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Further Muonic-Atom Test of Vacuum Polarization

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We have accurately measured eleven muonic-atom transition energies in five elements sensitive to the vacuum-polarization corrections. We find good agreement between the theoretical and experimental values, the average difference being $(0.18\pm 0.15)\%$ of the vacuum-polarization correction.

Recently, the energy levels in muonic atoms have been used to test the high-order vacuum-polarization corrections.[1-6] The latest results[6] of theory and experiment are in agreement to better than 0.7%. Our final results, presented here, from a recent measurement at the Space Radiation Effects Laboratory (SREL),[7] improve the agreement to better than 0.2%. This verification of the calculations increases the confidence in the recently reported[8-11] π^- mass.

Our increased accuracy was made possible by recent precise energy measurements by Deslattes *et al.*[12] of the 412-keV γ ray of ^{198}Au and the γ rays of ^{192}Ir which were used in the calibration. Further, the intrinsic Ge spectrometer system had high resolution, symmetrical line shape, and no measurable effect due to incident γ-ray angle, and was unaffected by high count rates. Finally, good stability was achieved over the duration of the experiment. This, combined with many tests for possibly systematic shifts, substantially reduced the uncertainty of this experiment. We used a 3.1-cm^3 intrinsic Ge diode,[13] with a resolution of 870 eV at 316 keV. The spectrometer system was stabilized using the 122-keV γ ray of ^{57}Co and the 477-keV γ ray of ^{7}Be for zero and gain stabilization respectively. The energy calibration was based on ten standard γ rays.[14] A quadratic relation between γ-ray energy and channel number gave a rms deviation for the ten lines of less than 1 eV with a χ^2/N of 1.2. Calibrations for different time periods showed no shifts and agreed with each other within statistical errors for the sample (2 eV) over the whole

TABLE I. Comparison of the experimental and calculated transition energies for several muonic atoms which are sensitive to the vacuum-polarization correction. The theoretical numbers are from Ref. 6. The weighted deviation of all transitions, as a fraction of the vacuum-polarization correction, is 0.0018 ± 0.0015.

Transition		Experiment (eV)	Theory (eV)	Vacuum polarization (eV)	Theory − experiment (eV)
Ca	$3d$-$2p$	156 842.4 ± 5.2	156 845 ± 3	716	2.6 ± 6.0
		158 171.6 ± 6.6	158 182 ± 4	734	10.4 ± 7.7
Ba	$5g$-$4f$	199 917.3 ± 4.8	199 907 ± 3	748	−10.3 ± 5.7
		201 274.7 ± 7.1	201 274 ± 3	762	−0.7 ± 7.7
Sn	$4f$-$3d$	345 253.7 ± 6.9	345 254 ± 5	1731	0.3 ± 8.5
		349 975.4 ± 4.9	349 978 ± 5	1795	2.6 ± 7.0
Tl	$5g$-$4f$	420 757.3 ± 3.7	420 765 ± 7	2038	7.7 ± 7.9
Pb	$5g$-$4f$	431 327.6 ± 3.4	431 336 ± 7	2106	8.4 ± 7.8
		437 749.4 ± 13.7	437 751 ± 7	2190	1.6 ± 15.3
Ba	$4f$-$3d$	433 904.8 ± 9.6	433 910 ± 7	2328	5.2 ± 11.9
		441 361.7 ± 5.1	441 365 ± 7	2435	3.3 ± 8.7

energy range. The calibration uncertainty was determined to be between 0.7 and 3 eV, depending on the energy, by use of subsets of the standard γ rays. Full details of the experimental techniques will be given in a forthcoming paper on precision measurements of muonic-atom transitions.

We report here the eleven transition energies in five elements in the energy range 150–450 keV used to test the theory. The energies derived using two independent line-fitting functions, the ratio of two quartics[15] and a Gaussian with exponential tails, gave the same results within 1 eV. The calibration lines were initially fitted to determine the variation of the line-shape parameters as a function of channel number. Then the source peaks were refitted with the new parameters to give the energy calibration. The muonic x-ray peaks were then fitted using the appropriate parameters with a Lorentzian folded in to account for the natural x-ray linewidth. From the positions thus determined and the calibration, the energies were calculated. No further corrections were made.[16] The results are shown in Table I, along with the theoretical energies[17] and vacuum polarization as quoted by Vuilleumier *et at.*[6] The error quoted includes the statistical error, the error due to interfering lines, the calibration error (0.7 to 3 eV), and a constant systematic error of 1.7 eV. This last error accounts for the uncertainty in the ^{198}Au line (1.2 eV), the fitting error (1 eV), angular effect (0.7 eV), and timing effect (0.1 eV). All errors were added in quadrature. We also show the difference between the measured and calculated energies.

