = kx/l_s$ causes the electrons to experience a Doppler frequency ω_d. When ω_d is greater than the growth rate γ, the electrons receive an ac, rather than a dc, acceleration and $\tilde{J}_z$ is small. Δ is then roughly defined by $\omega_d = \gamma$. In the collisionless regime $\gamma \gg \nu$, so $\omega_d = k_\parallel v_e$ and $\Delta = \gamma l_s/kv_e$, where $\nu = \eta n_e e^2/m$, η is the Spitzer-Harm resistivity, and v_e is the electron thermal velocity. Since $\tilde{J}_z = -k_0^2 c\tilde{A}_z/4\pi$ in the collisionless limit, $k_0^{-1} = c/\omega_{pe}$ being the skin depth, (2b) gives the growth rate and tearing width,

$$\gamma_k \approx \Delta' k v_e/k_0^2 l_s; \quad \Delta_k \approx \Delta'/k_0^2. \qquad (3)$$

In the "semicollisional" tearing instability $\gamma \ll \nu$, so the electrons diffuse along $\vec{B}_0$ and $\omega_d \simeq k_\parallel^2 v_e^2/\nu$. Since $\tilde{J}_z = -k_0^2 c\tilde{A}_z\gamma/4\pi\nu$, (2b) yields

$$\gamma_{sc} \approx \nu\Delta'/k_0^2\Delta_{sc} \approx \gamma_k^{2/3}\nu^{1/3}; \qquad (4a)$$

$$\Delta_{sc} \approx \Delta_k(\nu/\gamma_k)^{2/3}. \qquad (4b)$$

The discussion of the collisionless and semicollisional modes should be contrasted with the "collisional" instability where the electrons "short out" $\tilde{E}_z$ before the Doppler effects are important.

We now consider the nonlinear evolution of a single tearing mode—a model which is representative of the tokamak discharge whenever the rational surfaces where $\vec{k}\cdot\vec{B}_0 = 0$ are spatially separated. The harmonics are assumed to be heavily damped and hence negligible.[6] Under the assumption that $w \ll a$, where w is the half-width of the magnetic island, then (1) $|\tilde{J}_z| \ll |J_{z0}|$, which implies $B_y \simeq B_{y0}$; and (2) electron heating in the layer can be neglected. For this model,

$$A_z(x,y,t) = -B_{z0}x^2/2l_s + \tilde{A}_z\cos(ky), \qquad (5)$$

where $\tilde{A}_z$ is essentially constant across the layer. Note that this model is not appropriate for the $m=1$ mode in tokamak applications, where the "constant-ψ" approximation is invalid. A_z is constant along a field line, so

$$w = 2(\tilde{A}_z l_s/B_{z0})^{1/2}. \qquad (6)$$

The linear theory is valid as long as the magnetic-field perturbations within the region of particle acceleration are small, i.e., $w \ll \Delta$. When $w \gtrsim \Delta_L$ (where Δ_L is the linear layer width), the electron orbits are strongly altered by the new magnetic-field configuration, the electrons within the separatrix being constrained to move around the islands. For $w \gg \Delta_L$, the electrons within the separatrix still experience a nonzero time-averaged electric field $\langle E_z\rangle_t$ as can be seen in Fig. 1. $\langle E_z\rangle_t < 0$ for the electrons near the center of the island, while $\langle E_z\rangle_t > 0$ for electrons closer to the separatrix. Nonlinearly, we therefore expect $\Delta \simeq w$ since $\langle E_z\rangle_t \simeq 0$ for electrons well outside the separatrix where the magnetic-field lines are only weakly distorted.

