essentially the same hypothetical fusion nucleus as the system $^{56}Fe + ^{238}U$), implying that quasifission and quasielastic transfer might constitute almost all of the reaction cross section. This indicates that for any very massive heavy-ion system to exhibit a reasonable fusion cross section, a large degree of asymmetry between target and projectile masses is required.

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[1]See, for example, Marc Lefort, Phys. Rev. C 12, 686 (1975), and references contained therein.

[2]J. V. Kratz, J. O. Liljenzin, A. E. Norris, and G. T. Seaborg, Phys. Rev. C 13, 2347 (1976).

[3]J. V. Kratz, A. E. Norris, and G. T. Seaborg, Phys. Rev. Lett. 33, 502 (1974).

[4]J. Péter, C. Ngô, and B. Tamain, Nucl. Phys. A250, 351 (1975).

[5]K. L. Wolf, J. P. Unik, J. R. Huizenga, J. Birkeland, H. Freiesleben, and V. E. Viola, Phys. Rev. Lett. 33, 1105 (1974).

[6]See, for example, J. R. Nix, in *Annual Review of Nuclear Science, 1972,* edited by E. Segrè, J. R. Grover, and H. P. Noyes (Annual Reviews, Inc., Palo Alto, Calif., 1972), Vol. 22, p. 65.

[7]J. V. Kratz, J. O. Liljenzin, and G. T. Seaborg, Inorg. Nucl. Chem. Lett. 10, 951 (1974).

[8]W. Wätzig, F. Picard, and C. Lissing, Université Paris–Sud Report No. IPNO-75-03, 1975 (unpublished).

[9]J. B. Cumming, National Academy of Sciences, National Research Council, Nuclear Science Series Report No. NAS NS-3107, 1962 (unpublished).

[10]U. Reus and W. Westmeier, compilation from the literature of γ-ray energies and intensities ordered by nuclide and energy, 1976 (unpublished).

[11]J. H. Shea, J. L. Durell, G. S. Foote, and I. S. Grant, to be published.

[12]L. C. Northcliffe and R. F. Schilling, Nucl. Data, Sect. A 7, 233 (1970).

"Correlated Clusters" and Inclusive Spectra of Energetic Protons at 180° in Proton-Nucleus Collisions

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This paper presents a model based on the assumption of the existence of "correlated clusters" which stay as they are during a fast collision. The model can explain remarkably well the inclusive spectra of energetic protons at 180° in the proton-nucleus experiments by Frankel *et al.*

Recently, Frankel *et al.*[1] have measured the inclusive cross section for 180° production of high-energy protons ($E_p = 150-450$ MeV) in proton-nucleus collisions with an incident proton energy E_i of 600 and 800 MeV. To understand the data, Amado and Woloshyn[2] proposed a model based on a single-scattering mechanism and showed that the model gives results which are a few orders of magnitude smaller than the experiment if one uses for the Fermi-momentum distribution in the nucleus that corresponding to a zero-temperature, noninteracting Fermi-gas model. To explain the data, they employed a phenomenological Fermi-momentum distribution which is quite different in the higher-momentum region from the one usually adopted in low-energy nuclear physics,[3] even though the Fermi-momentum distribution in nuclei is not well investigated in such a high-momentum region ($k \gtrsim 700$ MeV/c).

In this Letter, I consider the backward-scattering problem as being caused by the reaction between the incident proton and a group of nucleons. I assume the existence of a group of nucleons which stay as they are during the fast collision. I call these nucleons "correlated clusters." The number of the correlated clusters (hereafter referred to as CC) to be found in the nucleus may be expressed by

$$G_N = \binom{A}{N} \frac{1}{A^{N-1}} P_N \quad (A \gg N), \qquad (1)$$

where A is the mass number of the target nucleus, and P_N denotes the probability of finding N nucleons in the CC state. P_N consists of two parts: One is the probability of finding N nucleons in a small volume V_c, whose radius is an order of the correlation length l_c ($l_c \sim 0.5-0.7$ fm), and the other is the probability ξ_N that the CC state, composed of N nucleons, does not break up during the fast collision. Therefore, I write P_N as

$$P_N = (V_c/V)^{N-1}\xi_N, \qquad (2)$$

where the radius of V is the mean distance l_m ($l_m \sim 1.4$ fm) between two nucleons inside nucleus.

