Interactions between Two Superconducting Weak Links in the Stationary ($V = 0$) States

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Effects of interaction between two superconducting weak links (SWL) at $V=0$ have been calculated using the Ginzburg-Landau theory. Variations of the critical current of one SWL affected by a dc current in a neighboring SWL are found in good qualitative agreement with a recent experiment. The current-phase relation of the combined system is computed for various separations between the two SWL's; it is shown explicitly that the system behaves as a single SWL when the spacing between links is comparable to the coherence length.

Physical properties of the superconducting weak link (SWL), which consists of two superconductors separated by a small region with weakened pair potential, have been extensively studied for many years. This system exhibits many interesting phenomena virtually identical to the Josephson effects found in tunnel junctions. One outstanding feature of this macroscopic quantum structure is that its fundamental characteristics and the interaction between two weak links can actually be controlled by variation of some macroscopic dimensions in the system. Experiments[1,2] have now revealed that coupling between two SWL's can indeed be changed by varying, among other parameters, the separation between the weak links. In this note we give a theoretical analysis of the interaction effects between two SWL's in the zero-voltage region. It is hoped that this result will clarify some interpretations of the complicated experimental observations.

When a SWL is current-biased below its critical current I_c, $V=0$, the order parameter is a stationary function of position only. Its general behavior can be quite well described by the Ginzburg-Landau (GL) equation.[3] Solutions of this equation subject to appropriate boundary conditions have appeared in the literature.[4,5] The nearly sinusoidal current-phase relation derived from these solutions is in accord with general expectation and with experiment.[6] We consider a two-SWL system in which each SWL can be described by a one-dimensional model with solutions obtainable from the previous work. Hence, aside from the nonlinear term in the GL equation, our problem is similar to solving a wave equation in a double-well region by making use of the known solutions of the equation in a single potential well. Preliminary results for the special case of two identical SWL's have appeared in publication.[7]

Our model of the weak links is schematically represented in Fig. 1(a). The regions L-a_1-a_2 constitute one SWL (A) in which the pair potential in a_1 is weakened. Likewise a_5-a_6-R form another SWL (B). These two SWL's are joined together by a superconductor a_3-a_4. Each zone is, in general, characterized by a set of constants defined in terms of the well-known parameters in the usual GL theory (ξ, H_c, χ, n, T_c, etc.). Except for the two ends (L and R), each zone has a fixed thickness a_i ($i=1,2,\ldots,6$). The cross-sectional area of the whole system is considered so small that effects due to the magnetic field can be neglected. At each interface of two different superconductors, the Zaitsev boundary conditions[8] are applied so that continuity of the pair potential and normal current density is satisfied. This arrangement is most suitable for calculating interaction effects on the critical current. When the current-phase (J-Φ) relation is calculated, regions 2, 3, 4, and 5 are taken to be the same material and we simply denote

$$\bar{a} = \sum_{i=2}^{5} a_i$$

as the distance separating the two SWL's.

If we express the reduced potential Δ/Δ_0 as $f(x)e^{i\varphi(x)}$, the GL equation to be solved in each zone with subscript k (e.g., $k=1,2,\ldots$) takes the following general form:

$$\eta_k \frac{d^2 f_k(y)}{dy^2} - \frac{j_k^2}{\zeta_k f_k^3(y)} + f_k(y) - f_k^3(y) = 0, \qquad (1)$$

where η_k and ζ_k are parameters pertaining to the kth zone normalized with respect to the corre-

