

## Isospin Dependence of Experimental Nuclear Deformations

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Isospin effects in nucleon inelastic scattering have been studied by comparing quadrupole deformation parameters deduced from measured  $(n, n')$  cross sections with values for  $(p, p')$  and values for electromagnetic excitation found in the literature. Proton-vibrational nuclei with  $N = 50$  and neutron-vibrational nuclei with  $Z = 50$  were studied. Predicted effects of core polarization are observed. Difficulties in interpreting detailed numerical relationships are discussed.

In the vibrational model, the excitation of the lowest collective  $2^+$  state of even-even nuclei is related to a deformation parameter  $\beta$  that is a characteristic geometrical property of the state.<sup>1</sup> Its evaluation for specific nuclei has been a goal of many experiments employing a variety of techniques. One may designate the results as  $\beta_{em}$  or  $\beta_{nuc}$ , depending on whether the transition is stimulated by the electromagnetic interaction (e.g., Coulomb excitation) or by the nuclear interaction (inelastic nucleon scattering). It is frequently supposed that  $\beta_{em}$  should equal  $\beta_{nuc}$ . Although there have been some theoretical reasons to the contrary,<sup>2</sup> there has been little convincing experimental evidence.

Recently Brown and Madsen<sup>3</sup> have investigated core-polarization effects in detail, and argue that they produce effective transition amplitudes,  $\beta_{eff}$ , that are not the same as the intrinsic deformation parameters and, in particular, that  $\beta_{em} \neq \beta_{nuc}$ . Each  $\beta$  has contributions from isoscalar and isovector transition operators, where the latter is related to the neutron excess or the Lane potential. These are further subdivided into contributions from the model nuclear states and from the external field. An isospin dependence of the external field will produce different  $\beta_{eff}$ . The ideas were extended<sup>4</sup> into specific predictions for  $\beta_{eff}$  for Coulomb-excitation,  $(n, n')$ , and  $(p, p')$  reactions. Some evidence was found in support of the predictions for  $\beta_{em}$  and  $\beta_{pp}$ , from the literature.

It is difficult to compare  $\beta_{em}$  and  $\beta_{pp}$ , in detail because their extraction from data is subject to quite different sets of assumptions and procedures. On the other hand,  $\beta_{nn'}$  for single nuclei have generally not been available. We present here some new and detailed evidence in favor of

an explicit isospin dependence of  $\beta_{nuc}$ .<sup>4</sup> Specifically, we have obtained  $\beta_{nn'}$  for  $(n, n')$  reactions on a number of single-closed-shell nuclei and compared the values with recent  $\beta_{em}$  and  $\beta_{pp}$ , values from the literature. The predicted relationships among all three quantities are found to hold in very reasonable detail.

The evidence can be presented most clearly for single-closed-shell nuclei. According to Ref. 4, one should expect  $\beta_{eff}$  for nuclei with neutron closed shells (proton-vibrational nuclei) to be in relationship

$$\beta_{em} > \beta_{nn'} > \beta_{pp}.$$

The inequalities are reversed for nuclei with proton closed shells (neutron vibrators). Various models of core polarization produce numerical ratios that bridge the inequalities.

Data are presented here for  $^{90}\text{Zr}$  and  $^{92}\text{Mo}$ , with  $N = 50$ , and for  $^{118,120,122,124}\text{Sn}$ , with  $Z = 50$ . These are generally accepted as being good single-closed-shell nuclei. In  $^{90}\text{Zr}$ , for example, the  $\nu(d_{5/2}g_{9/2}^{-1})$  configuration forms a multiplet that lies 4–5 MeV in excitation energy.<sup>5</sup> The  $2^+$  member could mix with the collective  $2^+$  state at 2.19 MeV, but the evidence is that it does not do so appreciably.<sup>5</sup>

An 11-MeV neutron beam was produced via the reaction  $D(d, n)^3\text{He}$  in a gas cell. The deuteron beam was obtained from the Ohio University tandem Van de Graaff accelerator and was pulsed to less than a nanosecond width with a 5-MHz repetition rate and an average current of 2 to 4  $\mu\text{A}$ . The scattering samples were enriched in the isotopes of interest.

