clusion of McCartan and Farr[10] that the $B\,^2\Sigma$ well depth is 0.5 cm^{-1} is inconsistent with our observation of a level bound by 2.4 ± 0.8 cm^{-1} in the $B\,^2\Sigma$ potential. The failure of York, Scheps, and Gallagher[9] to observe a pressure dependence of the far red wing fluorescence of Na-Ne suggests to us that their lowest pressure (40 Torr) was not low enough to avoid three-body collisions which formed bound NaNe*. This implies that the systems NaHe, LiNe, and LiHe may also have D_{eA}'s, which are a larger fraction of kT than concluded by Gallagher and co-workers.[9,22] It remains to be seen if more sophisticated theories of line broadening can explain the line shapes observed by the preceding workers using potentials consistent with the findings of this work.

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Nonlinear Dynamics of Drift-Cyclotron Instability

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Nonlinear shift of the ion gyrofrequency by the electrostatic ion cyclotron wave is shown to detune the resonance between the ion cyclotron wave and the ion drift wave, thereby stabilizing the drift-cyclotron instability. Solution to the resulting wave equation exhibits nonlinear oscillations.

Large-amplitude ion cyclotron waves have been observed in a variety of plasmas including mirror machines,[1] tokamaks,[2] Q-machines,[3] and the magnetosphere.[4] In an inhomogeneous plasma, ion cyclotron wave may become unstable when interacting with the ion diamagnetic-drift wave[5] and this drift-cyclotron instability has been observed in a multipole device.[6] In mirror machines, a single mode near the ion cyclotron frequency propagating in the direction of the ion diamagnetic drift is observed, characteristic of the drift-cyclotron mode for a nearly filled loss-cone distribution. (Catto[7] has pointed out that a temperature gradient, which we do not consider here, may importantly affect the linear theory of the mode.) This mode is a common occurrence in mirror machines, dominantly affecting the plasma containment.[1] It is also an important microinstability in the post-implosion phase of θ-pinch experiments when the plasma density gradient becomes diffused.[8] In this Letter, we study the nonlinear behavior of a large-amplitude ion cyclotron wave in an inhomogeneous, Maxwellian plasma, propagating in a direction perpendicular to the magnetic field. It is shown that the ion gyrofrequency is shifted by the electrostatic ion cyclotron wave. Because the drift-cyclotron instability is due to the resonant interaction of the ion Bernstein wave and the ion drift wave,[5] this nonlinear frequency shift can detune the resonance once the mode grows to a sufficiently large amplitude, leading to the saturation of the instability. We then solve the nonlinear wave equation to find a relaxation oscillation of the amplitude, possibly similar to the bursting instability observed in mirror machine.[1]

Consider an electrostatic ion cyclotron wave in an inhomogeneous plasma with density $N(y) = N_0(1+y/L)$ immersed in a magnetic field $\vec{B}=B\vec{z}$. The potential of the wave is of the form $\varphi=\Phi \times\exp[i(kx-\omega t)]$, where x is the direction of the ion diamagnetic drift $\vec{v}_d=(cT_i/eB^2N)\vec{B}\times\nabla N$, $\omega \simeq n\Omega = neB/mc$ (ion cyclotron frequency), and T_i is the ion temperature. The Hamiltonian equations of the particle motion with canonical variables $\mu = Mv_\perp^2/2\Omega$ (the magnetic moment) and $\theta = \arcsin(v_y/v_\perp)$ (the gyroangle) are[9]

$$\dot{\mu} = -\partial H/\partial\theta \text{ and } \dot{\theta} = \partial H/\partial\mu. \tag{1}$$

The Hamiltonian H can be decomposed into that of the unperturbed motion—gyration in the magnetic field $H_0 = \mu\Omega$—and that of the interaction with the electrostatic wave,

$$H_I = e\varphi(x,y,t) = e\Phi\sum_l J_l(k\rho)\exp[i(l\theta-\omega t)+ikX],$$

where $\rho = v_\perp/\Omega$ is the gyroradius, J_l are the Bessel functions, and $X = x-\rho\sin\theta$ is the guiding-center position. For $H_I \ll H_0$, Eq. (1) can be solved perturbatively by expanding $\mu=\sum_l\epsilon^l\mu_l$ and $\theta=\sum_l\epsilon^l\theta_l$ with the small parameter $\epsilon = e\varphi/Mv_\perp^2$. Because $\omega\simeq n\Omega$, we need to keep only the nth term in H_I and obtain, in the second order, the following shift in the gyrofrequency:

