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[14] ^{198}Au: 411 805.2 ± xxx eV; ^{192}Ir: 2057 795.82 ± 0.19 eV, 295 958.6 ± 0.28 eV, 308 457.09 ± 0.34 eV, 316 508.72 ± 0.23 eV, 468 072.29 ± 0.39 eV; R. D. Deslattes, private communication; the Ir values include data from G. L. Borchert *et al.*, unpublished. All errors quoted are relative to Au. ^{228}Th: 238 631 ± 3 eV, R. L. Graham *et al.*, Can. J. Phys. 43, 171 (1965). ^{57}Co: 122 066.3 ± 2 eV, 136 476.7 ± 2 eV; ^{139}Ce: 165 857.5 ± 6 eV; R. G. Helmer, private communication.

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Intermultiplet Mixing of the Vector Mesons in a Nonperturbative Approach to Broken SU(4)

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We predict the mass spectrum of the $2\,^3S_1$, $J^{PC}=1^{--}$ vector mesons, the D^*-$D^{*\prime}$ mixing angle, and the F^*-$F^{*\prime}$ mixing angle (defined to be equal to the D^*-$D^{*\prime}$ mixing angle) in a nonperturbative approach to broken SU(4) symmetry.

In this Letter, we demonstrate the necessity of intermultiplet (i.e., configuration) mixing between the ground-state $1\,^3S_1$ and excited-state $2\,^3S_1$ (our notation is $N^{2S+1}L_J$), $J^{PC}=1^{--}$ vector mesons. The (radial) mixing involves only the charmed sector ($c=\pm 1$) of the mass spectrum, i.e., the D^*-$D^{*\prime}$ and F^*-$F^{*\prime}$ (in our rather obvious notation, $\psi', F^{*\prime}, D^{*\prime}, \rho', \ldots$, are the radially excited counterparts to the ground-state $\psi, F^*, D^*, \rho, \ldots$) and is essentially zero for the uncharmed sector.[1]

We use a *nonperturbative approach* to broken SU(4) symmetry[2,3]—the method of asymptotic SU(4) and algebraic realization—since the large mass differences present in SU(4) multiplets raise serious doubts as to the validity of the usual perturbation-theoretic arguments.

In asymptotic SU(4), creation and annihilation operators of physical particles transform linearly under SU(4), but only in the infinite momentum limit. The ground-state mixing parameters are defined (in the zero charm sector) among the physical fields φ, ω, and ψ, and the SU(4) representation fields a_8, a_0, and a_{15}, in the infinite momentum limit (we suppress helicity indices) by

$$\begin{pmatrix}\varphi\\ \omega\\ \psi\end{pmatrix}=\begin{pmatrix}\alpha_8 & \alpha_0 & \alpha_{15}\\ \beta_8 & \beta_0 & \beta_{15}\\ \delta_8 & \delta_0 & \delta_{15}\end{pmatrix}\begin{pmatrix}a_8\\ a_0\\ a_{15}\end{pmatrix}. \quad (1)$$

The excited-state mixing parameters are similarly defined with φ replaced by φ', $\alpha_8 \to \alpha_8'$, $\alpha_0 \to \alpha_0'$, $\alpha_{15} \to \alpha_{15}'$, $a_8 \to a_8'$, etc. In the charmed sector for ground and excited states, we define

$$\begin{pmatrix} D^* \\ D^{*\prime} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_D \\ a_D' \end{pmatrix}, \tag{2a}$$

and

$$\begin{pmatrix} F^* \\ F^{*\prime} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_F \\ a_F' \end{pmatrix}. \tag{2b}$$