The spectra were obtained with a time-of-flight spectrometer and had an overall energy resolu-

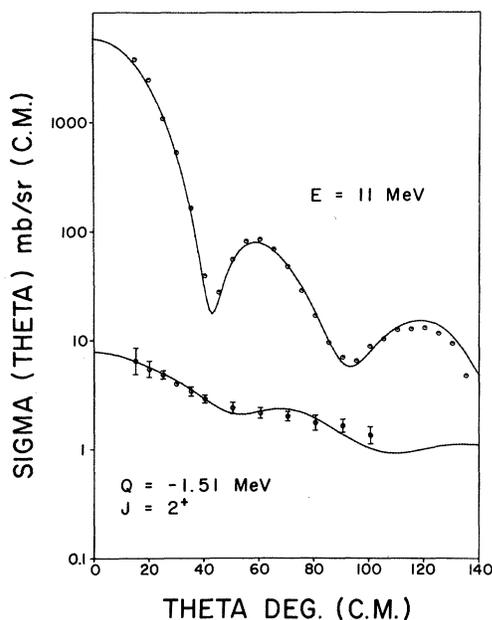


FIG. 1. Scattering of neutrons from  $^{92}\text{Mo}$ . Errors due to counting statistics, unless indicated otherwise, are smaller than the plotting symbol. Solid curves are optical models and distorted-wave Born-approximation calculations described in the text.

tion of less than 300 keV. The data were normalized by rotating the shielded detector to zero degrees, removing the scattering sample, and observing the neutron flux per monitor count which would have been incident upon the scattering sample had it been in place. The absolute cross-section normalization of the data is believed to be correct to better than 5%.

These measurements were made as an extension of an ongoing program of neutron scattering

measurements.<sup>6-8</sup> Optical potentials have been obtained from the data with a search code. The details are presented elsewhere.<sup>7,8</sup> Figure 1 shows the elastic-scattering angular distribution for  $^{92}\text{Mo}$  along with the optical-model calculations. The computed total cross section is in good agreement with existing data.

Calculations of the cross sections for the lowest  $2^+$  states were made with the distorted-wave code DWUCK4.<sup>9</sup> The form factors were those given by the macroscopic model with amplitudes adjusted to fit the data. These amplitudes are the deformation lengths  $\delta = \beta R$ , where  $R$  is the nuclear radius. The interaction form factors were complex. Fits to the data for  $^{92}\text{Mo}$  are shown in Fig. 1. The  $2^+$  state in  $^{90}\text{Zr}$  is unresolved from a  $5^-$  state. This was fitted simultaneously and subtracted from the angular distribution for presentation.

Generally, the calculated inelastic-scattering angular distributions represented the data very well. Where there were deviations, normalization to the data was normally made in the range  $30-90^\circ$ . These data usually had the best statistical accuracy and were least subject to possible systematic errors resulting from the tails of the elastic peaks.

The values of  $\beta_{nn'}$  obtained from the present data are given in Table I. Also listed are the values of  $\beta_{em}$  and  $\beta_{pp}$ , taken from the literature. Since all analyses used an  $R \approx 1.2A^{1/3}$  fm, values of  $\beta$  rather than  $\delta$  may be listed. The tabulation of Stelson and Grodzins<sup>16</sup> for  $\beta_{em}$  has been updated from the literature<sup>10,15,17-19</sup> where appropriate. The  $\beta_{pp}$  values have been taken directly from analyses in the literature.<sup>10-14,20</sup> One exception is for  $^{92}\text{Mo}$ . In the work of Lutz, Heikkinen, and

TABLE I. Experimental deformation parameters. The theoretical values are discussed in the text. [Numbers in parentheses represent experimental uncertainties in the last digit(s)]

|                                  | $^{90}\text{Zr}$      | $^{92}\text{Mo}$      | $^{118}\text{Sn}$      | $^{120}\text{Sn}$      | $^{122}\text{Sn}$     | $^{124}\text{Sn}$     |
|----------------------------------|-----------------------|-----------------------|------------------------|------------------------|-----------------------|-----------------------|
| $\beta_{pp}$                     | 0.070(5) <sup>a</sup> | 0.080(6) <sup>b</sup> | 0.134(10) <sup>c</sup> | 0.119(10) <sup>c</sup> | 0.112(7) <sup>c</sup> | 0.108(7) <sup>c</sup> |
| $\beta_{nn'}$                    | 0.085(8)              | 0.099(5)              | 0.109(7)               | 0.106(5)               | 0.100(6)              | 0.092(6)              |
| $\beta_{em}$                     | 0.094(5) <sup>d</sup> | 0.116(8) <sup>e</sup> | 0.108(2) <sup>f</sup>  | 0.106(2) <sup>f</sup>  | 0.102(2) <sup>f</sup> | 0.096(2) <sup>f</sup> |
| $\beta_{nn'}/\beta_{pp}$ (model) | 1.33                  | 1.31                  | 0.888                  | 0.896                  | 0.903                 | 0.911                 |
| $\beta_{nn'}/\beta_{pp}$ (expt.) | 1.2                   | 1.3                   | 0.81                   | 0.90                   | 0.90                  | 0.85                  |
| $\beta_{em}$ (calc.)             | 0.093                 | 0.109                 | 0.093                  | 0.099                  | 0.094                 | 0.084                 |

