# Radiative Muon Capture in Calcium

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The branching ratio relative to ordinary muon capture and the photon asymmetry relative to the muon-spin direction were measured for radiative muon capture in $^{40}$Ca. For ~1200 photon events, the partial branching ratio is $R_{k>57\ \mathrm{MeV}} = (21.1 \pm 1.4)\times 10^{-6}$, and the asymmetry for $k > 63.5$ MeV is $+0.90 \pm 0.50$. A fit of the photon spectrum to the theory of Rood, Yano, and Yano gives the value $g_P{}^{\mu} = (6.5 \pm 1.6) g_A$ for the pseudoscalar-coupling constant. These results are in disagreement with earlier experiments.

In weak-interaction theory, the rate for radiative muon capture (RMC),

$$\mu^- + p \rightarrow n + \nu + \gamma, \tag{1}$$

is sensitive to the induced pseudoscalar-coupling constant $g_P{}^{\mu}$ in the weak hadron current.[1] Also, it was predicted by Cutkosky[2] that the asymmetry $\alpha_\gamma$ of the emitted photon relative to the muon spin has the value $+1.0$ when only the muon radiating diagram is considered. Fearing[3] showed that other diagrams contribute corrections to $\alpha_\gamma$ of order $(m_\mu/m_N)^2$, $m_\mu$ and $m_N$ being, respectively, the muon and nucleon masses, so that the asymmetry should have a value not much less than unity.

The rarity of process (1) has thus far limited experiments to complex nuclei, e.g.,

$$\mu^- + {}^{40}\mathrm{Ca} \rightarrow {}^{40}\mathrm{K}^* + \nu + \gamma, \tag{2}$$

which is the process investigated in the experiment described here. In the theoretical analysis of process (2) by Rood, Yano, and Yano[4] (hereafter referred to as RYY), harmonic-oscillator wave functions were used to describe the nuclear states, and the closure approximation was used to reduce the matrix elements. The branching ratio $R$ of RMC to ordinary muon capture was rather insensitive to choice of nuclear model. RYY showed that a change in the value[5] of $g_P{}^{\mu}$ from $+7g_A$ to $+14g_A$ should increase $R_{k>57\ \mathrm{MeV}}$ by ~50% in $^{40}$Ca. For $g_P{}^{\mu} \simeq 7g_A$, which is the Goldberger-Treiman value,[6] RYY predicted that the photon asymmetry varies slightly with energy, decreasing from $+0.83$ at $k = 0.5k_{\max}$, to $+0.74$ at $k = 0.9k_{\max}$, where $k_{\max}$ is the maximum photon momentum averaged over final states.

The results of previous experiments have not been in good agreement with predictions. Conversi, Diebold, and di Lella[7] detected about 430 RMC photons from calcium, and obtained $R = (3.1 \pm 0.6)\times 10^{-4}$ and $g_P{}^{\mu} = (13.3 \pm 2.7) g_A$ by fitting their observed photon spectrum above 55 MeV to the then-current theory[8] and extrapolating to low energies. In the most recent published RMC experiment di Lella, Hammerman, and Rosenstein,[9] using a technique similar to that of Conversi, Diebold, and di Lella,[7] obtained values of $R = (1.14 \pm 0.09)\times 10^{-4}$ and $g_P{}^{\mu} = -5.9 g_A$ from a fit to the theory of Ref. 8. di Lella, Hammerman, and Rosenstein[9] identified a large background in the NaI crystal which apparently came from high-energy neutrons emitted in *ordinary* muon capture, and which they interpreted as explaining the higher value of $R$ obtained in Ref. 7. di Lella, Hammerman, and Rosenstein[9] also made the first measurement of the photon asymmetry $\alpha_\gamma$, and obtained $\alpha_\gamma \leq -0.32 \pm 0.48$, although theory predicts a positive result.[2-4]

The unsettled experimental situation led us to perform a new measurement of RMC in $^{40}$Ca with a technique which is insensitive to high-energy neutrons. The experimental geometry is shown in Fig. 1. The 85-MeV/$c$ "backward" muon beam at the Space Radiation Effects Laboratory (SREL)[10] was stopped in a calcium target measuring $13 \times 18 \times 2.5$ cm$^3$, the last dimension being along the beam direction. An acceptable muon stop was signified by $(1\cdot 2\cdot \overline{C}1\cdot 3\cdot \overline{4})$ with the further requirement that no other $(1\cdot 2)$ coincidence occurred within $\pm 1$ $\mu$sec of the muon-stop signal. This requirement reduced the average muon-stop rate from $\approx 30\,000$ sec$^{-1}$ to a "clean" stop rate of $18\,000$ sec$^{-1}$. The photon detector consisted of scintillation counters 5 and 6, a Pb converter 0.3 cm thick, scintillator 7, Čerenkov counter $C2$, scintillator 8, and a 25-cm-diam × 25-cm-thick NaI detector. The signature for a photon was $(\overline{1} + \overline{2} + \overline{3} + \overline{5} + \overline{6})(7\cdot C2\cdot 8\cdot \mathrm{NaI})$, and for a decay elec-

