London 88, 563 (1966).
[8]G. Baum *et al.*, Phys. Rev. A 5, 1073 (1972); U. Heinzmann *et al.*, Z. Phys. 240, 42 (1970).
[9]P. F. Wainwright *et al.*, to be published.
[10]See, for example, V. W. Hughes *et al.*, Phys. Rev. A 5, 195 (1972).
[11]V. I. Ochkur, Zh. Eksp. Teor. Fiz. 47, 1746 (1964) [Sov. Phys. JETP 20, 1175 (1965)].
[12]D. F. Gallaher, J. Phys. B 7, 362 (1974).
[13]H. Klar and W. Schlecht, J. Phys. B 9, 1699 (1976).

Upper-Hybrid-Resonance Absorption of Laser Radiation in a Magnetized Plasma

C. Grebogi, C. S. Liu, and V. K. Tripathi
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 19 May 1977; revised manuscript received 18 July 1977)

A magnetic field in a laser-irradiated plasma is shown to have an important effect on the resonant absorption of the laser radiation normally incident on the inhomogeneous plasma. For typical paramaters in laser-fusion experiments, the absorption coefficient is above 50% with $\alpha \equiv \pi^2(\omega_c/\omega)^2(L\omega/c)^{4/3}$ between 1.5 and 10 and a maximum of $\sim 70\%$ is achieved for $\alpha = 4$.

In laser-pellet fusion experiments, a magnetic field of a few megagauss is generated near the critical surface by a variety of sources such as $\nabla n \times \nabla T$, ponderomotive force, etc.[1-5] This self-generated magnetic field has been observed experimentally[6,7] and in computer simulations. Because the magnetized plasma can support an upper hybrid wave, linear conversion of the laser radiation into this wave constitutes an anomalous absorption which has also been explored in numerical simulations.[8] In this Letter we present an analytic theory of linear wave transformation whereby the normally incident laser radiation is converted into an upper hybrid wave at the resonance layer. Significant absorption with absorption coefficient $\gtrsim 10\%$ is attained for the parameter $\alpha = \pi^2(\omega_c/\omega)^2(L\omega/c)^{4/3}$ in the range $0.2 < \alpha < 75$. Here α/π^2 is the square of the ratio of the distance between the cutoff and the resonance layers $x_0 = L\omega_c/\omega$ and the scale of variation of the electromagnetic wave near the cutoff $x_{\rm em} = (Lc^2/\omega^2)^{1/3}$, L is the density scale length, $\omega_c = eB/mc$, and ω is the laser frequency. Absorption of 67% is achieved for $\alpha = 4$ and a broad maximum (>50%) absorption coefficient is found for typical parameters in laser-fusion experiments, i.e., $1.5 \lesssim \alpha \lesssim 10$.

It is well known that whenever an electromagnetic wave has a singularity near the point where the dielectric function for electrostatic waves vanishes, the electromagnetic wave can be converted into an electrostatic wave and anomalous absorption of the electromagnetic wave energy takes place with the generation of large-amplitude electrostatic waves. In an unmagnetized, inhomogeneous plasma, an obliquely incident electromagnetic wave with polarization in the plane of incidence can drive a density oscillation, because of the component of the electric field along the density gradient, giving rise to a nonvanishing $\nabla\cdot(n\vec{v}_{0s})$, where $\vec{v}_{0s} = -e\vec{E}_0/im\omega + \text{c.c.}$ At the critical density, where $\omega_p(x) = \omega$, $\omega_p = (4\pi ne^2/m)^{1/2}$, the plasma wave is resonantly driven by the electromagnetic wave tunneling through the reflection point (cutoff).[9-12] Piliya[9] has given a very elegant analysis of this process. Because of the self-generated magnetic field in laser-produced plasmas, two new features are introduced into this process of resonant absorption. First, the resonant frequency is now the upper hybrid frequency $\omega_{\rm uh} = (\omega_p^{\,2} + \omega_c^{\,2})^{1/2}$, instead of the plasma frequency ω_p; second, the Lorentz force provides coupling between the electromagnetic and electrostatic waves so that, even for normal incidence, mode conversion into an upper hybrid wave takes place and significant absorption of the incident laser energy results. This upper-hybrid-wave conversion has been experimentally studied by Dreicer.[13]

