Three-Body Mechanism for Narrow Resonances

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A new three-body mechanism is proposed, which leads naturally to narrow states at energies easily calculable via a simple analytic formula. This (zero-parameter) formula predicts over a score of narrow resonances in remarkable coincidence with experiment.

Before recent events intensified theoretical interest in the quark model, a number of attempts were made to directly link two- and three-body resonances.[1,2] Despite some notable successes,[3] it would appear that this approach can "explain" only a limited fraction of the hadron spectroscopy, and is intrinsically incapable of generating *narrow* states (say, $\Gamma < 30$ MeV). However, in this Letter I describe a new type of three-body mechanism which leads naturally to narrow states, at energies trivially calculable in terms of a simple analytic formula. Remarkably, this (zero-parameter) formula predicts over a score of narrow resonances in surprising coincidence with the hadron spectroscopy. Moreover, some unusual properties associated with this mechanism could resolve some long-standing difficulties in this field.

The proposed mechanism arises from a singularity in the diagram shown in Fig. 1. This singularity corresponds to the diagram being realized as an *on-shell sequential rescattering;* i.e., to k_1', p_2, and k_3 describing a physical intermediate state of invariant (three-body) energy $\sqrt{s}$. The situation in which the subenergies s_{12}, s_{23}' coincide with resonant energies of the two-body subsystems has been discussed previously,[2] and gives rise to a singularity first noted by Peierls.[1] The situation here is distinct in that, while particles 1 and 2 are assumed to resonate at energy s_{12}^0, s_{23}' is taken near the subenergy threshold; i.e., $\sqrt{s_{23}'} \simeq m_2' + m_3'$.

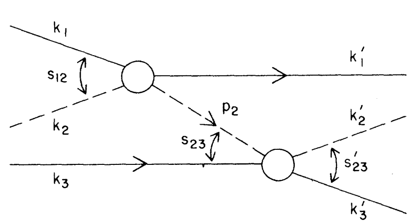

FIG. 1. Rescattering diagram which generates the singularity discussed in the text. The vertex blobs correspond to off-shell scattering amplitudes.

Although one could proceed along the lines of Ref. 2 in deriving the result, it is simpler to argue as follows. Thus, consider the diagram as a function of s_{23}', for fixed s and $\cos\theta_{k_3 k_1'} = -1$. It is apparent that the corresponding amplitude develops a pole (arising from an intermediate-state propagator) at that value of s_{23}' which satisfies the on-shell conditions

$$(k_1' + p_2)^2 = s_{12}^0;$$
$$(p_2 + k_3)^2 = s_{23}'; \quad (k_1' + p_2 + k_3)^2 = s, \tag{1}$$

for a mass-shell intermediate state ($k_1'^2 = m_1'^2$, $p_2^2 = m_2^2$, $k_3^2 = m_3^2$). Note that this is true regardless of whether the state described by k_1, k_2, k_3 is on-shell or whether $s_{12} = s_{12}^0$. The singular value of s_{23}' (s_{23}^0) will be near threshold providing that s is near the critical value

$$s_c = (m_2 + m_3)^2 + m_1'^2 + (m_2 + m_3)(s_{12}^0 - m_1'^2 - m_2^2)/m_2, \tag{2}$$

which is obtained by solving Eq. (1) for s with particles 2 and 3 at rest.

In practice, s_{12}^0 is *complex* (since the input resonance is taken to have its physical width) and $s = s_c$ will not be achieved for physical values of s. However, in certain cases (depending on s_{12} and the mass ratios) s_{23}^0 passes much closer to the subenergy threshold than one might suspect. As an example consider the $NN\pi$ system, taking particle 2 to be the pion, and $\sqrt{s_{12}^0}$ to be the complex mass ($\Gamma/2 = 50$ MeV) of the $\Delta(1236)$. This situation is illustrated in Fig. 2, where the dashed line shows the location of s_{23}^0 in terms of the kinetic energy $T = \sqrt{s_{23}^0} - (m_2' + m_3')$. Note that its position varies rapidly with s, and is only 3.6 MeV below threshold for $\sqrt{s} = \sqrt{s_c} = 2640$ MeV.

