and experiment is quite good. Spectra computed with plane-wave and constant matrix elements gave poor agreement with experiment. We note that agreement with experiment would be improved if the theoretical spectra were convoluted with an asymmetric energy-broadening function.

One notable success of the present model is the predicted strength of the peak near 5 eV of binding energy occurring in Fig. 2. We find that this peak is 80% $d$ like, but it occurs weakly since it has mostly $m=2$ character. The relative strength of this peak should increase when the photon energy and angle of emission are chosen so that $\vec{k}_i$ and $\vec{k}_f$ are perpendicular to each other.

We have thus demonstrated that the details of the final-state band structure are not terribly important and that a simple model based upon the free-electron–like energy dispersion and atomic-like dipole selection rules provides a good description for the angle-resolved photoemission spectra of Cu and possibly of other noble and transition metals. Complications due to many-body effects, the surface potential, multiple scattering, and surface bands seem to be unimportant for describing the spectra.[17] The success of the model introduced here opens the possibility of using experimental peak positions to map out the energy bands, and the experimental peak strengths and the polarization dependence to determine the orbital composition of the initial states.

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# Voltage Measurements within the Nonequilibrium Region near Phase-Slip Centers

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We have used complementary sets of normal and superconducting probes to study the nonequilibrium region near phase-slip centers in narrow superconducting strips. An exponentially decaying branch imbalance potential, $\mu_Q$, is observed and the decay length, the quasiparticle diffusion length, is measured directly and compared to the phase-slip resistance.

In recent studies[1-5] of the current-voltage ($I$-$V$) characteristics of narrow superconducting strips and microbridges, a significant role is attributed to the quasiparticles generated at localized resistive regions or "phase-slip centers" in the superconductor. It is concluded that $R_D$, the differential resistance of a phase-slip center above the critical current, and the range of phase-slip center interactions are determined by the quasiparticle diffusion length. This length is $\lambda_Q \cong (v\tau_Q l)^{1/2}$, where $v$ and $\tau_Q$ are the velocity and relaxation time for the quasiparticles and $l$ is the electron mean free path.[1,6] In explaining the role of $\lambda_Q$, Skocpol, Beasley, and Tinkham[1] have described the nonequilibrium region near a phase-slip center in terms of potentials $\mu_P$ and $\mu_Q$ for the paired electrons and quasiparticles. While $\mu_P$ varies spatially only over a distance of order $\xi$ (the coherence length) near a phase-slip center, $\mu_Q$ decays exponentially toward $\mu_P$ with characteristic length $\lambda_Q > \xi$. The gradient in $\mu_Q$ corresponds to a quasiparticle current, $\vec{j}_Q = -\nabla\mu_Q/\rho e$, in a sample of resistivity $\rho$; the phase-slip resistance is $R_D = 2\lambda_Q dR/dx$ for a uniform strip of resistance per unit length, $dR/dx$. Because a net charge transport is associated with $\nabla\mu_Q$, it seems to us that $\mu_Q$ must be interpreted as the "branch imbalance potential," i.e., $\mu_Q \equiv \mu_> - \mu_<$, where $\mu_>$ and $\mu_<$ are the electrochemical potentials for the "electronlike" ($k > k_F$ and "holelike" ($k < k_F$) branches of the quasiparticle excitation spectrum.[7-10] Such a potential has been observed directly in quasiparticle-injection experiments.[7,10] However, a much larger branch imbalance is required to explain the phase-slip experiments and, in this case, the mechanism for the generation of the imbalance is not understood. Moreover, the reported[1,3] temperature independence of $R_D \propto \lambda_Q \propto \tau_Q^{1/2}$ implies a conflict with the well-understood injection experiments which show that, near $T_c$, the branch imbalance relaxes by inelastic phonon scattering with characteristic time

$$\tau_Q(T) = \tau_Q(0)(1 - T/T_c)^{-1/2}.$$

The experiments described below provide a direct and detailed test of the ideas of Skocpol, Beasley, and Tinkham and show that $\mu_Q$ is the appropriate potential. This is possible because $\mu_Q/e$ is detectable as a voltage (setting $\mu_Q = \mu_P = 0$ at a remote location where the sample is in equilibrium) using probes of a normal material while superconducting probes measure $\mu_P/e$.[7-10] We have used both kinds of probes to measure $\mu_P$ and $\mu_Q$ near isolated phase-slip centers in thin film strips of tin and indium. This is done with sufficient spatial resolution to plot the spatial variation of $\mu_Q$ and measure $\lambda_Q$ directly. We verify that $\mu_Q$ is the same potential measured in quasiparticle-injection experiments and has the expected temperature dependence. Finally, the relationship between $R_D$ and $\lambda_Q$ is examined.