In the present Letter we have investigated the wave conversion of an extraordinary mode in a magnetoplasma around the point of upper hybrid resonance taking thermal effects into account. For the sake of mathematical convenience, the direction of wave propagation is taken along the density gradient, perpendicular to the magnetic field. This well-known problem was first studied theoretically by Budden[14] and recently re-examined by White and Chen,[15] both using the cold-plasma approximation. To render the electromagnetic wave equation in the form of a Whittaker equation, they chose a very specific density profile of the form $\tanh(x/L)$ (shown in Fig. 14 of Ref. 15). This density profile allows finite transmission and is not realistic for the case of the laser-produced plasmas, particularly for relatively weak magnetic fields, i.e., for $\omega_c/\omega < 0.1$ where the cutoff and the resonance surfaces occurring at the same density. Here we consider the more realistic case of a plasma with linear density profile, $n(x) = n_0(1 + x/L)$ immersed in a uniform magnetic field. Thermal convection of the upper hybrid wave is also taken into account. The absorption coefficient is found to be substantially higher than that previously obtained from the cold-plasma model with Budden's profile, with a maximum more than two times higher. The maximum absorption coefficient is also about twice the value found for obliquely incident radiation on unmagnetized plasmas.

We consider the propagation of an extraordinary mode in a collisionless magnetoplasma in the direction of the density gradient, the x axis. The static magnetic field is aligned along the z axis and, therefore, the electric vector of the wave is contained in the x-y plane. Using the equations of motion and continuity, and the wave equation, we obtain the following equations for the field components:

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{(\omega^2 - \omega_{uh}^2)(\omega^2 - \omega_c^2)}{\omega_p^2 v_{th}^2} E_x = i\frac{\omega_c}{\omega}\frac{\omega^2 - \omega_c^2}{v_{th}^2}\left(E_y - \frac{v_{th}^2}{\omega^2 - \omega_c^2}\frac{\partial^2 E_y}{\partial x^2}\right), \tag{1}$$

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\omega^2}{c^2}\frac{\omega^2 - \omega_{uh}^2}{\omega^2 - \omega_c^2} E_y = -i\frac{\omega_c}{\omega}\frac{\omega^2}{c^2}\frac{\omega_p^2}{\omega^2 - \omega_c^2}\left(E_x - \frac{v_{th}^2}{\omega^2 - \omega_c^2}\frac{\partial^2 E_x}{\partial x^2}\right) \tag{2}$$

where $v_{th} = (k_B T_0/m)^{1/2}$. In writing Eq. (2) we have assumed $v_{th}^2\omega_c^2\omega_p^2/c^2(\omega^2 - \omega_c^2)^2 \ll 1$ which is satisfied for laser fusion parameters.

We specify the density profile such that $x = 0$ corresponds to the upper hybrid resonance where $\omega_p^2 = \omega^2 - \omega_c^2$ and the density varies linearly with x, i.e., $\omega_p^2(x) = (\omega^2 - \omega_c^2)(1 + x/L)$. Equations (1) and (2) then become, respectively,

$$\frac{\partial^2 E_x}{\partial x^2} - \frac{x}{x+L}\frac{\omega^2 - \omega_c^2}{v_{th}^2} E_x = i\frac{\omega_c}{\omega}\frac{\omega^2 - \omega_c^2}{v_{th}^2}\left(E_y - \frac{v_{th}^2}{\omega^2 - \omega_c^2}\frac{\partial^2 E_y}{\partial x^2}\right), \tag{3}$$

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{\omega^2}{c^2}\frac{x}{L} E_y = -i\frac{\omega_c}{\omega}\frac{\omega^2}{c^2}\left(1 + \frac{x}{L}\right)\left(E_x - \frac{v_{th}^2}{\omega^2 - \omega_c^2}\frac{\partial^2 E_x}{\partial x^2}\right). \tag{4}$$