In order to see how this subenergy effect leads to a singularity in s, consider the consequences of unitarity. For a three-body system it is ad-

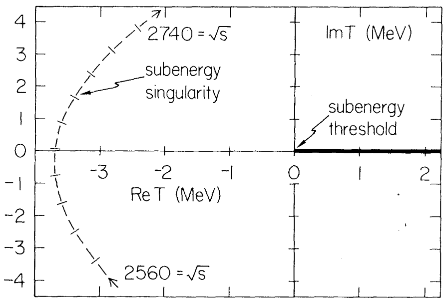

FIG. 2. Location of subenergy singularity in the kinetic energy (T) for the $N\bar{N}\pi$ system (dashed line) as a function of s (ticks indicate 20-MeV intervals of $\sqrt{s}$).

vantageous to expand the full amplitude T_3 in the form $T_3 = \sum_{\alpha,\alpha'} \tau_{\alpha\alpha'}$, where α (α') labels the pair of particles interacting first (last). The unitarity relation for the $\tau_{\alpha\alpha'}$ (channel amplitudes) takes the form $\Delta\tau_{\alpha\alpha'} = -\sum_\beta \tau_{\alpha\beta}\Delta G_0(s)\tau_{\beta\alpha'}{}^*$, where $G_0(s)$ is a suitable (mass-shell) propagator; i.e., $\Delta G_0(s) \propto \delta(\epsilon_1 + \epsilon_2 + \epsilon_3 - \sqrt{s})$ in the center-of-mass system. In a partial-wave decomposition $\tau_{\alpha\alpha'}$ depends only on s and the subenergies $s_{\beta\gamma}$, $s_{\beta'\gamma'}$ ($\alpha \neq \beta \neq \gamma$, $\alpha' \neq \beta' \neq \gamma'$). Thus, for a subenergy pole in channel $\beta = 1$, the corresponding contribution to $\Delta\tau_{\alpha\alpha'}$ takes the form

$$\Delta^{(1)}\tau_{\alpha\alpha'}(s_{\beta\gamma}, s_{\beta'\gamma'}) \simeq -2\pi i c_1{}^2 \tilde{\tau}_{\alpha 1}(s_{\beta\gamma})\tilde{\tau}_{1\alpha'}{}^*(s_{\beta'\gamma'}) I_1, \tag{3}$$
$$I_1 = \int_0^{\kappa_1{}^m} d\kappa_1 \kappa_1{}^2/(\kappa_1{}^2 - \kappa_0{}^2)(\kappa_1{}^2 - \kappa_0{}^{*2}).$$

Here I have taken the (pair) c.m. momentum (κ_1) as the variable and incorporated slowly varying kinematic factors into a constant, $c_1{}^2$; $\tilde{\tau}_{\alpha 1}(s_{\beta\gamma})$ is the residue of $\tau_{\alpha 1}(s_{\beta\gamma}, s_{23}')$ at the pole ($s_{23}' = s_{23}{}^0$). Ignoring the upper limit ($\kappa_1{}^m \gg \kappa_0$), I have $I_1 = (\pi/4)|\,\mathrm{Im}\kappa_0|^{-1}$, depending solely on s.

Thus I have shown that $\mathrm{Im}\tau_{\alpha\alpha'}$ contains a term proportional to $|\,\mathrm{Im}\kappa_0|^{-1}$. However, for the typical situation illustrated in Fig. 2, $\mathrm{Im}\kappa_0$ actually varies rather slowly with s. The circumstance which promotes this singularity into a significant effect is that I need *not* have $m_2' = m_2$, $m_3' = m_3$. For example, if the circle at the (23) vertex represents $n\pi^+ \to p\pi^0$ the threshold in T is shifted by 5.9 MeV to the left in Fig. 2; for $p\pi^- \to n\pi^0$ the shift is 3.3 MeV. Thus, the close proximity to threshold permits a final charge exchange to shift the singularity into, or very near, the physical region; correspondingly, the "width" associated with the effect is essentially zero. In the $NN\pi$ example, this implies that $\mathrm{Im}\tau_{\alpha\alpha'}$ is sharply peaked at $\sqrt{s} \simeq 2640$ MeV. Although I have cheated slightly in taking $\cos\theta_{k_3k_1'} \simeq -1$ in the partial-wave argument (one must integrate over $\theta_{k_3k_1'}$), a careful treatment (involving logarithms in I_1) yields substantially the same result, the net effect being to broaden the s dependence very slightly (to the order of several MeV).