Figure 1 shows a sketch of the sample geometry and a scanning electron micrograph of a tin sample with attached electrodes of silver ($N$) and a Pb alloy ($S$). Most samples studied were evaporated onto silicon substrates and had thickness, $d$, and mean free path, $l$, in the range 0.1 to 0.2 $\mu$m. The strips were 2 $\mu$m wide over most of their 50 $\mu$m length. A central section either 16 or 20 $\mu$m long narrowed to 1 $\mu$m in width and had a small notch (20–30% further width reduction) to facilitate phase-slip at the notch location. Two arrays of seven electrodes each were attached to the central section as shown in Fig. 1. The electrode spacing was either 2.0 or 2.5 $\mu$m. The electrodes were deposited after the sample itself was deposited and oxidized so that oxide-barrier tunnel junctions were formed at the joints. This feature is necessary to minimize distortions of the samples' metalurgical and electronic properties and to provide a well-defined electrode-sample interface. The patterning of the sample was achieved using a photolithographic technique described elsewhere.[11]

$I$-$V$ characteristics were obtained directly using a high-impedance amplifier to measure the voltage while sweeping the bias current at low frequency. The amplifier output was signal averaged to obtain curves like those shown in Fig. 2. In these low-frequency measurements, only the time-averaged quantities $\bar{\mu}_P$ and $\bar{\mu}_Q$ are measured. This distinction is important only within $\xi$ of the phase-slip center.

Figure 2 compares the $I$-$V$ characteristics of 2.5-$\mu$m segments of an indium strip to the char-

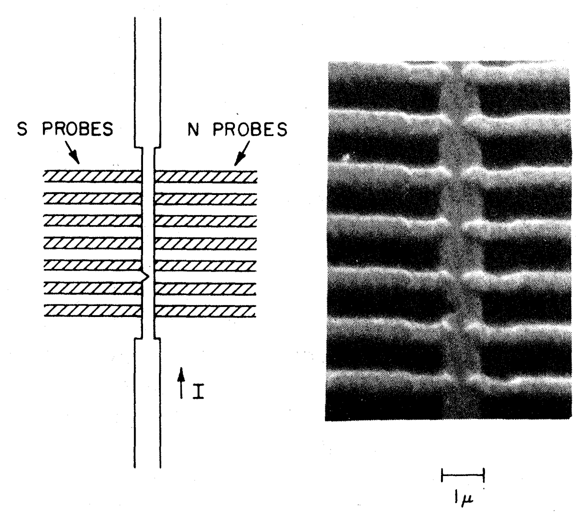


FIG. 1. A sketch of the sample geometry and a scanning electron micrograph of a tin sample with attached electrodes of a normal ($N$) and superconducting ($S$) material. The electrode-sample interfaces are oxide-barrier tunnel junctions.

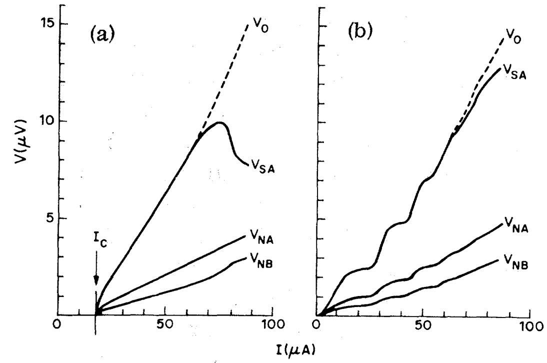


FIG. 2. $I$-$V$ characteristics for a phase-slip center in an indium strip. (a) The curve $V_0$ is the characteristics for the entire strip. The curve $V_{SA}$ is obtained using superconducting probes spanning only a 2.5-$\mu$m strip segment which contains the phase-slip center. Normal probes are used to obtain the curve $V_{NA}$ for this segment and the curve $V_{NB}$ for an adjacent segment for which no voltage is detected with superconducting probes. (b) Same as (a), except that a 1.1-GHz rf signal is present.

acteristic, $V_0$, of the entire strip. A phase-slip center nucleates at current $I_c$ at a position located between the probes used to obtain $V_{SA}$ and $V_{NA}$. The superconducting probes ($V_{SA}$) show that the entire change in $\bar{\mu}_P$ ($\Delta\bar{\mu}_P = eV_{SA} = eV_0$) occurs across the 2.5-$\mu$m-long sample segment (the drop in $V_{SA}$ at high current results from sample heating). In contrast, only a fraction of this change, $\Delta\bar{\mu}_Q = eV_{NA}$, is detected using normal probes. The remaining change in $\bar{\mu}_Q$ occurs across other parts of the strip. The curve $V_{NB}$ shows the normal-probe signal across an adjacent 2.5-$\mu$m sample segment. In Fig. 2(b) an rf signal has been applied to induce current steps at the Josephson voltage and harmonics in $V_0$ and $V_{SA}$. The steps are of reduced amplitude in $V_{NA}$—an effect proposed by Clarke[7] as a possible source of error in voltage standards based on the Josephson step height.