From Eq. (3), we find that, well away from the resonance layer, thermal corrections are not important and E_x can be written as a superposition of particular solutions corresponding to the transverse wave and to the outgoing electrostatic wave

$$E_x = -i(\omega_c/\omega)[(x+L)/x]E_y + \eta E_{x_-}. \tag{5}$$

Here E_y is a linear combination of the incoming and outgoing electromagnetic waves $E_y = E_{y_+} + \xi E_{y_-}$ and E_{x_-} is the WKB solution of the homogeneous equation found by setting the right-hand side of Eq. (3) equal to zero, i.e.,

$$E_{x_-} \sim [-(x+L)/x]^{1/4}\exp\{i(\omega^2 - \omega_c^2)^{1/2}/v_{th}\int^x[-x/(x+L)]^{1/2}dx\}. \tag{6}$$

Equation (6) is valid for $|x/L| > (v_{th}/L\omega)^{2/3} \simeq (\lambda_D/L)^{2/3}$, where $\lambda_D = v_{th}/\omega_p(0)$. $E_{y_\pm}$ are the WKB solutions of the electromagnetic wave equation obtained from Eq. (4) by substituting the first term on the right-hand side of Eq. (5) into Eq. (4) and neglecting thermal corrections:

$$\partial^2 E_y/\partial x^2 + (\omega^2/c^2)\epsilon_x E_y = 0 \tag{7}$$

and

$$E_{y_\pm} \sim \epsilon_x^{-1/4}\exp[\mp i(\omega/c)\int^x \epsilon_x^{1/2}dx], \tag{8}$$

where $\epsilon_x = [(\omega_c^2/\omega^2)(1 + L/x)^2 - 1](x/L)$. The turning point for the electromagnetic wave is $x/L \simeq -\omega_c/\omega$ ($\epsilon_x = 0$) and, therefore, the solution (8) is not valid over a width $\Delta x = 5(\omega_c/\omega)^{1/2}(c/\omega)$ around the turning point.

These faraway solutions are now used as the boundary conditions for the solutions of Eqs. (3) and (4) in the resonance region. In order to solve Eq. (3) around $x=0$, we assume that the right-hand side is constant, because the scale length of variation of the electromagnetic wave is much longer than that of the upper hybrid wave there. Hence,

$$\partial^2 E_x/\partial x'^2 - x'E_x = x_{es}^2 P(0); \tag{9}$$

$$P(0) = i(\omega_c/\omega)[(\omega^2-\omega_c^2)/v_{th}^2][E_y(0) - v_{th}^2/(\omega^2-\omega_c^2)\partial^2 E_y/\partial x^2|_0],$$

and x is normalized to the scale of variation of the electrostatic field $x' = x/x_{es}$, $x_{es} = [Lv_{th}^2/(\omega^2-\omega_c^2)]^{1/3}$. The solution of Eq. (9) may be written as

$$E_x = x_{es}^2 P(0) Y(x'), \tag{10}$$

where $Y(x') = -i\int_0^\infty \exp[-i(tx'+t^3/3]\,dt$. In order to match the near-field solution (10) with the far-field solution (6) we expand $Y(x')$ for $-x' \gg 1$ as

$$Y(x') \sim (-x')^{-1/4}\pi^{1/2}\exp\{-i[\pi/4 + \tfrac{2}{3}(-x')^{3/2}]\} - 1/x' \tag{11}$$

and compare with Eq. (5) to get $\eta \sim P(0)\pi^{1/2}x_{es}^2(L/x_{es})^{1/4}\exp(-i\pi/4)$. Equation (11) is valid for $|x/L| \ll 1$ and the assumption of constant E_y is valid for $|x/L| < \omega_c/\omega$. The latter assumption is in compliance with the condition $-x' \gg 1$ which requires $x_{es} \ll L\omega_c/\omega$.

In order to determine $E_y(0)$ we need to solve Eq. (4) in the resonance region. On using Eq. (10), Eq. (5) takes the form

$$\partial^2 E_y/\partial x''^2 - x''E_y = x_{em}^2 Q(x'); \tag{12}$$

$$Q(x') = -i(\omega_c\omega/c^2)x_{es}^2 P(0)\{Y(x') - [v_{th}^2/(\omega^2-\omega_c^2)x_{es}^2][1 + x'Y(x')]\},$$

and x is normalized to the scale of variation of the electromagnetic field $x'' = x/x_{em}$, $x_{em} = (Lc^2/\omega^2)^{1/3}$. The general solution of Eq. (12) may be written as