Given a production mechanism which emphasizes the geometry of Fig. 1, one would thus expect to see a sharp peak in the corresponding differential cross section. In addition, a true resonance may develop in one or more partial waves. Consider, for example, a two-channel model in which channel 1 corresponds to the pair at threshold, and channel 2 to the pair at resonance. In the zero-width approximation, the $\beta = 2$ contribution to $\Delta\tau_{\alpha\alpha'}$ takes on a form similar to that of Eq. (3), but with I_2 essentially equal to the c.m. momentum of the resonance-spectator system. In the spirit of the isobar model, one may factor out the subenergy singularities from $\tau_{\alpha\alpha'}$ and work with reduced (isobar) amplitudes $t_{\alpha\alpha'}$. The latter permit the simple unitary representation $t_{\alpha\alpha'} = N_{\alpha\alpha'}/D$, where

$$N_{\alpha\alpha'} = \lambda_{\alpha\alpha'}(1 - \delta_{\alpha\alpha'}\gamma\rho_\beta), \quad \beta \neq \alpha;$$
$$D = 1 - \rho_1 - \rho_2 + \gamma\rho_1\rho_2, \tag{4}$$

and $\gamma = 1 - \lambda_{12}\lambda_{21}/\lambda_{11}\lambda_{22}$. Here ρ_α is given by a dispersion integral with $\mathrm{Im}\rho_\alpha = \pi\lambda_{\alpha\alpha}I_\alpha$; the functions $\lambda_{\alpha\alpha'}(s)$ have only left-hand cuts in s. Empirically, an exact (numerical) treatment shows that $I_1 \simeq [(\sqrt{s} - \sqrt{s_c})^2 + \mu^2]^{-1}$, where μ depends chiefly on the mass difference $(m_2 + m_3) - (m_2' + m_3')$. For the $NN\pi$ example, $\mu = 2$ MeV for $\pi^+ n \to \pi^0 p$, and $\mu = 22$ MeV for $\pi^- p \to \pi^0 n$. One may then evaluate ρ_1 analytically (near s_c), and verify that both $\mathrm{Re}\rho_1$ and $\mathrm{Im}\rho_1$ are rapidly varying functions for real $s \simeq s_c$, and that $\rho_1 \to 2\pi i\lambda_{11}I_1$ as $\sqrt{s} \to \sqrt{s_c} - i\mu$.

Observing that a resonance pole corresponds to a complex zero of D, and that $D \propto 1 - \lambda\rho_1$, with $\lambda = (1 - \gamma\rho_2)/(1 - \rho_2)$, inspection of Eq. (4) leads to the following conclusions: (1) If λ/μ is small, a pole will develop near $\sqrt{s} = \sqrt{s_c} - i\mu$; however, the amplitudes $t_{\alpha\alpha'}$ (or T_3) will *not* exhibit simple Breit-Wigner behavior (due to the cut in s) as s varies over physical values; (2) as $|\lambda|/\mu$ increases toward unity, the pole position will be shifted (e.g., for the $NN\pi$ system with $\lambda = \mu = 2$ MeV, numerical studies give $M_{\mathrm{res}} = 2660$ MeV, $\Gamma_{\mathrm{res}} = 40$ MeV); (3) for $|\lambda| \gg \mu$ there will be no associated pole; (4) whether or not a pole is generated, it is quite possible to see a sharp peak in the elastic coupled-channel reaction $(2 \to 2)$ arising from the presence of ρ_1 in N_{22}, without a comparable peak in $(1\to 1)$. One would expect these qualitative aspects to survive in a more rigorous treatment at the three-body level, and suitably generalize to a situation with other types of inelastic channels.