<sup>a</sup>Refs. 10, 11.

<sup>b</sup>Reanalysis of data of Ref. 12 by the present authors.

<sup>c</sup>Ref. 13. Values in Ref. 14 are typically about 10% larger.

<sup>d</sup>Ref. 15.

<sup>e</sup>Ref. 16.

<sup>f</sup>Ref. 17.

Bartolini<sup>12</sup> the optical potential had an imaginary geometry quite different from that of other analyses. In particular, the product of  $W_D a_i$  in Ref. 12 was about 50% larger than in other  $(p, p')$  analyses. This is known to have an effect on the value of  $\beta$ .<sup>21</sup> We have reanalyzed the data of Ref. 12 with an optical potential which was obtained by searching for values of the real and imaginary well depths which best fit the elastic proton scattering data at 15 MeV. Geometrical parameters were held constant at the values used for the neutron-scattering analysis. Equally satisfactory fits to the proton-scattering cross sections were obtained with a reduction in  $\beta$  from 0.105 to 0.080. This is in good agreement with independent determinations.<sup>20</sup>

The results for  $\beta_{nn}$  and  $\beta_{pp}$  are entirely in accord with the predicted inequalities discussed above. We note particularly the reversal of the inequality between the  $N=50$  isotones and  $Z=50$  isotopes.

Madsen, Brown, and Anderson<sup>4</sup> also obtained numerical relationships between the  $\beta_{eff}$  by the adoption of a no-parameter schematic model. We have evaluated Eqs. (13) and (14) of Ref. 4 for the nuclei of interest. The model ratios  $\beta_{nn}/\beta_{pp}$  are also given in Table I along with the ratios of the experimental values. The overall agreement is very good— $\beta_{nn}$  is (20–30)% larger than  $\beta_{pp}$ , for the  $N=50$  isotones, but about 5% lower for the  $Z=50$  isotopes.

It should be emphasized that, although the comparison between  $\beta_{nn}$  and  $\beta_{pp}$  is perhaps the most reliable method for investigating the core-polarization effects, neither  $\beta_{nn}$  nor  $\beta_{pp}$  is free from uncertainty. Data normalizations [which are frequently uncertain by (10–20)% in  $(p, p')$  studies], the effects of Coulomb-excitation corrections, the influence of different optical-model potentials and coupled channels, the proper interpretation of the complex form factor, and the contributions of neglected interactions all provide a certain amount of caution. Nevertheless, our reading of the literature, and our own conventional calculations, lead us to conclude that if neutron and proton potentials have similar geometries and reproduce the data, then the inequalities we have observed will remain, although the precise numerical ratios might be modified. It is not yet clear whether the data for all isotopes can be fully understood within a Lane-model consistent optical potential, especially with regard to the imaginary terms. The problem is made difficult since one cannot always obtain numerical values for pub-

lished  $(p, p)$  and  $(p, p')$  data. Specific investigations are being made of some of the above problems.

It is instructive also to make comparisons with  $\beta_{em}$ . The data for  $\beta_{nn}$  and  $\beta_{pp}$  may be used to calculate  $\beta_{em}$  in a manner not dependent on detailed models.<sup>4</sup> The results also appear in Table I. The agreement with experiment is entirely satisfactory for the  $N=50$  isotones. The calculated values seem to be systematically low for the Sn isotopes. This could arise from incorrect values for  $\beta_{em}$ , or from differences in the experimental  $\beta_{nn}$  and  $\beta_{pp}$ , that are too large, or from defects in the theory. Although the ratios  $\beta_{nn}/\beta_{pp}$  agree reasonably well with theory, the theoretical values are model dependent and may need certain corrections. The  $\beta_{pp}$  values from Ref. 14 would make the discrepancies worse.