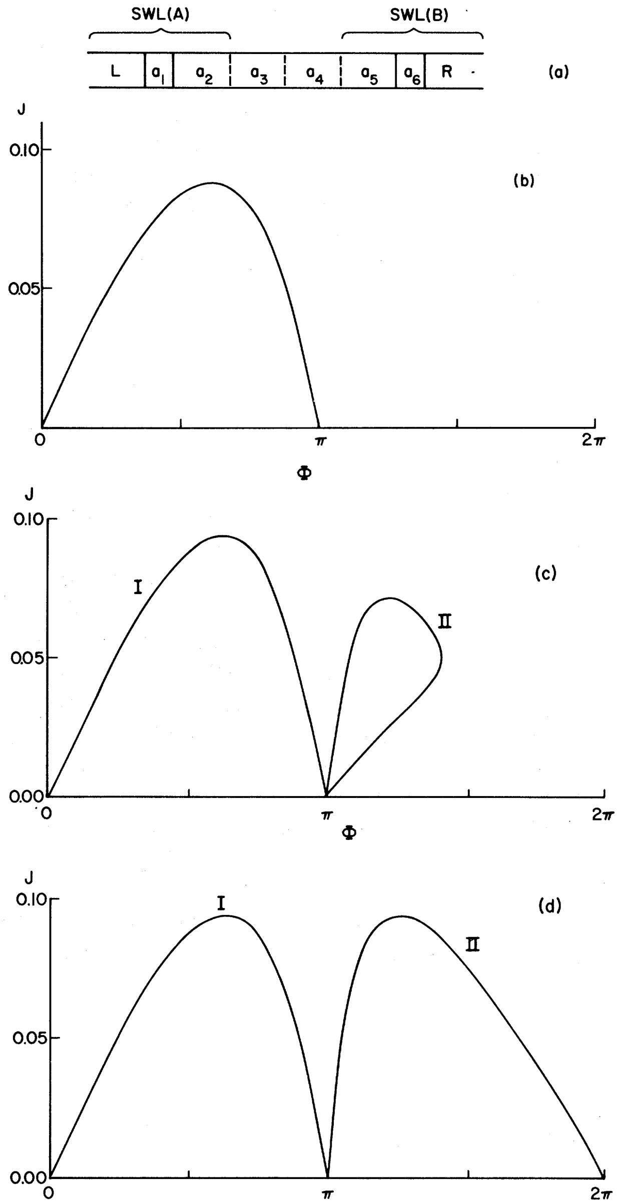

FIG. 1. (a) Schematic representation of the two-SWL model. (b)–(d) Current-phase relation at various separations: (b) $D = 0.3$; (c) $D = 0.8$; (d) $D = 3.0$. For all three plots, $\eta_1 = 0.01$, $\eta_6 = 0.02$, and $\eta = 1.0$ for all other regions; $d_1 = 0.1$, $d_6 = 0.1$,

$$D = \sum_{i=2}^{5} d_i.$$

sponding parameters in a reference superconductor, i.e., $\eta_k = \xi_k^2/\xi_r^2$, ξ_r being the coherence length of the reference superconductor, $y = \xi_r^{-1}x$, and j_k is the current density measured in units of $\xi_r^{1/2}$ which is equal to the quantity $cH_c^2\xi/\varphi_0$ defined in the reference superconductor. We normally take L to be the reference material. In dimensionless form, the thickness of each zone becomes $d_i = a_i/\xi_r$ $(i = 1, 2, \ldots, 6)$ and the separation between the two SWL's becomes $D = \bar{a}/\xi_r$.

The phase difference can be obtained from the integral

$$\Delta\Phi = \sum_k \int \frac{j_k dy}{(\eta_k \zeta_k)^{1/2} f_k^2(y)} \tag{2}$$

with appropriate limits in each zone. Our definitions are actually quite similar to those used by Baratoff and co-workers.[4] Equations (1) and (2) reduce to their simpler form given in Ref. 4 when we set $\eta_k = \zeta_k = \gamma$.

To calculate the J-Φ relation for the two-SWL system, we set j_k equal to a constant current J. For every given value of J, the whole spatial variation of $f(y)$ is obtained by starting with a trial solution and following by iterations. The total phase difference is then obtained by integration. Current-phase plots for three different separations are shown in Fig. 1. In these plots, Φ is defined the same way as in Ref. 4, which is the phase difference between results obtained from (2) for a given J with and without the weak links.