So far the assumptions made are (1) φ-φ', φ-ω', φ-ψ', ω-φ', etc., mixing can be neglected; (2) K^*-$K^{*\prime}$ mixing (i.e., the strange sector) is ignored; (3) D^*-$D^{*\prime}$ and F^*-$F^{*\prime}$ mixing is described by the *same* angle θ; and (4) SU(2) breaking is assumed to be insignificant. Assumption (1) is very plausible since the $1\,^3S_1$ is almost ideal; supposition (2) has been investigated previously for orbital configuration mixing [within an SU(3) context] and strange-sector mixing angles were found to be of the order of $1°$.[4] Assumption (3) is motivated by the possibility that the mass scale of the charmed-sector radial wave functions may be quite different from the usual wave functions.[5] Conjecture (4) is adopted for the sake of simplicity. It is certainly true that SU(2) cannot *a priori* be neglected when one deals with almost-ideal multiplets.[2,6]

$\alpha_8, \alpha_0, \alpha_{15}, \beta_8, \dots$, and $\alpha_8', \alpha_0', \alpha_{15}', \beta_8', \dots$, can be parametrized in terms of the Euler angles (χ, β, γ) and (χ', β', γ'). In fact, we have $\alpha_8 = \cos\chi \times \cos\gamma$; $\alpha_0 = -\cos\chi \sin\gamma \sin\beta + \sin\chi \cos\beta$; $\alpha_{15} = \cos\chi \times \sin\gamma \cos\beta + \sin\chi \sin\beta$; $\beta_8 = -\sin\chi \cos\gamma$; $\beta_0 = \sin\chi \times \sin\gamma \sin\beta + \cos\chi \cos\beta$; $\beta_{15} = -\sin\chi \sin\gamma \cos\beta + \cos\chi \sin\beta$; $\delta_8 = -\sin\gamma$; $\delta_0 = -\cos\gamma \sin\beta$; $\delta_{15} = \cos\gamma \cos\beta$ (and similarly, $\alpha_8' = \cos\chi' \cos\gamma'$, etc.). χ mixes φ and ω, β mixes ω and ψ, and γ mixes φ and ψ (also χ' mixes φ' and ω', etc.).

Upon considering asymptotic SU(4) and asymptotic algebraic realization, we find a number (fourteen in all) of independent, nonlinear constraint equations that the $1\,^3S_1$ and $2\,^3S_1$ vector mesons must obey. They are

$$\langle K^{*+} | [\dot{V}_{K^+}, V_{K^0}] | \overline{K^{*0}} \rangle = \langle K^{*0} | [\dot{V}_{K^0}, A_{\pi^-}] | \rho^+ \rangle = \dots = 0,$$

and

$$\langle \rho^+ | [A_{\pi^+}, A_{\pi^-}] | \rho^+ \rangle = 2\langle K^{*+} | [A_{\pi^+}, A_{\pi^-}] | K^{*+} \rangle = 0.$$

The first set of equations are examples of the well-known commutation relations (CR's) $[\dot{V}_\alpha, V_\beta] = [\dot{V}_\alpha, A_\beta] = 0$ with $\dot{V}_\alpha \equiv (d/dt) V_\alpha$, where (α, β) is any *exotic* combination of physical SU(4) indices [i.e., $(\alpha, \beta) = (K^+, K^0)$, (K^0, π^-), (D^0, π^-), (D^+, D^0), (F^+, π^+), etc.].[2] The second equation is obtained by using the hypothesis of level realization of asymptotic SU(4) in the CR $[A_{\pi^+}, A_{\pi^-}] = 2V_0$, where the intermediate states are the $L=0$, $N=1$, and $L=0$, $N=2$, $J^{PC}=1^{--}$ mesons.[2]