More quantitatively, we write the guiding-center equation for electrons with a model pitch-angle scattering operator as follows:

$$[\partial/\partial t + v_\parallel \partial/\partial s - \tfrac{1}{2}\nu(\partial/\partial\xi)(1-\xi^2)(\partial/\partial\xi)]\tilde{f} = 2e\tilde{E}_\parallel v_\parallel f_0/mv_e^2, \qquad (7)$$

where "$\parallel$" refers to the component of a vector along $\vec{B}$, $\xi = v_\parallel/v$, and f_0 has been approximated as a Maxwellian distribution. With neglect of collisions, (7) is easily inverted and $\tilde{J}_z$ is then calculated as follows:

$$\tilde{J}_z = (c/4\pi)k_0^2\int d^3v f_0(2v_\parallel^2/v_e^2) \times \int_{-\infty}^{t} d\tau \cos(ky)\partial\tilde{A}_z/\partial\tau \simeq (c/4\pi)k_0^2\tilde{A}_z\int_0^T d\tau \cos[ky(\tau)]/T, \qquad (8)$$

where the second equality is valid when $\gamma(t)T \ll 1$, T being the period of the electron motion along $\vec{B}$ and $\gamma(t) = \partial \ln A_z/\partial t$ being the local growth rate; $y(t)$ is the y coordinate of the particle as a function of time. Since $\tilde{J}_z$ in (8) is essentially given by the time-averaged electric field seen by the electrons, it is constant along a given field line. $\theta(t)=ky(t)$ satisfies the equation

$$d^2\theta/dt^2+\omega_b^2\sin\theta=0, \quad (9)$$

where $\omega_b=kv_\parallel w/2l_s$ is the "bounce" frequency of the "deeply trapped" electrons. The boundary condition $d\theta/dt=2\omega_b x/w$ at $\theta=0$ specifies the field line occupied by the electron. The present investigation is evidently very similar to O'Neal's calculation[9] of the damping of a large-amplitude wave, our x-y phase space being analogous to the v_z-z phase space in his work. We project out the k component of $\tilde{J}_z$ and combine this result with (2a) to obtain

$$w=\Delta'/2k_0^2G, \quad (10)$$

where

$$G=\int dx\int_0^{2\pi}d\theta\cos\theta\int_0^T dt\cos[\theta(t)]/2\pi Tw. \quad (11)$$

G is a constant which can be expressed as an integral over complete elliptic functions. Numerical evolution of this integral yields $G=0.410$. In the collisionless limit, the magnetic island saturates when $w\approx\Delta_k$, the linear tearing width. Note, however, that this result is somewhat artificial since the collisionless approximation breaks down when $\gamma<\nu$. The collisionless tearing mode then evolves into the semicollisional regime.

When $\gamma<\nu$, f is still constant along a given field line as long as the electrons complete many orbits around the magnetic island during a time γ^{-1}. This can be shown[10] in various limits by directly inverting the operator on the left-hand side of (7). We can therefore average (7) over a field line, eliminating the operator $v_\parallel\partial/\partial s$. Solving the resulting equation for f and calculating $\tilde{J}_z$, we find

$$\tilde{J}_z=(c/4\pi)k_0^2(\nu^{-1}\partial\tilde{A}_z/\partial t)\int_0^S ds\cos[ky(s)]/S, \quad (12)$$

where S is the length of one period of the field line. For (12) to be correct, we require $\langle\Delta s^2\rangle^{1/2}\gg S$, where $\langle\Delta s^2\rangle^{1/2}=v_e(\gamma\nu)^{1/2}$ is the average distance an electron diffuses during a time γ^{-1}. The average of $\cos(ky)$ over the field line physically corresponds to time-average field experienced by the electron, which is identical to the time average in (8) as long as the inequality is satisfied. Equation (12) is then identical to (8) except for the operator $\nu^{-1}\partial/\partial t$, so from (10)

$$\gamma(t)=\nu\Delta'/2k_0^2Gw(t)\approx\gamma_{sc}\Delta_{sc}/w(t). \quad (13)$$

A comparison of this result with (4) clearly indicates that the tearing width becomes the magnetic island width in the nonlinear regime. From (6), $dw/dt=\Delta'c^2/16G$, so the magnetic island grows algebraically in the nonlinear stage. This result is similar to that of Rutherford[6] in the collisional tearing mode, although the physical mechanisms are quite different. From the expression for w_b, $S\simeq l_s/kw$ for deeply trapped particles. The inequality $\langle\Delta s^2\rangle^{1/2}\gg S$ then simply reduces to

$$w\gg\Delta_{sc}. \quad (14)$$

This inequality, along with the restriction $w\ll a$, defines the range of validity of (13).