$$E_y = C_1\mathrm{Ai}(x'') + C_2\mathrm{Bi}(x'') - (x_{em}^2/W[\mathrm{Ai},\mathrm{Bi}])[\mathrm{Ai}(x'')\int_{-\infty}^{x''}\mathrm{Bi}(x'')Q(x')\,dx'' - \mathrm{Bi}(x'')\int_{\infty}^{x''}\mathrm{Ai}(x'')Q(x')\,dx''], \tag{13}$$

where Ai and Bi are the Airy functions and $W[\mathrm{Ai},\mathrm{Bi}] = \mathrm{AiBi}' - \mathrm{BiAi}' = \pi^{-1}$. For E_y to be well behaved at $x'' \to +\infty$, C_2 must be zero. Also, since $x_{em} \gg x_{es}$, $\mathrm{Ai}(x'')$ is a slowly varying function of x as compared to $Y(x')$ and $Q(x')$, hence,

$$E_y(0) \simeq C_1\mathrm{Ai}(0) - \pi x_{em}^2\mathrm{Ai}(0)\mathrm{Bi}(0)\int_{-\infty}^{+\infty} Q(x')\,dx'', \tag{14}$$

or

$$E_y(0) = C_1\mathrm{Ai}(0)/\{1 - i\pi^2 x_{em}x_{es}^3(\omega_c^2/c^2)[(\omega^2-\omega_c^2)/v_{th}^2]\mathrm{Ai}(0)\mathrm{Bi}(0)\},$$

where $\int_{-\infty}^{+\infty} Y(x')\,dx'' = -i\pi x_{es}/x_{em}$.

Now, for $-x'' \gg 1$, we may write the asymptotic form of E_y, using Eqs. (13) and (14), as

$$E_y \sim C_+(-x'')^{-1/4}\exp\{-i[\pi/4 + \tfrac{2}{3}(-x'')^{3/2}]\} + C_-(-x'')^{-1/4}\exp\{i[\pi/4 + \tfrac{2}{3}(-x'')^{3/2}]\}, \tag{15}$$

where $C_\pm = \pi^{-1/2}\{\mp C_1/2i + [E_y(0) - C_1\mathrm{Ai}(0)]/2\mathrm{Bi}(0)\}$. The reflection coefficient of the electromagnetic wave is given by

$$\xi = \{[1 - \alpha\mathrm{Ai}^2(0)]^2 + \alpha^2\mathrm{Ai}^2(0)\mathrm{Bi}^2(0)\}/\{[1 + \alpha\mathrm{Ai}^2(0)]^2 + \alpha^2\mathrm{Ai}^2(0)\mathrm{Bi}^2(0)\} \tag{16}$$

where $\alpha = \pi^2(\omega_c/\omega)^2(L\omega/c)^{4/3} = \pi^2 x_0^2/x_{em}^2$. Using Eq. (16), the absorption coefficient (or conversion coefficient) of the electromagnetic wave can be written as

$$A = 4\alpha\mathrm{Ai}^2(0)/\{[1 + \alpha\mathrm{Ai}^2(0)]^2 + \alpha^2\mathrm{Ai}^2(0)\mathrm{Bi}^2(0)\} \tag{17}$$

which is valid for $x_{es}/L \ll \omega_c/\omega$. A attains a maximum of ~ 0.7 for $\alpha = 4$. The large values of the absorption coefficient are due to thermal effects which cause (i) resonance broadening and (ii) convection of the energy of the electrostatic wave away from the resonance region. The variation of A with α is displayed in Fig. 1.

The coefficient of anomalous absorption of a magnetized plasma for a laser beam depends on x_0 and

