el, which predicts $R_{NC/CC}=0.43$ (0.36) for $\sin^2\theta_W = 0.33$ for neutrino energies below (above) charm threshold. Our measured value is in good agreement with these values but is consistent with other models also.

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Upper Bound for the Induced Pseudoscalar Form Factor in Muon Capture

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It is shown, on very general grounds, that the Goldberger-Treiman (GT) value for the induced pseudoscalar form factor, h_A, in muon-capture reactions represents an upper bound for any realistic h_A that takes into account corrections to (i) nuclear GT relations and (ii) the hypothesis of partial conservation of axial-vector current. The proof is independent of the particular model one might choose for these corrections. An application of this bound to muon capture on ^{12}C makes it difficult to understand the sign of the experimentally reported induced pseudotensor form factor in ^{12}B β decay.

The possibility that the weak hadronic current might contain second-class vector and/or axial-vector pieces is a long-standing problem that has deserved a great deal of experimental and theoretical consideration.[1] At present there is experimental indication of a rather large induced pseudotensor form factor (second-class axial-vector coefficient) for the mass-19 system[2] and for the mass-12 system.[3] In the latter case the experimental value is

$$g_T(0)/Ag_A(0) = -4.86 \pm 1.68, \tag{1}$$

where g_A and g_T are the axial-vector and induced pseudotensor form factors, respectively. Two recent analyses[4,5] of the reaction

$$\mu^- + {}^{12}\text{C} \to {}^{12}\text{B(g.s.)} + \nu_\mu \tag{2}$$

make use of the experimental information on the capture rate[6] and on the ^{12}B recoil polarization[7] with the aim of (i) deducing the weak-magnetism form factor, g_M, independently of the induced pseudoscalar form factor, h_A, and (ii) studying the range of compatibility of g_T for the given set of experimental data. As it turns out, the value so deduced for g_M is in good agreement with its counterpart measured in inelastic electron scattering on ^{12}C, thus supporting the conserved vector current (CVC) hypothesis.[8] On the other hand if g_T/Ag_A is to be negative, as required by present experiment,[3] it was found that f_p, defined as

$$f_p=\frac{m_\mu h_A(q^2=-0.74m_\mu^2)}{Ag_A(q^2=-0.74m_\mu^2)}, \tag{3}$$

should be $f_p>10-12$. In Eq. (3) m_μ is the μ^- mass and $q^2=-0.74m_\mu^2$ is the momentum transfer for Reaction (2). Such a value of f_p indicates a large deviation from the Goldberger-Treiman[9] (GT) point

$$f_p^{\text{GT}}=\frac{m_\mu}{A}\frac{(m_{^{12}\text{C}}+m_{^{12}\text{B}})}{\mu_\pi^2+0.74m_\mu^2}=7.1. \tag{4}$$

A priori this might not come as a surprise because departures from the chiral-symmetry limit as well as nuclear effects are expected to make a contribution to the corrections to (i) nuclear GT relations and (ii) the PCAC (partial conservation of axial-vector current) hypothesis.[10-13] Nevertheless, model-dependent estimates[11-13] suggest that $\tilde{f}_p<f_p^{\text{GT}}$, where $\tilde{f}_p$ includes the above-mentioned corrections to Eq. (4). Therefore, it would be very important to decide if this trend is of a more general nature, i.e., if it is expected to hold for any muon-capture process irrespective of the particular model one chooses for the for the corrections to GT relations and PCAC.

In this Letter we prove, on very general grounds, that f_p^{GT} indeed represents an upper bound for $\tilde{f}_p$ in the spacelike region ($q^2<0$), i.e.,

$$\tilde{f}_p(q^2)\leq f_p^{\text{GT}}(q^2)=m_\mu(m_i+m_f)/A(\mu_\pi^2-q^2), \tag{5}$$

where m_i and m_f are the masses of the initial and final nuclei involved in the muon capture reaction.

Defining the matrix element of the axial-vector current between initial (i) and final (f) nuclei as

$$\langle f|A_\mu^\pm|i\rangle=\bar{u}_f\tau^\pm[\gamma_5\gamma_\mu g_A(q^2)+\gamma_5 q_\mu h_A(q^2)]u_i, \tag{6}$$

and taking the divergence on both sides of (6) one obtains[14]

$$\frac{\mu_\pi^2 f_\pi\sqrt{2}\,g_{\pi if}(q^2)}{\mu_\pi^2-q^2}=(m_i+m_f)g_A(q^2)+q^2h_A(q^2), \tag{7}$$

where f_π is the pion decay constant, μ_π the pion mass, and $g_{\pi if}(q^2)$ the π-nucleus-nucleus strong coupling. We have already absorbed in $g_{\pi if}(q^2)$ all possible corrections to the GT relation, Δ_π, defined by[15]

$$\Delta_\pi=1-(m_i+m_f)g_A(0)/f_\pi\sqrt{2}\,g_{\pi if}(\mu_\pi^2). \tag{8}$$

For the nucleon the main contribution to Δ_π seems to arise from corrections to the PCAC hypothesis.[15-18] In the nuclear case one expects additional corrections to PCAC[13,19] as well as contributions from anomalous-threshold singularities.[10] All these effects can be absorbed in $g_{\pi if}(q^2)$ so that Eq. (7) is the most generally valid expression.