I now want to describe the proton inclusive cross section at 180° in proton-nucleus collisions. My mechanism for this process is similar to the Amado-Woloshyn model,[2] except that in this case the incident proton hits the CC, instead of the nucleon. Therefore, I write the cross section as

$$\frac{d\sigma}{d^3p_f} = \sum_N G_N \int \frac{1}{E_f E_{CC}} |m_{p\text{-CC}}|^2 W_N(k) \frac{d^3k}{(2\pi)^3} \frac{1}{(8\pi)^2 F} \delta(E_i + NM - E_f - E_{CC} - \bar{\epsilon}) \qquad (3)$$

with $E_{CC} = [(NM)^2 + (\vec{p}_i + \vec{k} - \vec{p}_f)^2]^{1/2}$ and $F = |\vec{p}_i| NM$, where $(E_i, \vec{p}_i)$ and $(E_f, \vec{p}_f)$ denote the initial and final energy and momentum, respectively. $\bar{\epsilon}$ is the average nuclear excitation. $w_N(k)$ is the momentum distribution of the N-correlated cluster (N-CC) state, related to the N-CC state wave function $[\rho_N(r)]^{1/2}$ by

$$w_N(k) = C|\int \rho_N^{N/2}(r) e^{i\vec{k}\cdot\vec{r}} d^3r|^2. \qquad (4)$$

Choosing a simple expression for $\rho_N(r)$, such as

$$\rho_N(r) = (\nu/\pi)^{3/2} e^{-\nu r^2}, \qquad (5)$$

one obtains the following form for $w_N(k)$:

$$w_N(k) = (4\pi/N\nu)^{3/2} e^{-k^2/N\nu}, \qquad (6)$$

which is normalized according to

$$\int w_N(k)[d^3k/(2\pi)^3] = 1. \qquad (7)$$

Finally, $m_{p\text{-CC}}$ denotes the p-CC invariant amplitude and is related to the cross section in the center-of-mass system[4] by

$$\left(\frac{d\sigma}{d\Omega_{\text{c.m.}}}\right)_{p\text{-CC}} = \frac{1}{(8\pi)^2} \frac{1}{s} |m_{p\text{-CC}}|^2, \qquad (8)$$

where s is the square of the center-of-mass energy. To determine the value of $m_{p\text{-CC}}$, I assume that $(d\sigma/d\Omega_{\text{c.m.}})_{p\text{-CC}}$ is the same as the proton-nucleon cross section in the center-of-mass system. Note, that, in Eq. (3), the off-shell effects in $m_{p\text{-CC}}$ are ignored.

In Fig. 1, I show the various contributions from the N-CC state to the proton inclusive cross section calculated by Eq. (3). One can see from Fig. 1 that the zeroth-order term ($N=1$ case) gives results which are a few orders of magnitude smaller than experiment, as has been already stated in Ref. 2. However, the $N=2$ case gives as large contributions around $E_p = 150$ MeV as the observed cross section. Further, the $N=3$ case becomes important around $E_p = 200$ MeV and the $N=4$ case around $E_p = 300$ MeV. The solid line shows a sum of the $N=1$ through $N=4$ contributions and yields an excellent description of the observed data. In Fig. 2, we compare the calculations with experiment for three typical cases with different masses and different incident energies. Again, the model shows a remarkable agreement with the experiment.

Concerning the parameters introduced in this model, I will comment on ν which appears in $\rho_N(r)$. The value used here is $0.5\nu = 0.419$ fm^{-2} which is independent of the mass number A. This choice may not cause any serious errors, since I am only interested in describing the data to within a factor 2. Secondly, I introduced P_N, the probability of finding the N nucleons in the CC state. Here the following value is used for (V_c/V),

$$(V_c/V) = 0.07,$$

which corresponds to the correlation length $l_c \simeq 0.6$ fm which is consistent with the one produced