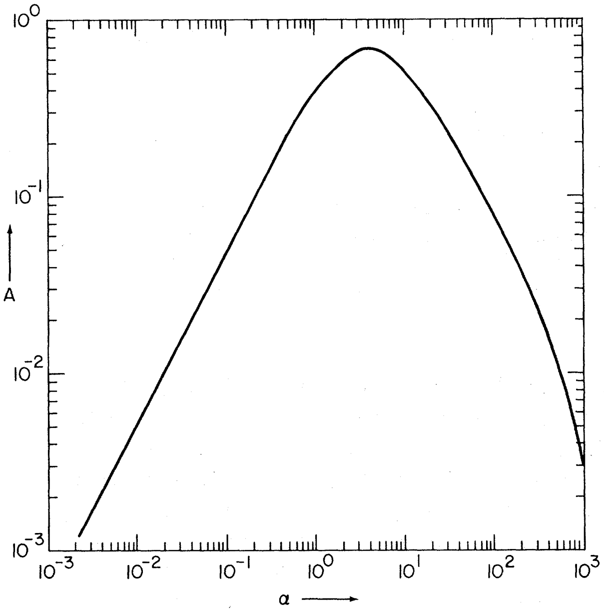

FIG. 1. Variation of the absorption coefficient of the electromagnetic wave with the parameter $\alpha \equiv \pi^2(\omega_c/\omega)^2 \times (L\omega/c)^{4/3}$.

x_{em} through $\alpha = \pi^2(\omega_c/\omega)^2(L\omega/c)^{4/3}$. As α increases the coefficient of anomalous absorption first increases, attains a maximum, and then decreases monotonically. For very large values of α, i.e., large density scale length, the separation between the reflection and the resonance layers becomes too large for the electromagnetic field to tunnel through the evanescent region and, therefore, the wave conversion process is diminished. For very small values of α, the magnetic field, which provides the coupling, is small and again, the absorption is reduced. It should be mentioned that in the case of Budden's profile the density scale length in the resonance region is effectively larger than in the case of a linear profile and, therefore, the absorption coefficient for Budden's profile would be less than that for the linear profile for $\alpha > 4$.

For realistic plasma parameters, considerable absorption of laser energy (up to ~70%) may be obtained, e.g., for a CO_2 laser (10.6 μm) in a 0.5-keV plasma with $L/\lambda_D \sim 20$ and $B \sim 2$ MG we get $\alpha \sim 2.9$ or, $A \sim 0.65$.

The amplitude of the electrostatic wave at the resonance layer is, from Eq. (10),

$$E_x(0) = i x_{es}^2(\omega_c/\omega)[(\omega^2 - \omega_c^2)/v_{th}^2]Y(0)E_y(0). \quad (18)$$

This work was supported by the Center for Theoretical Physics (University of Maryland), the U. S. Energy Research and Development Administration, the National Science Foundation, and the Conselho Nacional de Desenvolvimento Cientifico e Tecnologico (Brazil).

[1]D. A. Tidman and R. A. Shanny, Phys. Fluids 17, 1207 (1974).

[2]J. A. Stamper and D. A. Tidman, Phys. Fluids 16, 2024 (1973).

[3]J. A. Stamper, K. Papadopoulos, S. O. Dean, E. A. McClean, and J. M. Dawson, Phys. Rev. Lett. 26, 1012 (1972).

[4]J. J. Thomson, C. E. Max, and K. Estabrook, Phys. Rev. Lett. 35, 663 (1975).

[5]B. Bezzerides, D. F. DuBois, D. W. Forslund, and E. L. Lindman, Phys. Rev. Lett. 38, 495 (1977).

[6]W. F. DiVergilio, A. Y. Wong, H. C. Kim, and Y. C. Lee, Phys. Rev. Lett. 38, 541 (1977).

[7]J. A. Stamper and B. H. Ripin, Phys. Rev. Lett. 34, 138 (1975).

[8]W. L. Kruer and K. G. Estabrook, Lawrence Livermore Laboratory Report No. UCLR-79127, 1977 (unpublished).

[9]A. D. Piliya, Zh. Tekh. Fiz. 36, 818 (1966) [Sov. Phys. Tech. Phys. 11, 609 (1966)].

[10]N. G. Denisov, Zh. Eksp. Teor. Fiz. 31, 609 (1956) [Sov. Phys. JETP 4, 544 (1957)].

[11]V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasmas* (Pergamon, New York, 1964), p. 20.

[12]J. P. Freidberg, R. W. Mitchell, R. L. Morse, and L. I. Rudsinski, Phys. Rev. Lett. 28, 795 (1972).

[13]H. Dreicer, in *Plasma Waves in Space and in the Laboratory*, edited by J. O. Thomas and B. J. Landmark (University Press, Edinburgh, 1967), p. 157.

[14]K. G. Budden, *Radio Waves in the Ionosphere* (Cambridge Univ. Press, London, 1961), p. 476.

[15]R. B. White and F. F. Chen, Plasma Phys. 16, 565 (1974).