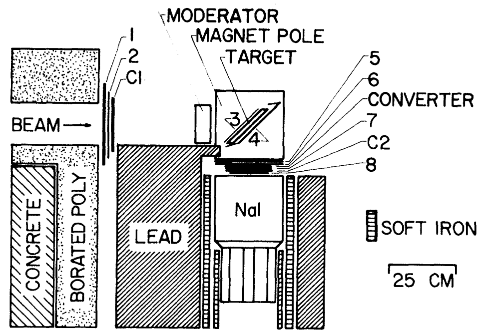


Fig. 1. Experimental arrangement. Counters 1–8 are scintillators; $C1$ and $C2$ are plastic Čerenkov counters.

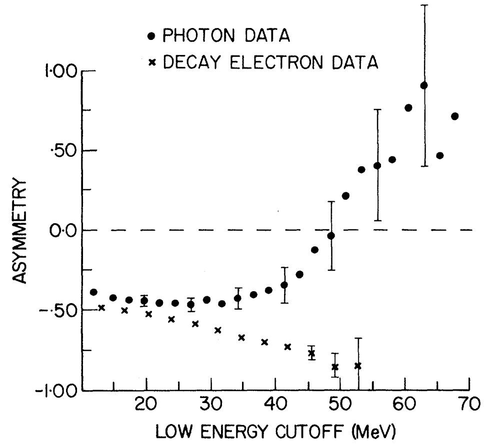


Fig. 2. Photon and electron asymmetries relative to muon-spin direction vs low-energy cutoff in NaI detector. The asymmetry for a given low-energy cutoff includes all the data for higher cutoffs.

tron $(\bar{1}+\bar{2}+\bar{3})(5\cdot 6\cdot 7\cdot C2\cdot 8\cdot \mathrm{NaI})$. The use of the external Pb converter and the Čerenkov counter $C2$ suppressed background neutrons.

Photon and electron events were timed relative to muon-stop signals using a time-to-amplitude converter (TAC) which was gated to allow observation of events following a stopped muon by 0–800 nsec ("foreground") or preceding the stopped muon by 0–500 nsec ("background"). A transverse (horizontal) field[11] of ~ 540 G throughout the target produced muon precession to allow measurement of the asymmetries of decay electrons and RMC photons with respect to the initial muon spin.

The resolution and energy calibration for the NaI crystal were determined from the prompt-photon spectrum produced by negative-pion capture in LiH and Li. The difference between these two spectra, corrected for target thickness, is produced by the pion processes in hydrogen $\pi^- p \to n\gamma$ and $\pi^- p \to n\pi^0 \to n\gamma\gamma$. A Monte Carlo calculation determined the absolute efficiency of the photon detector. The result agreed with the experiment on LiH-Li of Chabre *et al.*[12]

The intensity versus time of *electron* events in the interval between 100–800 nsec following the muon-stop signal was fitted to the form

$$I_e(t) = A\exp(-t/\tau_{\mathrm{Ca}})[1+S_{\mathrm{Ca}}\sin(\omega t+\varphi)] + B\exp(-t/\tau_{\mathrm{C}})[1+S_{\mathrm{sc}}\sin(\omega t+\varphi)] + G,$$

where $A$ and $B$ are the amplitudes for decay electrons originating in the calcium target and carbon (in scintillation counters), respectively; $\tau_{\mathrm{Ca}}$ and $\tau_{\mathrm{C}} = 2034$ nsec are the muon lifetimes in calcium and carbon[13]; $S_{\mathrm{Ca}}$ and $S_{\mathrm{sc}}$ are the effective asymmetries in Ca and scintillator, respectively ($S_{\mathrm{Ca}}/S_{\mathrm{sc}} \sim 2.2$ experimentally[14]); $G$ is the background. The quantities $A$, $B$, $S_{\mathrm{Ca}}$, $\omega$, $\varphi$, and $\tau_{\mathrm{Ca}}$ were free parameters in the analysis, while $G$ was determined from the negative time background.