From the curves shown in Fig. 1 we see that when the SWL's are very close to each other [$D = 0.3$, Fig. 1(b)], the current-phase relation takes the same form as a single SWL, signifying that the two-SWL system now behaves as a single coherent quantum entity. For a large separation [$D = 3.0$, Fig. 1(d)], there are two complete J-Φ branches, indicating that the two SWL's are essentially independent of each other.[9] In the intermediate region [Fig. 1(c)], the J-Φ plot shows evolution of the second branch which has split off from the single coherent system. This behavior of branching out can be followed step by step through gradual changes in the separation D.

When the second branch [II in Figs. 1(c) and (d)] starts to grow, the J-Φ curve may become multivalued. In order to decide which part on the curve should be energetically more favorable, we have calculated the GL free energy corresponding to each point on the J-Φ curve. The first branch [I in Figs. 1(c) and (d)] always has lower free energy F than II. F is a monotonic increasing function of Φ for $0 < \Phi < \pi$. It appears that F takes a discontinuous jump between I and II at $\Phi = \pi$. The lower part of II normally has higher free energy than the upper part. The energy structure in the F-Φ domain resembles the band structure of a periodic lattice; this point will be discussed in detail in a later communication. From these considerations we feel that it should be interesting to measure the entire branch I and part of the branch II by placing the two-SWL system in a superconducting ring. These experiments should provide information on the dc coupling of two SWL's and on the spatial distribution of the pair potential.

From the maximum of branch I, we determine the critical current of the system. This has been done for two different sets of parameters in the weakened regions (a_1 and a_6) with the separation D varying between 0 and 3.0. The critical current as a function of D is shown in Fig. 2.

It is interesting to note that the critical current decreases with decreasing separation D. The lower limit is set by $D=0$ when the two SWL's physically merge into one but with a larger thickness of the weakened region than that of the original SWL. If the separation is large, the two SWL's are independent, and thus the critical current of the twin approaches the smaller of the two individual critical currents. The variation of critical current for intermediate separations can be understood by considering the depression of order parameter in the region which connects the two SWL's [a_2-a_5, Fig. 1(a)]. Narrower separations will require more depression of f in order to satisfy the boundary conditions, thereby reducing the total critical current. This variation of I_c is in agreement with one experiment[10] in which locking of two different critical currents was observed at a value lower than either one of the individual critical currents.

The coherence effect arising from stationary interaction described above should become more pronounced for longer ξ. This is borne out in the observation of stronger locking of critical currents when T is closer to T_c.[10,11]

We have also calculated the effect on the critical current of one SWL due to a dc current in the neighboring SWL. This is done by considering a current sink (or source) at the interfaces marked with dashed lines in Fig. 1(a), thus corresponding to the experimental situation where the two SWL's are current biased separately. The boundary conditions applied to each interface ensure that the current density is continuous at every point. For simplicity, we have used "rigid boundary condition" (i.e., f is a constant determined by the current) in L and R for this calculation.

For every input current I_B flowing in SWL (B), the spatial variation of $f(y)$ is calculated over the whole system and from which the J-Φ relation for SWL (A) is obtained. The critical current I_c for SWL (A) is computed for two relative directions of I_B. Results are shown in Fig. 3. The current in B has the effect of lowering the critical current of A. This can be understood by considering the spatial distribution of $f(y)$. Higher

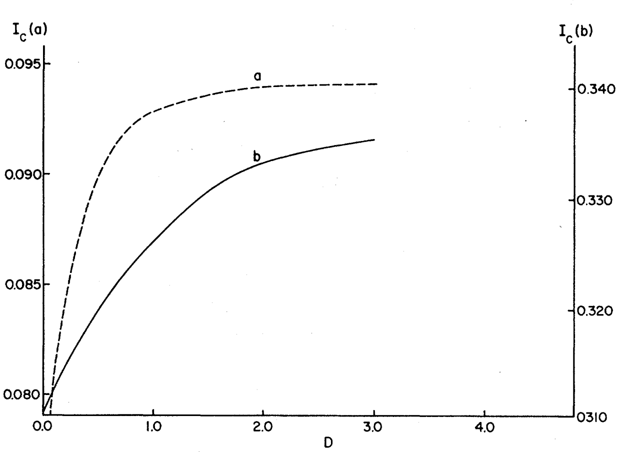

FIG. 2. Critical current of the two-SWL system as a function of D. $\eta=1.0$ in regions other than 1 and 6, $d_1=d_2=0.1$ for both curves. Curve a, $\eta_1=0.01$ and $\eta_6=0.02$; individual critical currents are $I_{cA}=0.094$, $I_{cB}=0.16$. Curve b, $\eta_1=0.1$ and $\eta_6=0.2$; individual critical currents are $I_{cA}=0.336$, $I_{cB}=0.369$.