We can simplify these equations considerably by obtaining estimates of d, l, l', δ_8, and δ_8' from experiment[7] ($d \equiv \langle \varphi | A_{\pi^+} | \rho^- \rangle$, $s \equiv \langle \omega | A_{\pi^+} | \rho^- \rangle$, $l \equiv \langle \psi | A_{\pi^+} | \rho^- \rangle$, $l' \equiv \langle \psi' | A_{\pi^+} | \rho^- \rangle$). For these estimates, we assume the partially conserved axial-vector current (PCAC) hypothesis in order to relate on-shell coupling constants to our off-shell axial-vector charge matrix elements[2] and the decay-rate formulas

$$\Gamma(V \to V' + P) = (q^3/12\pi) g_{VV'P}^2$$

and

$$\frac{\Gamma(V \to P_1 + P_2)}{\Gamma(V' \to P_1' + P_2')} = \left(\frac{q_{VP_1P_2}}{q_{V'P_1'P_2'}}\right)^3 \left(\frac{g_{VP_1P_2}}{g_{V'P_1'P_2'}}\right)^2.$$

With the input of $\Gamma(\varphi \to \pi^+\rho^-)$, $\Gamma(\psi \to \pi^+\rho^-)$, and $\Gamma(\psi' \to \pi^+\rho^-)$, we obtain $|d| \cong 1.18 \times 10^{-3}$ MeV^{-1}, $|l| \cong 1.89 \times 10^{-6}$ MeV^{-1}, and $|l'| \lesssim 7.26 \times 10^{-7}$ MeV^{-1}. From $\langle \psi | A_{K^+} | K^- \rangle = \langle \psi | [A_{\pi^+}, V_{K^0}] | K^- \rangle$ and $\langle \overline{K^{*0}} | [V_{K^0}, A_{\pi^-}] | \pi^- \rangle = 0$, we conclude that

$$\langle \psi | A_{K^+} | K^+ \rangle = -(\sqrt{\tfrac{1}{2}})(\sqrt{\tfrac{3}{2}}\delta_8) \langle \rho^0 | A_{\pi^+} | \pi^- \rangle. \tag{3}$$

Similarly, we find that

$$\langle \psi' | A_{K^+} | K^+ \rangle = -(\sqrt{\tfrac{1}{2}})(\sqrt{\tfrac{3}{2}}\delta_8') \langle \rho^{0\prime} | A_{\pi^+} | \pi^- \rangle. \tag{4}$$

Thus (with PCAC),

$$\delta_8 \equiv -\sin\gamma = \pm \left(\frac{-2}{\sqrt{3}}\right) \left(\frac{q_{\rho^0\pi^+\pi^-}}{q_{\psi K^+K^-}}\right)^{3/2} \left(\frac{\Gamma(\psi \to K^+K^-)}{\Gamma(\rho^0 \to \pi^+\pi^-)}\right)^{1/2}, \tag{5}$$

$$\delta_8' \equiv -\sin\gamma' = \pm \left(\frac{-2}{\sqrt{3}}\right) \left(\frac{q_{\rho'^0\pi^+\pi^-}}{q_{\psi K^+K^-}}\right)^{3/2} \left(\frac{\Gamma(\psi' \to K^+K^-)}{\Gamma(\rho'^0 \to \pi^+\pi^-)}\right)^{1/2}. \tag{6}$$

From experimental data[7] and $\Gamma(\rho^{0\prime} \to \pi^+\pi^-) \cong 50$ MeV, we compute that $|\gamma| \lesssim 3.66 \times 10^{-5}$ and $|\gamma'| \lesssim 1.16 \times 10^{-4}$. Thus it is a *good* approximation to take $\gamma \cong \gamma' \cong 0$.