For all electron energies above ~ 5 MeV, the analysis yields $B/A \approx 2.3\%$, $G/A \approx 7.9\%$, $\tau_{\mathrm{Ca}} = 365 \pm 8$ nsec, the last value being in some disagreement with previous results.[15] The asymmetry amplitude $S_{\mathrm{Ca}} = PD_{\mathrm{Ca}}\alpha_e$, where $P$ is the beam polarization ($\approx 0.62$), $D_{\mathrm{Ca}}$ is the depolarization factor for negative muons in calcium, and $\alpha_e$ is the decay-electron asymmetry parameter. By varying the low-energy cutoff in the analysis, we obtained the dependence of $S_{\mathrm{Ca}}$ on energy, and found it to be consistent with a Monte Carlo calculation of $\alpha_e$, with[14] $D_{\mathrm{Ca}} \approx 1/6$ and with the spectrum of decay electrons from negative muons bound in calcium of Johnson, O'Connell, and Mullin.[16]

Figure 2 shows the measured asymmetries versus low-energy cutoff. The photon asymmetry $\alpha_\gamma$ approaches the electron asymmetry $\alpha_e$ at lower energies, where external bremsstrahlung produced by decay electrons predominates, but changes sign at higher energies as the main source of photons becomes RMC. Taking all photon energies $k$ in the range 63.5 MeV $< k <$ 82 MeV, we find $\alpha_\gamma = +0.90 \pm 0.50$, in agreement with theory, but in disagreement with the value found by di Lella, Hammerman, and Rosenstein.[9]

The branching ratio $R = \lambda_{\mathrm{RMC}}/\lambda_{\mathrm{cap}}$ vs photon energy was determined from the number of RMC photons per MeV compared to the rate of emission of decay electrons from Ca, with use of the relation $1/\tau_{Ca} = \lambda_{cap} + \lambda_{decay}$, where $\lambda_{cap}$ and $\lambda_{decay}$ are the capture and decay rates in calcium. The electrons were detected in the same geometry used for the RMC photons. The electron-detection efficiency was obtained by sequentially removing various components of the counter array, extrapolating to zero target thickness (a 20% effect), and correcting for the part of the electron spectrum suppressed by the NaI-counter threshold (a 5% effect).

Reconstruction of the "true" RMC photon spectrum from the observed photon spectrum required unfolding the photon resolution function for the detection array from the observed photon spectrum $N^{ob}(E_i) \propto \sum_j T(E_i, E_j) N^t(E_j)$, where $T(E_i, E_j)$ is the resolution function, i.e., the probability that a photon of energy $E_j$ is detected in the pulse-height bin at $E_i$, and $N^t(E_j)$ is the true photon intensity at energy $E_j$. Inversion of the resolution matrix $T(E_i, E_j)$ by means of a standard computer algorithm failed to converge because of the similarity of adjacent columns of $T(E_i, E_j)$. Therefore an alternate approach was used to extract $N^t(E_j)$. The true spectrum $N^t(E_j)$ was assumed to be a power series in $E_j$ for $E_j > 57$ MeV, and a least-squares fit to be observed spectrum was obtained. A two-parameter fit of the form $N^t(E) = aE^{-2} + bE^{-3}$ gave a normalized $\chi^2 = 1.02$, and the result for $1229 \pm 44$ RMC events above 57 MeV was $R_{k>57\ \mathrm{MeV}} = (21.1 \pm 1.4) \times 10^{-6}$.

The discrepancy between the muon lifetime in calcium as measured by us and previous groups[15] was investigated carefully. Possible drift or calibration errors of the TAC system were sought, but none was found. We therefore used our value of $\tau_{Ca}$ to calculate the muon-capture rate $\lambda_{Cap}$ which enters directly into the value of $R$. Use of $\tau_{Ca} \sim 340$ nsec would lower the measured $R$ by $\sim 8\%$, but would give a considerably poorer fit to our decay-electron time distributions.

To compare our result for $R$ with that of Ref. 9, it is necessary to use the form of theory[8] which was fitted to the observed spectrum in that reference, since the "true" spectrum $N^t(E)$ (with resolution removed) was not given there. From Ref. 9 we extract the result $R_{k>57\ \mathrm{MeV}} = 15.4 \times 10^{-6}$, whereas fitting our data to the same theory[8] gives $R_{k>57\ \mathrm{MeV}} = (20.0 \pm 1.4) \times 10^{-6}$

The "true" differential photon spectrum $N^t(E)$ as obtained in our experiment for $k > 57$ MeV is given vs $k$ in Table I. The observed data were fitted to the theory of RYY, with $g_P^{\mu}$ and $k_{max}$, the maximum photon momentum averaged over final states, as free parameters. The average neutrino momentum $\nu_{av}$ was fixed[4] at $k_{max}/1.02$. All coupling constants other than $g_P^{\mu}$ were assigned standard values. The fit gave $g_P^{\mu} = (6.5 \pm 1.6) g_A$, and $k_{max} = 86.5 \pm 1.9$ MeV. Thus for both the asymmetry and the branching ratio we do not confirm the results of Refs. 7 and 9, but instead obtain agreement with the theory of RYY with the expected value[6] for $g_P^{\mu}$.