$$\Delta\Omega = \mathring{\theta}_2 = -\frac{ne^2|\varphi|^2}{\Delta\omega}\frac{d^2[J_n^2(k\rho)]}{d\mu^2}, \tag{2}$$

where $\Delta\omega = n\Omega-\omega$ is the frequency mismatch between the wave frequency and the nth harmonic of the cyclotron frequency and $|\Delta\omega|\gg\Delta\Omega$ is assumed. When this frequency shift $\Delta\Omega$, upon proper averaging over the distribution, becomes comparable to the linear growth rate of the drift-cyclotron mode, then the detuning of the resonance between the ion drift wave and the ion cyclotron wave is expected and the saturation of the instability results. To derive quantitatively the saturation level and the nonlinear dynamics of the drift-cyclotron instability, we solve the Vlasov equation with the canonical variables μ, θ, X, and Y (guiding-center coordinates), with $Y = y+\rho\cos\theta$:

$$\frac{\partial f}{\partial t}+\frac{\partial H}{\partial\mu}\frac{\partial f}{\partial\theta}-\frac{\partial H}{\partial\theta}\frac{\partial f}{\partial\mu}+\frac{1}{\Omega}\left(\frac{\partial H}{\partial X}\frac{\partial f}{\partial Y}-\frac{\partial H}{\partial Y}\frac{\partial f}{\partial X}\right)=0, \tag{3}$$

where we have used $\dot{X} = -\Omega^{-1}\partial H/\partial Y$ and $\dot{Y}=\Omega^{-1}\partial H/\partial X$ and $H = H_0+H_I$ with

$$H_I = e\Phi\exp(ikX-i\omega t)\sum_l J_l(k\rho)\exp^{il\theta} \simeq e\Phi\exp(ikX-i\omega t)J_n(k\rho)\exp(in\theta)$$

for $\omega\simeq n\Omega$. The unperturbed distribution function is taken to be $f_0(\mu, Y) = [N_0(Y)\Omega/\pi T]\exp(-\mu\Omega/T)$ for an inhomogeneous Maxwellian plasma. Expanding the perturbed distribution function in the powers of ϵ so that $\delta f = \sum_l\epsilon^l\delta f_l$, we find the first-order perturbed distribution

$$\delta f_1 = (\omega-n\Omega)^{-1}en\Phi\left[\frac{\partial}{\partial\mu}-\frac{k}{n\Omega}\frac{\partial}{\partial Y}\right]f_0\sum_l J_n(k\rho)J_l(k\rho)\exp[ikx-i\omega t+i(n-l)\theta], \tag{4}$$

where the expansion $\exp(-ik\rho\sin\theta) = \sum_l J_l(k\rho)\exp(-il\theta)$ is used in converting the guiding-center coordinate X to the particle coordinate x. In the second order, we obtain a dc term $\delta\bar{f}_2$, corresponding to the quasilinear modification of the unperturbed distribution, and a second-harmonic term $\delta\tilde{f}_2$, as follows:

$$\delta\bar{f}_2 = \frac{e^2|\varphi|^2n^2}{(\omega-n\Omega)^2}\left[J_n\left(\frac{\partial}{\partial\mu}-\frac{k}{n\Omega}\frac{\partial}{\partial Y}\right)+\frac{\partial J_n}{\partial\mu}\right]\left[J_n\left(\frac{\partial}{\partial\mu}-\frac{k}{n\Omega}\frac{\partial}{\partial Y}\right)\right]f_0, \tag{5}$$

$$\delta\tilde{f}_2 = -\frac{1}{2}\frac{n^2e^2\Phi^2}{(\omega-n\Omega)^2}\left[J_n\left(\frac{\partial}{\partial\mu}-\frac{k}{n\Omega}\frac{\partial}{\partial Y}\right)-\frac{\partial J_n}{\partial\mu}\right]\left[J_n\left(\frac{\partial}{\partial\mu}-\frac{k}{n\Omega}\frac{\partial}{\partial Y}\right)\right]f_0\exp[i2kX+2i(n\theta-\omega t)]. \tag{6}$$