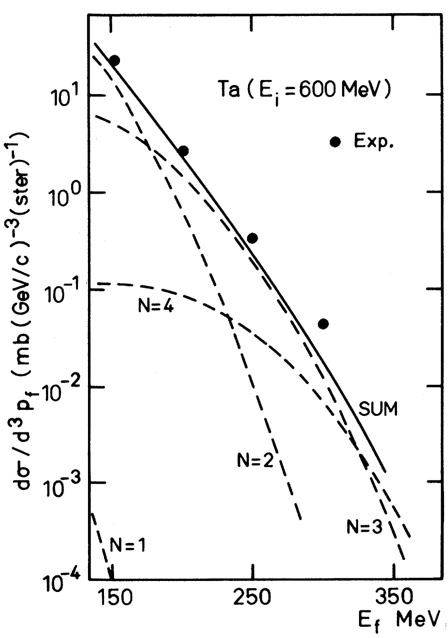

FIG. 1. The contributions of each N-CC state ($N=1-4$). The solid line shows the sum of the contributions and is compared with the data.

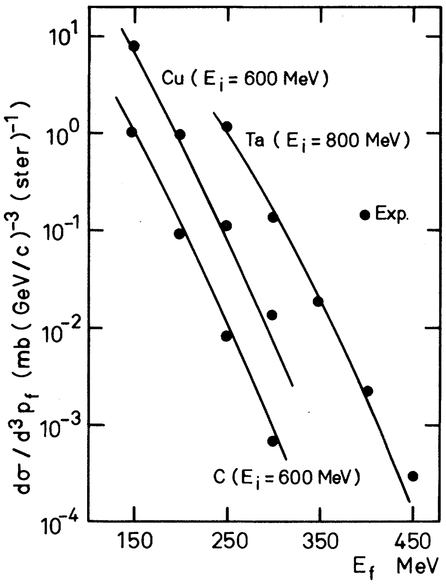

FIG. 2. A comparison of the present calculations with the experimental data from Ref. 1.

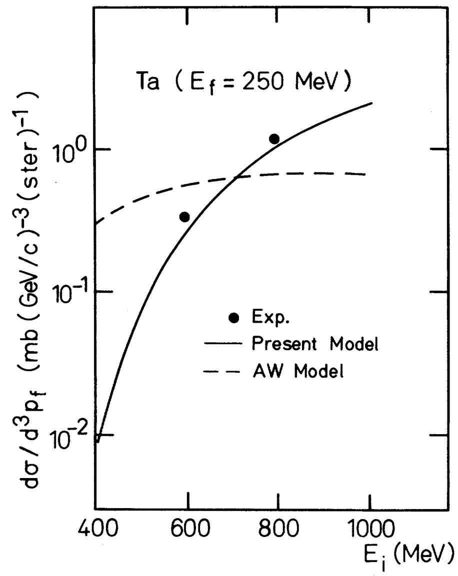

FIG. 3. The incident-energy dependence of the cross section at $E_F = 250$ MeV. Here, the incident-energy dependence of the proton-nucleon cross section in the c.m. system is ignored in both models. Further, I employ the elastic proton-nucleon cross section for $(d\sigma/d\Omega_{\text{c.m.}})_{p\text{-CC}}$.

by the short-range correlations in the nucleon-nucleon interaction.[5] Further, ξ_N of P_N in my calculation is found to be $\xi_N \simeq 1$, which means that during the fast collision the CC state has very little probability of breaking up. However, this result is not conclusive at all, since we put the p-CC cross section equal to the p-N cross section, which apparently underestimates the p-CC cross section, since the CC state has a larger volume than the proton.

Next, I will discuss other possible mechanisms that should be considered for the backward scattering process. First, I comment on some possible effects due to statistical processes. In this regard, I refer to the calculations by Fijita and Mantzouranis,[6] who studied the same problem in terms of the generalized exciton model,[7] which can describe the low-energy nucleon distribution quite well. However, it was found that the generalized exciton model cannot reproduce the observed behavior of the energy dependence of the inclusive cross section in such high-energy cases. Secondly, I comment on multiple-scattering processes (i.e., sequential scatterings), since they are known to be very important in forward scattering. However, one can easily check that the incident proton loses too much energy when it scatters at 180° through multiple scatterings.