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TABLE I. Differential photon spectrum $N^t(E)$ for radiative muon capture in calcium, relative to ordinary muon capture. This is the "true" spectrum, with instrumental resolution removed, as described in the text.

| Photon energy (MeV) | $10^6 N^t(E)$ (photons/capture MeV) |
|---|---|
| 57 | $1.92 \pm 0.16$ |
| 60 | $1.50 \pm 0.12$ |
| 65 | $0.98 \pm 0.07$ |
| 70 | $0.63 \pm 0.04$ |
| 75 | $0.39 \pm 0.03$ |
| 80 | $0.21 \pm 0.02$ |
| 85 | $0.09 \pm 0.02$ |
| 90 | $0.004 \pm 0.03$ |

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# Ponderomotive Force and Linear Susceptibility in Vlasov Plasma

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We have obtained an exact, simple, and general relation between the second-order Hamiltonian, representing the ponderomotive force on the oscillation center of a particle in a high-frequency field, and the standard linear Vlasov susceptibility.

The concept of ponderomotive force has been extremely useful in interpreting the nonlinear interaction of radiation with plasma. A survey of early work,[1] on the quasistatic force caused by a high-frequency field, was oriented toward the question of rf confinement of plasma. More recently, this concept has been applied to the parametric instabilities caused by radiation in unmagnetized[2] and magnetized[3-5] plasmas.

Since the ponderomotive force acts on the oscillation center of a particle, a systematic treatment for Vlasov plasma can be based on the Hamiltonian of an oscillation center. A canonical formalism for the latter was first developed by Dewar, to establish a rigorous theory of quasilinear diffusion.[7] It was then used by Johnston to study induced scattering[8] of waves in magnetized plasma, greatly simplifying the standard[9] Vlasov derivation; and by Johnston and Kaufman for the three-wave interaction[10] in a nonuniform magnetized Vlasov plasma.

We have used Lie transforms[11] to investigate a number of nonlinear problems; the details will be reported in a longer publication.[12] In this Letter we present the result for the Hamiltonian of an oscillation center in a (possibly electromagnetic) field of fixed frequency $\omega$:

$$\vec{E}(\vec{x}, t) = \vec{E}_\omega(\vec{x})e^{-i\omega t} + \text{c.c.}, \tag{1}$$

superimposed on a quasistatic (possibly nonuniform) magnetic and electric field. This Hamiltonian $K(\vec{R}, \vec{P})$ turns out to be expressible in terms of a Poisson bracket,[10,13] which we recognize as the same Poisson bracket that appears in the generalized expression[14] for the *linear* Vlasov susceptibility $\overleftrightarrow{\chi}_\omega{}^s(\vec{x}, \vec{x}')$. The latter is defined by the relation

$$(4\pi/i\omega)\langle\vec{j}\rangle_\omega{}^s(\vec{x}) = \int d^3x'\, \overleftrightarrow{\chi}_\omega{}^s(\vec{x}, \vec{x}')\cdot\vec{E}_\omega(\vec{x}'), \tag{2}$$

where $\langle\vec{j}\rangle$ is the Vlasov current density, and $s$ is the species label.

The relation obtained is

$$\int d\Gamma f(\Gamma) K_2(\Gamma) = -(4\pi)^{-1}\int d^3x \int d^3x'\, \vec{E}_\omega{}^*(\vec{x})\cdot\overleftrightarrow{\chi}_\omega{}'(\vec{x}, \vec{x}')\cdot\vec{E}_\omega(\vec{x}'), \tag{3}$$

where $\Gamma$ denotes $\vec{R}, \vec{P}$ (the oscillation-center variables), $f$ is the quasistatic phase-space density, $K_2$ is the quadratic (in $\vec{E}_\omega$) part of the Hamiltonian, and the prime on $\overleftrightarrow{\chi}$ represents Hermitian (reactive) part. Now the linear susceptibility $\overleftrightarrow{\chi}$ has been intensively studied in many problems of interest. Hence