Solving for $h_A(q^2)$ in Eq. (7) one obtains

$$h_A(q^2)=\frac{(m_i+m_f)}{q^2}g_A(0)\left[\frac{\mu_\pi^2}{\mu_\pi^2-q^2}\frac{K(q^2)}{K(0)}-\frac{g_A(q^2)}{g_A(0)}\right], \tag{9}$$

where $K(q^2)=g_{\pi if}(q^2)/g_{\pi if}(\mu_\pi^2)$, with $K(0)=1-\Delta_\pi$. On the other hand the cononical GT expression for h_A is

$$h_A^{\text{GT}}(q^2)=(m_i+m_f)g_A(q^2)/(\mu_\pi^2-q^2). \tag{10}$$

From taking the ratio of Eqs. (9) and (10) it follows that

$$\frac{h_A(q^2)}{h_A^{\text{GT}}(q^2)}=\frac{\mu_\pi^2}{-q^2}\left[1-\frac{K(q^2)}{K(0)}\frac{g_A(0)}{g_A(q^2)}\right]+1. \tag{11}$$

In muon capture $q^2<0$ and therefore $h_A(q^2)\leq h_A^{\text{GT}}(q^2)$ provided

$$\frac{K(q^2)/K(0)}{g_A(q^2)/g_A(0)}\geq 1.$$

Since q^2 is very small and since we recall that both $K(q^2)$ and $g_A(q^2)$ must be decreasing functions of q^2 in the spacelike region, one can replace the previous inequality by

$$\left[\frac{d}{dq^2}\frac{g_A(q^2)}{g_A(0)}\right]_{q^2=0}\geq\left[\frac{d}{dq^2}\frac{K(q^2)}{K(0)}\right]_{q^2=0}, \tag{12}$$

or, alternatively,

$$\langle r^2\rangle_A\geq\langle r^2\rangle_\pi, \tag{13}$$

where $\langle r^2\rangle_A$ is the axial-vector rms radius and $\langle r^2\rangle_\pi$ the pionic rms radius.[16,17] Therefore, Eq. (5) is proved provided Eq. (13) holds.

We shall give now several arguments that support the inequality (13). First of all we note that for the nucleon $\langle r^2\rangle_A$ is equal, within experimental errors,[20] to the isovector electromagnetic rms radius. In the case of nuclei the impulse approximation implies that this equality still holds and the experimental information sustains this view to a good degree of accuracy.[21] Therefore, one can safely replace $\langle r^2\rangle_A$ in Eq. (13) by the isovector rms radius,[22] $\langle r^2\rangle_{EM}$.

It has been shown[17,23] that in the chiral-symmetry limit ($\mu_\pi \to 0$), $\langle r^2\rangle_{EM}$ develops a logarithmic singularity while $\langle r^2\rangle_\pi$ remains finite and quite small.[24] This singular behavior affects both the Dirac and the Pauli rms radii. Therefore, Eq. (13) is trivially satisfied in this limit. In the real world one has to consider two types of corrections, namely departures from chiral-symmetry and nuclear effects. The first one is not expected to contribute significantly,[17] i.e., $\langle r^2\rangle_\pi$ remains very small while $\langle r^2\rangle_{EM}$, though free of the logarithmic singularity, receives a large contribution from the ρ-meson pole (recall that the pionic form factor has no pion pole in the same way as the electromagnetic form factor has no photon pole). Nuclear effects, on the other hand, tend to quench the pionic and the axial-vector form factors.[12,19] Though the exact amount of quenching is more or less model dependent, it has been shown[12,19] that it affects $\langle r^2\rangle_\pi$ and $\langle r^2\rangle_A$ exactly in the same proportion. In summary, the inequality (13) seems to be well supported in which case the bound (5) follows.

An application of the bound (5) to muon capture on ^{12}C, Eq. (2), combined with the analyses of Refs. 4 and 5 indicates that the reported value for the induced pseudotensor form factor,[3] Eq. (1), is in conflict with theoretical expectations. In fact one would expect that $g_T(0)/Ag_A(0)$ be positive instead of negative. Nevertheless, this conclusion should be handled with some care because the form-factor analyses[4,5] have been performed at the 1-standard-deviation level. An increase in the capture rate and/or the ^{12}B recoil polarization by more than 2 standard deviations could bring (1) into agreement with our bound for $\tilde{f}_p$.

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