We note that $\delta\tilde{f}_2$ is proportional to the ponderomotive potential $\psi = e^2k^2|\Phi|^2/M(\omega - n\Omega)^2$, representing the effect of the "fake temperature" associated with particle oscillatory energy in the wave. In an inhomogeneous plasma this "fake temperature" gives rise to a dc density modification:

$$\overline{\Delta n}_2 = \int \frac{d\mu\, d\theta}{2\pi}\,\delta\bar{f}_2 = \frac{e^2|\Phi|^2 nk}{M(\omega - n\Omega)^2 T}\,\frac{\partial N_0}{\partial Y}\left(1 - \frac{\omega_*}{n\Omega}\right) I_n(b)e^{-b}, \tag{7}$$

where $\omega_* = kV_d$ is the ion diamagnetic-drift frequency and $I_n(b)$ is the modified Bessel function with argument $b = (k\rho_i)^2 = k^2(T_i/M\Omega^2)$. For large $k\rho_i$, $I_n(b)e^{-b} \sim 1/\sqrt{b} = 1/k\rho_i$. Because the dc electron density is not as strongly affected by this single mode, a static electric field

$$E = \frac{4\pi e^3|\Phi|^2 nk}{M(\omega - n\Omega)^2 T} N_0 \;\; 1 - \frac{\omega_*}{n\Omega} \;\; I_n(b)e^{-b}\left(1 + \frac{4\pi N_0 Mc^2}{B^2}\right)^{-1} \tag{8}$$

will be set up and the plasma undergoes the $\vec{E}\times\vec{B}$ drift. For a linear density profile, this $\vec{E}\times\vec{B}$ drift is uniform and has no effect on the stability. For other density profiles, the $\vec{E}\times\vec{B}$ drift will be nonuniform, giving rise to an additional frequency shift which, however, is smaller than $\Delta\Omega$ by a factor $I_n(b)e^{-b}$ and will not be considered here.

In the third order, we obtain the nonlinear effect of the wave on itself corresponding to the nonlinear gyrofrequency shift through the third-order distribution having the same phase as the linear wave:

$$\delta f_3 = -\frac{e^3|\Phi|^2\Phi n^3 J_n}{(\omega - n\Omega)^3}\left(\frac{\partial^2 J_n^{\,2}}{\partial\mu^2}\right)\left(\frac{\partial}{\partial\mu} - \frac{k}{n\Omega}\frac{\partial}{\partial Y}\right) f_0 \exp[ikX + i(n\theta - \omega t)] = \frac{\partial(\delta f_1)}{\partial\Omega}\Delta\Omega. \tag{9}$$

The dominant contribution to Eq. (9) arises from the effect of $\delta\bar{f}_2$ [see Eq. (5)]. It may easily be shown that the effect of $\delta\tilde{f}_2$ and the corresponding $\tilde{E}_2$ is much smaller in the large-$k\rho$ limit primarily because in $\delta\tilde{f}_2$ the operator $\partial/\partial\mu$ does not act on the rapidly varying Bessel factors. The perturbed ion density evaluated at the particle coordinate x and in phase with the wave potential, i.e., varying as $\exp(ikx - i\omega t)$, is therefore

$$\delta n_i = \int \frac{d\mu\, d\theta}{2\pi}(\delta f_1 + \delta f_3) = -\frac{N_0 e\Phi}{T_i}\left\{1 - \frac{\omega - \omega_*}{\omega - n\Omega} I_n(b)e^{-b} - \frac{\omega - \omega_*}{\omega - n\Omega}\frac{\psi}{T}\beta\right\} e^{ikx - i\omega t}, \tag{10}$$

where the last term is the nonlinear density fluctuation with $\beta = n^2\int_0^\infty x^{-1}[dJ_n^{\,2}(x)/dx]^2 dx = 4n^2[\pi^2(2n+1)(2n-1)]^{-1}$, and change to the particle coordinate x from the guiding-center coordinate X is made by setting $X = x - \rho\sin\theta$ and expanding $\exp ikX = \sum_l J_l(k\rho)\exp(-il\theta + ikx)$. The perturbed electron density is the usual linear response to the flute mode:

$$\delta n_e = \frac{e\varphi}{T_i}\left(-\frac{\omega_*}{\omega} + k^2\rho_i^{\,2}\frac{m}{M}\right). \tag{11}$$