Up to now, I have described the CC model. In what follows, I compare my model, first, with the Amado and Woloshyn (AW) model and, second, with the model presented by Burov *ét al.*[8]

As mentioned before, the AW model can also reproduce the data quantitatively. A question may arise to whether one can discriminate between the AW model and the present one. This may be achieved when one considers the dependence of the cross section on the incident proton energy. This is qualitatively quite clear because in the AW model the main features of the spectra are determined by the momentum distribution in nucleus whereas in my model when the incident energy becomes higher, high-energy protons at 180° become more available, since the outgoing protons are the ones that hit the CC. Figure 3 confirms numerically this tendency. At the present stage, where there are only two experimental points available, one may not draw any conclusions as to which model is more reasonable, since both models are only reliable to within a factor 2 or 3.

Next, I discuss the connection of the CC model with the so-called "fluctuon" model that has recently been developed by Burov, Lukyanov, and Titov and that has turned out to explain quite successfully the pion inclusive spectra in 8-GeV proton-nucleus collision. The fluctuons are some special state, related to a fluctuation of the nuclear density, composed of N nucleons ($N=2, 3, 4, \ldots$). However, the nucleons composing the fluctuons are assumed to be confined to a very small volume whose radius is smaller than the radius of a nucleon. Although such a state is unknown in nuclear physics, the picture has one advantage over my model, since the fluctuons automatically satisfy the only condition which exists in my model, namely, that the CC does not break up during the fast collision. In other words, the fluctuons are geometrically so small that the nucleons in each fluctuon interact with the incident protons simultaneously. On the other hand, it is found in my model that the mean distance between the nucleons composing the CC is almost the same as the correlation length which is usually found in nuclear structure calculations. In this respect, it may be said that the CC has more reality in the nucleus than the fluctuon.

I have shown that a model which takes into account collisions between the incident proton and correlated clusters is quite successful for explaining the backward-scattering problem. Further studies of this kind would be quite useful for exploring new features concerning correlations in nuclei.

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[1]S. Frankel, W. Frati, O. Van Dyck, R. Werbeck, and V. Highland, Phys. Rev. Lett. 36, 642 (1976).
[2]R. D. Amado and R. M. Woloshyn, Phys. Rev. Lett. 36, 1435 (1976). See also references therein.
[3]K. Nakamura, S. Hiramatsu, T. Kamae, H. Muramatsu, N. Izutsu, and Y. Watase, Nucl. Phys. A268, 381 (1976).
[4]V. Chaloupka *et al.*, Phys. Lett. 50B, 1 (1974).
[5]H. A. Bethe, Annu. Rev. Nucl. Sci. 21, 93 (1971).
[6]T. Fujita and G. Mantzouranis, unpublished.
[7]G. Mantzouranis, H. A. Weidenmüller, and D. Agassi, Z. Phys. A276, 145 (1976).
[8]V. V. Burov, V. K. Lukyanov, and A. I. Titov, Phys. Lett. 67B, 46 (1977).

Universal Fragment-Momentum Distribution in High-Energy Nucleus-Nucleus Collisions[(a)]

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Fragments with $1 \leq Z \leq 12$, some with energies as low as 3 MeV/nucleon and some with energies as high as 120 MeV/nucleon, have been analyzed in high-energy nucleus-nucleus reactions with C, Ne, and Ar projectiles. For a given projectile the invariant cross sections *of all fragments* appear to define a "universal" curve that is exponential in momentum, provided momentum is evaluated in a frame in which the distribution is isotropic. The low speed of the frame ($\sim 0.01c$ to $\sim 0.1c$) suggests emission from the recoiling target.

Recently we reported[1] energy and angular distributions of complex nuclei ($3 \leq Z \leq 9$) with energies ~15 to ~60 MeV/nucleon produced in high-energy nucleus-nucleus interactions at the Lawrence Berkeley Laboratory Bevalac. The angular distributions were consistent with isotropic emission from a source moving in the beam direction with a very low velocity, $\beta_0 \approx 0.08 \pm 0.02$. The energy distributions in the moving frame were about equally consistent with Maxwellians with a very high temperature, $\tau \approx 50$ to 70 MeV, or with exponentials in momentum, the latter implying a nonthermal process. We pointed out that a source in thermal equilibrium at such a high temperature and low velocity would be incompatible with energy-momentum conservation and concluded that most of these complex nuclei must have been emitted nonthermally.