With the input $\varphi = 1.0197$ GeV, $\omega = 0.7827$ GeV, $K^* = 0.894$ GeV, $\psi = 3.098$ GeV, $D^* = 2.010$ GeV, $\rho' = 1.600$ GeV, and $\psi' = 3.684$ GeV,[7] plus the estimates for d, l, l', δ_8, and δ_8', it proves possible to solve our nonlinear set of equations, albeit by computer. For the ground state, we obtain $\rho = 0.7604$ GeV,[8] $\chi = 218.684°$, $\beta = 209.997°$, $F^* = 2.064$ GeV, and $s = 1.976\times10^{-2}$ MeV^{-1}. For the excited-state mesons, we find *two distinct* solutions. Solution I: $K^{*\prime} = 1.668$ GeV, $\varphi' = 1.661$ GeV, $\omega' = 1.560$ GeV, $D^{*\prime} = 2.258$ GeV, $F^{*\prime} = 2.306$ GeV, $\theta = 90.487°$, $\chi' = 202.860°$, $\beta' = 242.572°$, $d' = 3.25\times10^{-7}$ MeV^{-1}, and $s' = -1.21\times10^{-6}$ MeV^{-1}; solution II: $K^{*\prime} = 1.668$ GeV, $\varphi' = 1.634$ GeV, $\omega' = 1.926$ GeV, $D^{*\prime} = 2.258$ GeV, $F^{*\prime} = 2.306$ GeV, $\theta = 90.241°$, $\chi' = 173.861°$, $\beta' = 250.138°$, $d' = 8.56\times10^{-7}$ MeV^{-1}, and $s' = -7.54\times10^{-7}$ MeV^{-1}. Here $d' \equiv \langle\varphi'|A_{\pi^+}|\rho^-\rangle$ and $s' \equiv \langle\omega'|A_{\pi^+}|\rho^-\rangle$. The wave functions for the states (φ, ω, ψ) in terms of the eighth, zeroth, and fifteenth ground-state SU(4) components are as follows:

$$\begin{pmatrix}\varphi\\ \omega\\ \psi\end{pmatrix} = \begin{pmatrix}-0.7806 & 0.5413 & 0.3125\\ 0.6250 & 0.6760 & 0.3903\\ 0 & 0.4999 & -0.8661\end{pmatrix}\begin{pmatrix}a_8\\ a_0\\ a_{15}\end{pmatrix}. \tag{7}$$

The quark content is thus

$$\begin{pmatrix}\varphi\\ \omega\\ \psi\end{pmatrix} = \begin{pmatrix}0.0596 & 0.9982 & 0\\ 0.9982 & -0.0596 & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\sqrt{\tfrac{1}{2}}(u\bar{u}+d\bar{d})\\ s\bar{s}\\ c\bar{c}\end{pmatrix}. \tag{8}$$

For solution I, we find that

$$\begin{pmatrix}\varphi'\\ \omega'\\ \psi'\end{pmatrix}_{\mathrm{I}} = \begin{pmatrix}-0.9215 & 0.1789 & 0.3448\\ 0.3885 & 0.4245 & 0.8179\\ 0 & 0.8876 & -0.4606\end{pmatrix}\begin{pmatrix}a_8'\\ a_0'\\ a_{15}'\end{pmatrix} \tag{9}$$

and

$$\begin{pmatrix}\varphi'\\ \omega'\\ \psi'\end{pmatrix}_{\mathrm{I}} = \begin{pmatrix}-0.2647 & 0.9414 & -0.2091\\ 0.8583 & 0.1311 & -0.4961\\ 0.4396 & 0.3108 & 0.8427\end{pmatrix}\begin{pmatrix}\sqrt{\tfrac{1}{2}}(u\bar{u}+d\bar{d})\\ s\bar{s}\\ c\bar{c}\end{pmatrix}. \tag{10}$$

For solution II, we obtain

$$\begin{pmatrix}\varphi'\\ \omega'\\ \psi'\end{pmatrix}_{\mathrm{II}} = \begin{pmatrix}-0.9943 & -0.0363 & -0.1006\\ -0.1069 & 0.3378 & 0.9351\\ 0 & 0.9405 & -0.3398\end{pmatrix}\begin{pmatrix}a_8'\\ a_0'\\ a_{15}'\end{pmatrix} \tag{11}$$

and

$$\begin{pmatrix}\varphi'\\ \omega'\\ \psi'\end{pmatrix}_{\mathrm{II}} = \begin{pmatrix}-0.6408 & 0.7646 & 0.0689\\ 0.5589 & 0.5262 & -0.6409\\ 0.5263 & 0.3722 & 0.7645\end{pmatrix}\begin{pmatrix}\sqrt{\tfrac{1}{2}}(u\bar{u}+d\bar{d})\\ s\bar{s}\\ c\bar{c}\end{pmatrix}. \tag{12}$$