The growth rate in (13) has a simple physical interpretation in terms of the energetics of the tearing mode. From linear theory the rate of magnetic energy release per unit area in the y-z plane is given by $\gamma\Delta'A_z^2/4\pi$.[11] This energy is dissipated by Joule heating in the tearing layer, $\Delta\tilde{J}_z\tilde{E}_z\simeq\Delta\gamma^2k_0^2/4\pi$. As Δ increases with w in the nonlinear phase, the growth rate of the instability must decrease so that the released magnetic energy can heat the larger number of electrons in the tearing layer.

That the semicollisional tearing mode does not saturate within the present model is not really surprising. Since $\tilde{E}_z=0$ at saturation, $\tilde{J}_z=0$ in the central layer. This result is only self-consistent if $\tilde{B}=0$, so the semicollisional tearing mode must continue to evolve. In the collisionless limit, on the other hand, $\tilde{J}_z$ remains finite even when $\tilde{E}_z=0$ and saturation can therefore occur. The inclusion of finite w/a corrections, which has been shown by White *et al.*[7] to saturate the collisional tearing mode, will be considered in a separate publication.

We have found that the collisionless tearing mode nonlinearly evolves into the semicollisional regime. In the long-mean-free-path conditions of present tokamak discharges, $\Delta_{sc}\approx\rho_i$, the ion Larmor radius.[4] The inequality in (14) describing the validity of the semicollisional theory therefore implies that the collisional tearing mode also evolves into the semicollisional regime. The dynamics of the "singular layer" in present discharges is therefore dominated by electron diffusion along the perturbed magnetic surfaces.

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Electron Transport in Methane Gas

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We propose a kinetic theory for electron–drift-velocity maxima in polyatomic gases. The case of methane is considered in detail, and good agreement with experiment is obtained with use of model cross sections. The Boltzmann equation is solved directly by applying an iterative numerical technique, which converges well when inelastic scattering effects are important.

We consider the drift velocity V_d (average velocity) of a swarm of electrons in a steady determined by the action of a uniform electric field $\vec{E}$ and scattering from the molecules of a gas. For several polyatomic gases, V_d exhibits a maximum as a function of E.[1,2] It has been clear for a long time that inelastic scattering including a vibrational transition is involved (Cottrell and Walker[1]). However, no satisfactory kinetic theory has been presented to explain this feature. In this Letter, we give a general theory for it. We consider methane in particular, since it has been extensively studied from both an experimental and theoretical viewpoint[3] and its behavior is typical of several nonpolar polyatomic gases (e.g., CD_4, SiH_4, SiD_4, C_2H_4, C_2H_6). We propose that the velocity maximum is due to a strong "streaming" anisotropy in the electron velocity distribution $f(\vec{V})$. This results from the combined effects of elastic and inelastic scattering near the Ramsauer minimum in a way not previously understood in gas kinetic theory. To check our model quantitatively, we solve the appropriate Boltzmann equation for $f(\vec{V})$ for model scattering cross sections appropriate to methane. This is done by applying an iterative numerical technique originally developed by Rees[4] for what is essentially the same transport problem in semiconductors.

The iterative technique is numerically exact and converges well when $f(\vec{V})$ is anisotropic. Previous calculations of $f(\vec{V})$ in similar circumstances are all based on equations appropriate when anisotropy is small (see Huxley and Crompton[1] for a review of this work and references). Apparently the general condition for this is that the total inelastic cross section be much smaller than the total elastic cross section.[5] In the present model, this is not the case for most energies of interest [see Eq. (5)]. A strong anisotropy is also consistent with Cottrell and Walker's[1] demonstration that V_d is of the order of the rms electron speed for the fields considered here. Another consequence is that the elastic *energy loss* (and, in this context, the elastic momentum-transfer cross section) plays almost no role since it is far smaller than the inelastic energy loss. (The direction-changing effects of elastic scattering are crucial, however; see below.) Setting $m/M = 0$ in Eq. (2) at a typical value for E in the calculation described below affected only the fourth significant digit in $f(\vec{V})$.

It is worth emphasizing that the iterative technique[4] is very general and widely applicable to the problem of calculating $f(\vec{V})$ in similar electron- or ion-transport problems from given cross sections. It should prove especially useful