An inspection of the table shows that there is good agreement between theory and experiment. Further, if we assign the difference totally to the vacuum-polarization correction, we find that the average difference is (0.18 ± 0.15)%.

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[13]The Ge detector was fabricated by the Ge Detector Division of the Lawrence Berkeley Laboratory, University of California, Berkeley, Calif.

[14] ^{198}Au: 411 805.2 ± xxx eV; ^{192}Ir: 2057 795.82 ± 0.19 eV, 295 958.6 ± 0.28 eV, 308 457.09 ± 0.34 eV, 316 508.72 ± 0.23 eV, 468 072.29 ± 0.39 eV; R. D. Deslattes, private communication; the Ir values include data from G. L. Borchert *et al.*, unpublished. All errors quoted are relative to Au. ^{228}Th: 238 631 ± 3 eV, R. L. Graham *et al.*, Can. J. Phys. 43, 171 (1965). ^{57}Co: 122 066.3 ± 2 eV, 136 476.7 ± 2 eV; ^{139}Ce: 165 857.5 ± 6 eV; R. G. Helmer, private communication.

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[16]In previous experiments (Refs. 1 and 4) corrections were made for angular or timing effects. In this experiment they were determined to be consistent with zero (0 ± 0.7 eV and 0 ± 0.1 eV, respectively) and thus no corrections were applied.

[17]Our Pb measurements, also from this experiment, indicate that the electron screening calculations are accurate to about 2%. See P. Vogel *et al.*, Phys. Rev. A 15, 76 (1977). The electron screening correction in our case is between 10 and 200 ppm.

Intermultiplet Mixing of the Vector Mesons in a Nonperturbative Approach to Broken SU(4)

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We predict the mass spectrum of the $2\,^3S_1$, $J^{PC}=1^{--}$ vector mesons, the D^*-$D^{*\prime}$ mixing angle, and the F^*-$F^{*\prime}$ mixing angle (defined to be equal to the D^*-$D^{*\prime}$ mixing angle) in a nonperturbative approach to broken SU(4) symmetry.

In this Letter, we demonstrate the necessity of intermultiplet (i.e., configuration) mixing between the ground-state $1\,^3S_1$ and excited-state $2\,^3S_1$ (our notation is $N^{2S+1}L_J$), $J^{PC}=1^{--}$ vector mesons. The (radial) mixing involves only the charmed sector ($c=\pm 1$) of the mass spectrum, i.e., the D^*-$D^{*\prime}$ and F^*-$F^{*\prime}$ (in our rather obvious notation, $\psi', F^{*\prime}, D^{*\prime}, \rho', \ldots$, are the radially excited counterparts to the ground-state $\psi, F^*, D^*, \rho, \ldots$) and is essentially zero for the uncharmed sector.[1]

We use a *nonperturbative approach* to broken SU(4) symmetry[2,3]—the method of asymptotic SU(4) and algebraic realization—since the large mass differences present in SU(4) multiplets raise serious doubts as to the validity of the usual perturbation-theoretic arguments.

In asymptotic SU(4), creation and annihilation operators of physical particles transform linearly under SU(4), but only in the infinite momentum limit. The ground-state mixing parameters are defined (in the zero charm sector) among the physical fields φ, ω, and ψ, and the SU(4) representation fields a_8, a_0, and a_{15}, in the infinite momentum limit (we suppress helicity indices) by

$$\begin{pmatrix}\varphi\\ \omega\\ \psi\end{pmatrix}=\begin{pmatrix}\alpha_8 & \alpha_0 & \alpha_{15}\\ \beta_8 & \beta_0 & \beta_{15}\\ \delta_8 & \delta_0 & \delta_{15}\end{pmatrix}\begin{pmatrix}a_8\\ a_0\\ a_{15}\end{pmatrix}. \quad (1)$$