It is thus reasonable to anticipate a resonance near $s = s_c$ in those three-hadron systems for which $s_{23}^{\,0}$ lies in or near the physical region (*and* in coupled inelastic channels). Furthermore, since the properties of the (12) resonance are presumed known, and the (23) system must be in an s wave to take full advantage of the process illustrated in Fig. 1, it is possible to predict the quantum numbers of the effect with reasonable accuracy. Now consider a variety of experimental evidence in support of these notions, beginning with the $N\bar{N}\pi$ system produced in $\pi^- p \to p_F(p\bar{p}\pi^-)$. From the above, one would expect a narrow resonance in $p\bar{p}\pi^-$ near 2640 MeV. In fact, this work was originally motivated by the apparent observation of such a state at 2660 MeV ($\Gamma < 20$ MeV) in a recent experiment at Stanford Linear Accelerator Center involving π^- on deuterium (preliminary results were reported by Rogers *et al.*[4]). A detailed analysis of this experiment is well underway, and it now appears certain that virtually all significant features of the data can be well understood on the basis of this mechanism (including a 5π decay mode via $p\bar{p} \to 4\pi$, especially rapid t dependence relative to background, a strong preference for backward Jackson angles, a marked tendency of $p\bar{p}$ to emerge back-to-back, etc.). Thus, the data appear quite consistent with the predictions $JPG = 1+-$, $I = 1$ or 2.

In view of its simplicity, one might expect that the general application of Eq. (2) would generate a multitude of spurious predictions, in which case the 2660 state could be dismissed as coincidence. However, if we restrict ourselves to stable particles (pseudoscalar nonet, baryon octet) and well-established resonances, the corresponding predictions are actually in remarkable agreement with experiment; there are *no* obvious contradictions. A simple calculation (including finite-width corrections) yields the mass values displayed in Table I[5]; the following points should be emphasized:

(A) *The 1^+ nonet.*—It appears significant that the *only* states which arise from purely mesonic systems correspond precisely to the 1^+ mesons, with masses in excellent agreement with either the established values (D,E) or the sharp (lower-mass) edge of the associated peaks (A,Q). In the absence of spin one can make virtually unique predictions, and verify both the 1^+ character and the correct isospin and hypercharge assignments. Moreover, the unusual analytic properties discussed subsequent to Eq. (4) may explain some of the difficulties experienced in analyzing related experiments.[8] Note that recent nondiffractive experiments indicate a narrower, lower-mass (1050 MeV) A_1 than has been obtained in diffractive analyses.[9]

(B) *The $N\bar{N}\pi$ system.*—It is amusing that higher N and Δ resonances in place of $\Delta(1236)$ generate a mass sequence in striking correspondence with the χ states observed in $\psi(3700)$ decays, and the

TABLE I. Mass predictions for meson states.

A. Mesonic Systems				B. $N\bar{N}\pi$ System		
System	Input Resonance	Predicted Mass	Experiment	$N\pi$ State	Predicted Mass	Experiment
3π	$\rho(770)$	1095	$A_1(1100)$	$\Delta(1236)$	2640	2660[a]
$K\pi\pi$	$K^*(892)$	1180	$Q_{1,2}(1200\text{–}1400)$	N(1470)	3440	$\chi(3415)$[b]
$K\bar{K}\pi$	$K^*(892)$	1430	E(1420)	N(1470)	3440	$\chi(3455)$[b]
$\eta\pi\pi$	$\delta(970)$	1270	D(1285)	N(1535), (1500)*	3650, (3540)*	$\chi(3510)$[b]
				N(1520), (1510)*	3600, (3570)*	$\chi(3550)$[b]
				$\Delta(1650)$	4020	$\psi(4100)$ region
C. $S\bar{S}M$ Systems (S=0)				$\Delta(1670)$	4090	
System	Input Resonance	Predicted Mass	Experiment	N(1670)	4090	
				N(1688)	4150	
$\bar{N}K\Sigma$	$\Lambda(1520)$	2790	2800, 2820	N(1700)	4180	
$\bar{\Sigma}\pi\Sigma$	$\Lambda(1405)$	2870	2850[c]	N(1780)	4430	$\psi(4400)$ region
$\bar{\Sigma}\pi\Sigma$	$\Sigma(1385)$	3055	3050[c]	N(1810)	4520	