Normally, experiments do not determine  $\beta_{em}$  directly, but rather determine the  $B(E2)$  from which  $\beta_{em}$  is calculated. It is common for this to be done with the assumption of a uniformly charged sphere of radius  $1.2A^{1/3}$  fm, a procedure also adopted here. The effects of a nonuniform distribution can produce corrections that might be very significant for the comparisons in Table I.<sup>22</sup>

There are additional uncertainties even in the  $B(E2)$  values, since different measurement techniques can produce different results. Typically, Coulomb excitation is used to determine the  $B(E2)$  values since any model dependence does not have a strong influence and can be evaluated reasonably well. On the other hand,  $(e, e')$  experiments are more strongly model dependent, while resonance-fluorescence  $(\gamma, \gamma')$  experiments are more difficult and less frequently performed, although they are in principle model independent. In <sup>90</sup>Zr, for example, two  $(e, e')$  experiments<sup>15,23</sup> produced larger  $B(E2)$ 's than a  $(\gamma, \gamma')$  determination,<sup>24</sup> and all were substantially larger than an older Coulomb-excitation result.<sup>16</sup> The best fit to the  $(e, e')$  and  $(\gamma, \gamma')$  experiments<sup>15</sup> is used in Table I. For the Sn isotopes, "absolute" values from a recent Coulomb-excitation study<sup>17</sup> were used in Table I. These are lower than another independent determination of "absolute" values.<sup>19</sup> It is pertinent to note that  $(e, e')$  data<sup>25</sup> give evidence for values still lower.

The data presented here give clear and satisfying support to the suggestion<sup>4</sup> that there is an isospin dependence to experimental values of  $\beta_{nuc}$  and that one can expect  $\beta_{em} \neq \beta_{nuc}$ . Theoretical values of the numerical relations are model dependent. Experimentally deduced values are sub-

ject to experimental details and assumptions of analysis that need to be studied more closely.

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## Determination of the Static Quadrupole Moment of the 1.98-MeV $2^+$ State in $^{18}\text{O}$

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The static quadrupole moment of the first  $2^+$  state in  $^{18}\text{O}$  has been determined using the reorientation effect in the Coulomb excitation of  $^{18}\text{O}$  projectiles. For constructive interference from the coupling through the second  $2^+$  state, the values obtained are  $Q_{2^+} = -(0.073 \pm 0.027)e \cdot b$  and  $B(E2, 0^+ \rightarrow 2_1^+) = (0.00453 \pm 0.0025)e^2 \cdot b^2$ . Our result for  $Q_{2^+}$  strongly disagrees with a recent measurement [ $Q_{2^+} = -(0.19 \pm 0.02)e \cdot b$ ] but is consistent with theoretical calculations [ $Q_{2^+} = (0.03 - 0.06)e \cdot b$ ].

A great deal of interest has been generated by a recent measurement<sup>1</sup> of the static quadrupole moment of the first  $2^+$  state of  $^{18}\text{O}$ , which utilized the reorientation effect in the excitation of  $^{18}\text{O}$  projectiles. The quadrupole moment measured in that work is approximately three times the rotational value [based on the  $B(E2)$  measured in the same work], and is strongly inconsistent with calculations arising from current models<sup>2-5</sup> of the structure of  $^{18}\text{O}$ . Another measurement of the quadrupole moment is of great interest since if this large value could be confirmed, either drastic revisions of current nuclear-structure ideas would be required, or the reliability of the reorientation effect would be questioned.

In this Letter we report on a new measurement of the  $Q_{2^+}$  and  $B(E2, 0^+ \rightarrow 2_1^+)$  values of the first excited state of  $^{18}\text{O}$ , using the reorientation effect<sup>6</sup> in Coulomb excitation in which the  $^{18}\text{O}$  projectiles were excited by Coulomb scattering from a Au target. The 60-MeV  $^{18}\text{O}$  beam, produced by the Brookhaven National Laboratory (BNL) Model MP7 tandem accelerator, varied in intensity between 20 and 40 particle nA. This bombarding energy was chosen because it is the highest energy which maintains a 6-fm separation between target and projectile surfaces (using  $R = 1.25A^{1/3}$  fm), a condition which has been suggested<sup>7</sup> to insure negligible probability of nuclear excitation. Calculations<sup>8</sup> have verified that, for this bom-