What is striking about both of these solutions is, of course, $\theta \cong 90°$ and $D^{*\prime} = 2.258$ GeV.[9] These values indicate that any model of meson spectroscopy based on a potential should explicitly include the interaction between charmed and uncharmed quarks, which appears to be significantly different from the charmed-charmed and uncharmed-uncharmed quark interactions. The values of θ, D^*, and $D^{*\prime}$ obtained also explain quite nicely why previous theoretical models tend to predict $D^* = 2.2$–2.26 GeV rather than 2.01 GeV.[10]

The decays $\Gamma(\psi\to\rho\pi)$, $\Gamma(\psi\to KK)$, $\Gamma(\psi'\to\rho\pi)$, and $\Gamma(\psi'\to KK)$ in our model are, of course, in agreement with experiment since they are the input data. The decay $\Gamma_{\mathrm{theory}}(\psi\to K^\pm K^{*\mp})$ is perhaps too large $\{[\Gamma(\psi\to K^\pm K^{*\mp})/\Gamma(\psi\to\rho^\pm\pi^\mp)]_{\mathrm{theory}} \cong 0.85\}$[7,11] while $\Gamma(\psi'\to\psi\eta)$ is predicted to be zero [whereas the experimental branching ratio $R(\psi'\to\psi\eta) \cong (4.1\pm0.7)\%$]. However, these discrepancies are *not* surprising, for in our model $\Gamma(\psi\to K^\pm K^{*\mp})$ and $\Gamma(\psi'\to\psi\eta)$ are very sensitive to the values of δ_8 and δ_8'. In fact, any decay which is controlled by δ_8 and/or δ_8' will be consistently overestimated or underestimated in our model. $\Gamma(\psi'\to\psi\eta)$ is acutely affected by the approximations $\delta_8 = \delta_8' = 0$ since

$$\sqrt{3}\, g_{\psi'\eta\psi} = \sqrt{\tfrac{3}{2}}\,\delta_8' g_{\psi K^+K^{*\prime -}} + \sqrt{\tfrac{3}{2}}\,\delta_8 g_{\psi' K^- K^{*+}}.$$

It should be noted that the large contamination of

ψ' (when compared to ψ) by uncharmed quarks is expected since the quantity $(\psi'^2 - \psi^2)/(\rho'^2 - \rho)$ has the approximate value of 2 rather than its "ideal" value of 1. This in turn implies that the assumption of a universal Regge slope is too naive. Furthermore, by utilizing level realization in the commutator $V_{K^+} = [A_{\pi^+}, A_{K^0}]$ sandwiched between the states $\langle \psi'|$ and $|K^-\rangle$, we can show that

$$R(\psi' \to \pi^+ \rho'^-) \cong \frac{5.06\times 10^2}{\Gamma(\rho'^0 \to \pi^+\pi^-)} R(\psi' \to \pi^+\rho^-),$$

where $\Gamma(\rho'^0 \to \pi^+\pi^-)$ is measured in MeV. Thus if $\Gamma(\rho'^0 \to \pi^+\pi^-) \cong 50$ MeV, we find that $R(\psi' \to \pi^+\rho'^-) \leq 1\%$, which is consistent with experiment. Thus ψ' is "stable."

As of now, we have not been able to ascertain theoretically whether solution I or solution II is physical. Experimentally, the situation is not better. In that regard, a study of $1^{--} \to 0^{-+} + \gamma$, $0^{-+} \to 1^{--} + \gamma$, and the leptonic decays of the neutral vector mesons may be enlightening.

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