[a]Ref. 4. [b]Ref. 6. [c]Ref. 7.

spikes seen (e.g., at 4030, 4100, 4410) in the ratio $R(e^+e^- \to \text{hadrons}/\mu^+\mu^-)$,[6] especially if one slightly shifts the $N(1500)$ masses within experimental uncertainties. Of course, such an identification is highly speculative, especially since the mechanism need not always produce an observable effect. However, it is interesting that a noncharm mechanism might also be contributing to this complicated region.

(C) *$S\bar{S}M$ systems.*—Using the relatively elastic (lower mass) strange resonances, three additional strangeness $S=0$ states are predicted; in each case there is a strong experimental candidate.

If one considers the $\bar{N}\pi\Sigma$ system, the $\Lambda(1405)$ resonance in $\Sigma\pi$ generates a state with $S=1$ at 2600 MeV, which may already have been seen,[10] whereas the $N\pi$ resonances generate a sequence of $S=1$ states comparable to those in (B); e.g., $\Delta(1236)$, $N(1470)$, $N(1520)$,... lead to effects at 2955, 3860, 4020,... (no experimental evidence at present). Additional possibilities include a possible $N\bar{N}$ "bootstrap," via an $N\bar{N}\pi$ system with a narrow $N\bar{N}$ resonance at the (12) vertex (e.g., input states at 1890, 2020, 2190 MeV generate effects near 2030, 2170, 2350 MeV, respectively; all are seen experimentally[11]). Finally, some of the N, Δ states employed as input may themselves be generated via the $N\pi\pi$ system; e.g., taking the $\Delta(1236)$ with particle 3 a pion in Fig. 1 yields a mass of 1488 MeV, which may correspond to the $N(1470)$ and explain its odd properties. Taken in conjunction with more conventional three-body calculations[3] these results suggest that the hadron spectrum may be largely generated by threshold conditions involving observed physical hadrons. This would mean that multiquark forces dominate, as opposed to the conventional, pair-wise models.

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[1]G. F. Chew, Phys. Rev. Lett. 4, 142 (1960); R. F. Peierls, Phys. Rev. Lett. 6, 641 (1961).

[2]D. D. Brayshaw and R. F. Peierls, Phys. Rev. 177, 2539 (1969).

[3]R. Aaron and R. D. Amado, Phys. Rev. Lett. 27, 1316 (1971); D. D. Brayshaw, Phys. Rev. D 11, 2583 (1975).

[4]A. Rogers *et al.* (Purdue-Indiana-Vanderbilt Collaboration at Stanford Linear Accelerator Center). Recent experimental results have been taken from Proceedings of the Fifth International Conference on Experimental Meson Spectroscopy, Northeastern University, Boston, Massachusetts, 29–30 April 1977 (to be published).

[5]Unless otherwise noted, experimental references are to be taken from T. G. Trippe *et al.*, Rev. Mod. Phys. 48, No. 2, Pt. 2, S51 (1976).

[6]The experimental situation is reviewed in SLAC Report No. SLAC-198, 1976 (unpublished).

[7]H. Braun *et al.*, Phys. Lett. 60B, 481 (1976).

[8]For a discussion of the pitfalls related to neglecting a proper analytic representation, see D. D. Brayshaw, Phys. Rev. Lett. 37, 1329 (1976).

[9]New experimental results, and diffractive analyses were summarized, respectively, by R. Hemingway and R. Longacre, in Proceedings of the Fifth International Conference on Experimental Meson Spectroscopy, Northeastern University, Boston, Massachusetts, 29–30 April 1977 (to be published).

[10]A. Apostolakis *et al.*, Phys. Lett. 66B, 185 (1977).

[11]The status of $N\bar{N}$ resonances was summarized by L. Montanet, in Proceedings of the Fifth International Conference on Experimental Meson Spectroscopy, Northeastern University, Boston, Massachusetts, 29–30 April 1977 (to be published).