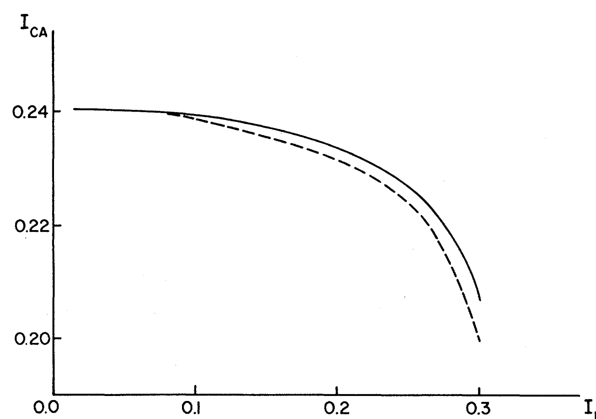

FIG. 3. Critical current of SWL (A) as a function of current I_B in SWL (B). $\eta_1=0.03$, $\eta_6=0.05$, and $\eta=1.9$ in all other regions. $d_1=d_3=d_4=d_6=0.1$, $d_2=d_5=0.4$. Rigid boundary conditions are used in this calculation. Dashed curve, currents in the same direction; solid curve, currents in opposite directions. Current sinks (sources) are introduced at boundaries shown by dashed lines in Fig. 1(a).

current flowing in B results in a further depression of f and the continuity conditions imposed at the boundaries require that f in A should also be depressed further, thereby leading to a decrease in the critical current I_{cA}.

The difference in the reduction of I_{cA} due to I_B flowing in two relative directions arises from the different pair density distributions in the region which connects the two SWL's. When currents in the two SWL's are opposite and meet in the middle of the system, their influence on the depression of $f(y)$ in the middle is less effective than in the case when the two currents are in the same direction. Hence, the curve for currents flowing against each other should lie above that for currents flowing in the same direction. The curves shown in Fig. 3 are in good qualitative agreement with experimental results.[1,11] The stationary interaction thus explains the symmetric part[1,2] of critical-current depression in the $V=0$ region.

In conclusion, we have shown explicitly that the stationary interaction between two SWL's gives rise to coherent behavior of the system when the separation is around one coherence length. The decrease in critical current of one SWL due to pair-density depression originating from a current in a neighboring SWL is in good agreement with experiment. Details of the present calculation will be presented in a separate publication.

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Orbital Model of Thermochemical Parameters

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A thermochemical scheme containing two adjustable variables per element has been developed by Miedema and co-workers which correctly predicts the signs of the heats of formation of 500 binary metallic alloys. Here we propose a simple but l-dependent orbital quantum model based on the valence energies of hydrogenic ions which contains *no adjustable elemental variables* and which fits the Miedema variables for 25 simple metals with an rms accuracy of 3%.

Some of the central questions in condensed-matter physics are thermochemical in nature and involve the existence and the crystal structures of binary compounds and alloys. Although quantum mechanics provides us with a general recipe for answering these questions (at least in principle) for any individual system, a central theoretical goal has been the development of atomistic concepts which will predict global trends in all possible binary interactions in terms of separable elemental parameters. If we consider the global ensemble of all such binary combinations, the number of possibilities is so great and the distinctions in energy so fine as to preclude the development of a deductive, quantum-mechanical solution from "first principles." Thus the literature abounds with atomistic empirical schemes of one kind or another, and often there seem to be no criteria which can distinguish among them. While it may be difficult or impossible to use