Substituting Eqs. (10) and (11) into the Poisson equation, we find the following nonlinear dispersion relation:

$$\left(1 - \frac{\omega_*}{\omega}\right)\left(1 - \frac{[I_n(b)e^{-b} + \psi\beta/T]\omega}{\omega - n\Omega}\right) = -k^2(\lambda_D^{\,2} + \rho_e^{\,2}), \tag{12}$$

where $\lambda_D = (T/4\pi ne^2)^{1/2}$ the Debye length and $\rho_e = (m/M)^{1/2}\rho_i$, the electron gyroradius. Setting $\psi = 0$ and $\omega = \omega_*(1+\delta) = n\Omega[1 + I_n(b)e^{-b} + \delta]$ with $\delta \ll 1$, we find that the instability exists for $\rho_i/L > (m/M + \Omega^2/\omega_{pi}^{\,2})$ and the linear growth rate $\gamma = (k/\rho_i)^{1/2}(\lambda_D^{\,2} + \rho_e^{\,2})^{1/2} n\Omega$ with $k = nL/\rho_i^{\,2}$ where $L^{-1} = d\ln N/dY$ and $\omega_{pi} = (4\pi ne^2/M)^{1/2}$. With the nonlinear term included, i.e., $\psi \neq 0$, we find the equation for δ to be $\delta^2 - \delta\Delta = -\gamma^2/n^2\Omega^2$, where $\Delta = \beta\psi/T$. Thus nonlinear stabilization of the original mode is achieved when δ is real or $\Delta^2/4 \geq \gamma^2/n^2\Omega^2$. The resulting amplitude at saturation is

$$e\Phi_s/T_i \approx (m/M)^{1/4}(\rho_i/L)^{7/4}. \tag{13}$$

For $n = 1$, this is of the order of a few percent for $\rho_i/L \approx 1/2$, consistent with the value observed in 2X2B.[1] In larger devices with $\rho_i/L \ll 1$ the nonlinear frequency shift would limit $e\varphi/T$ to very small values, probably well below the threshold for violation of super-adiabatic confinement.[10] Whether such stabilization would be effective against the more rapid drift-cone modes characteristic of non-Maxwellian distributions remains to be investigated.

Although this nonlinear frequency shift detunes the resonance for the original wave, one may question whether a new resonance may be satisfied for a new mode with slightly different k, i.e., $k\rho_i^2/L = n[1 - I_n(b)e^{-b} - \beta\psi/T]$ which in turn will be unstable. This is not the case if the quantization condition allows only one discrete k number, and more importantly, if $\psi \propto |\Phi|^2$ has rapid nonlinear oscillation around the saturation level, which is indeed the case as shown below. The detuning of the resonance due to the nonlinear shift of the cyclotron frequency not only stops the growth but also generates nonlinear oscillation in the wave amplitude because the overshot of the amplitude above the saturation level given by (13) causes the wave to be damped. The wave damping eventually restores the resonance and the instability sets in again provided the free-energy source of the instability—the density gradient is not relaxed over this time scale.

To examine the nonlinear temporal behavior of the wave amplitude, we derive the following nonlinear wave equation for the amplitude by setting $\delta\omega = \omega - \omega_* = \omega - n\Omega(1+\Gamma) \rightarrow i\partial/\partial t$ in Eq. (12) with a slowly varying $\Phi(t)$:

$$\frac{\partial^2\Phi}{\partial t^2} + i\alpha|\varphi|^2\frac{\partial\varphi}{\partial t} = \gamma^2\Phi \tag{14}$$

where $\alpha = \beta e^2k^2/Mn\Omega I_n^2(b)e^{-2b}T_i$. [To include spatial variation, one would put $\omega_* \rightarrow \omega_* - (i/k)\partial/\partial y$.] Letting $\Phi = re^{i\chi}$ (r, χ real), we find

$$\tfrac{1}{2}(\dot{r}^2 + r^2\dot{\chi}^2) - r^2\gamma^2 = C_1, \tag{15}$$

$$\dot{\chi} + \tfrac{1}{4}\alpha r^2 = C_2/r^2, \tag{16}$$

where C_1 and C_2 are constants of motion corresponding to the energy and angular momentum. Because the wave initially grows from the thermal level, we may set $C_1 = C_2 \simeq 0$. The resulting equation becomes

$$\int(-\tfrac{1}{4}\alpha r^6 + 2\gamma^2r^2)^{-1/2}\,dr = t. \tag{17}$$

The period implied by Eq. (17) is infinite. With a small level of thermal noise, i.e., $C_1, C_2 \neq 0$ but small, the period would be $\gamma^{-1}\ln(r_{\max}/r_0)$ i.e., perhaps 10 times the inverse growth time. The maximum amplitude is given by

$$r_{\max}{}^2 = 2\sqrt{2}\,\gamma/\sqrt{\alpha} \tag{18}$$

in agreement with Eq. (13). Such nonlinear oscillations are somewhat similar to the bursting phenomenon observed in mirror machine.[1] Eventually collisions or wave coupling will damp the nonlinear oscillation and a steady-state saturation level given by Eq. (13) may be then reached. However a definitive resolution of this question awaits the solution of the spatially dependent nonlinear problem.

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Analytic Solutions of the Two-Dimensional Eigenvalue Problem for the Trapped-Electron Instability in Tokamaks

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The radial localization of the destabilizing trapped-electron term that is caused by the differences in the pitch of the magnetic field and the mode structure is shown to result in a completely new form of the dispersion relation for the trapped-electron instability.

The trapped-electron instability is basically a drift wave that is driven unstable by the presence of trapped electrons.[1,2] Because the frequencies of interest are much less than the electron bounce frequency, the trapped electrons respond to the pitch of the tokamak magnetic field, $\vec{B}=(B_0R_0/R)\times[\hat{\zeta}+(\epsilon/q)\hat{\theta}]$, while the ions and adiabatic electrons respond to the pitch of the mode structure. The quantities, r,θ, and ζ are the radial, poloidal, and toroidal variables, respectively; $\epsilon=r/R_0$, $R=R_0(1+\epsilon\cos\theta)$, q is the safety factor, and R_0 is the major radius of the magnetic axis. The response of the trapped electrons to the magnetic field results in a coupling of poloidal modes; the full eigenvalue equation is a second-order differential equation in the radial coordinate and an integral equation in the poloidal variable.

Because of the important effects that the trapped-electron instability may have on the transport in tokamaks, a complete understanding of the two-dimensional eigenvalue equation is essential. Numerical solutions of this equation are currently being pursued and preliminary results are available.[3,4] Previous analytic investigations have been on one-dimensional ones only, retaining either the radial[1] or the poloidal[2] variation. However, to retain only the poloidal variation is to neglect completely the stabilizing influence of shear and, perhaps more importantly, the radial localization of the destabilizing trapped-electron-induced coupling term. On the other hand, retention of just the radial variation has been possible only in the limit in which the mode extent, x_i, is much less than the spacing, $\Delta=(l\,\partial q/\partial r)^{-1}$, between rational surfaces of the same toroidal-mode number l, but different poloidal-mode numbers m. This isolated–rational-surface model completely neglects, therefore, the coupling of rational surfaces and thus only treats radial variations caused by shear. In particular, the maximum growth rates in this isolated–rational-surface model are found to occur for $m\rho_i/r\sim 1$, where $\rho_i=(M_iT_i)^{1/2}c/eB_0$ is the ion gyration radius and m/r the poloidal wave-vector component. For $m\rho_i/r\gtrsim 1$, however, the mode centered about the rational surface at which $q(r_m)=m/l$ overlaps the neighboring few rational surfaces having the same l. As a result, contribution from neighboring rational surfaces cannot be neglected. Furthermore, because the radial variation of the destabilizing trapped-electron term is on the scale Δ and because $\Delta\lesssim x_i$, the radial localization of the trapped-electron contribution must be retained.

In this Letter, an analytic solution of the full two-dimensional eigenvalue equation is presented which does not suffer from the shortcomings of previous analytic treatments and which provides needed insight into the preliminary numerical results.[3,4] In fact, the differential equation has been solved analytically by two techniques. Only the simpler perturbation-theory solution will be sketched in this Letter. Further mathematical details and the more rigorous solution by a method of matched asymptotic expansions[5] will be presented in an expanded article.[6]