Figure 3 shows a plot of the quantities $\bar{\mu}_Q(X)/e$ and $\bar{\mu}_P(X)/e$ obtained from curves like those in Fig. 2(a) but for a tin strip. The voltage detected by each probe at a fixed current is plotted at the probe position. The notch in this sample was poorly defined (very little width or resistance variation) and the initial phase-slip occurred very near one of the probe locations. The variation in the voltage $V_S$ ($V_S = \bar{\mu}_P/e$) is too rapid to be resolved in any detail but is consistent with a variation governed by a length $\lesssim \xi$ (we estimate that $\xi \sim 1.0$ $\mu$m for this sample and temperature).

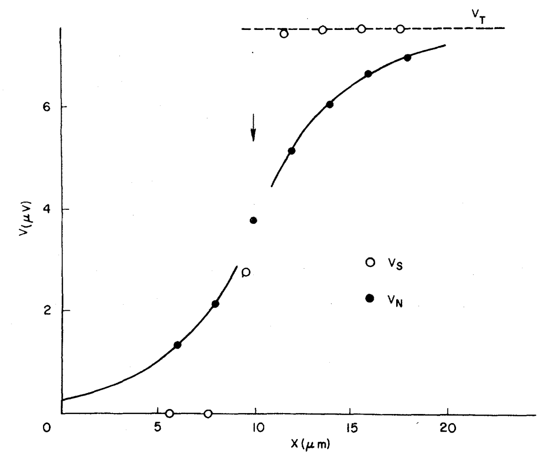


FIG. 3. The spatial variation of the quantities $V_S$ ($V_S = \bar{\mu}_P/e$) and $V_N$ ($V_N = \bar{\mu}_Q/e$) near a phase-slip center (arrow) in a tin strip.

The variation in $V_N$ ($V_N = \bar{\mu}_Q/e$) is quite well resolved and has been fitted to a simple exponential decay on either side of the phase-slip center: On the left-hand side $V_N = \frac{1}{2}V_T \exp[(X - X_0)/\lambda]$ and on the right-hand side $V_N = V_T\{1 - \frac{1}{2}\exp[(X_0' - X)/\lambda]\}$, where $V_T$ is the total phase-slip voltage (i.e., $V_0$) and $\lambda = 4.0 \pm 0.2$ $\mu$m. The outer points have been adjusted slightly (~10%—a simple application of the diffusion equation[1] is involved) to correct for the increase in sample width just outside the outer probe positions. Because $X_0 \neq X_0'$, it is clear that the length extracted from measurements of $R_D$ can, at best, only approximate $\lambda_Q$. Nevertheless, Fig. 3 is strikingly similar to the sketch of the nonequilibrium region by Skocpol, Beasley, and Tinkham in Ref. 1 and convincingly supports their description.

Figure 4 shows the temperature dependence of $\lambda_Q$ obtained from plots like that in Fig. 3 but for a different tin strip. The solid circles are the data for this decay length. The open circles are decay lengths obtained for the case when a branch imbalance is produced by injecting quasiparticles into the strip through one of the tunnel-junction electrodes; their agreement with the phase-slip data confirms that the same quantity is involved in both kinds of experiment.[12] The solid line is a least-squares fit to the phase-slip data and is given by $\lambda_Q(T) = (1.55 \pm 0.05)(1 - T/T_c)^{-n}$ $\mu$m, with $n = 0.23 \pm 0.03$. Assuming[1,6] $\lambda_Q = (\frac{1}{3} v\tau_Q l)^{1/2}$ (using the value $v = v_F = 0.65 \times 10^8$ cm/sec for the quasiparticle velocity[10] and the value $l = 0.13$ $\mu$m obtained from the strip resistivity), one obtains for this film $\tau_Q(0) = (0.9 \pm 0.2) \times 10^{-10}$ sec. This re-

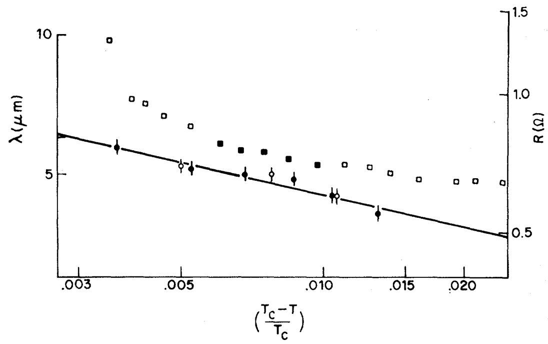


FIG. 4. Log-log plot showing the temperature dependence of the quasiparticle diffusion length (circles) and the minimum differential resistance (rectangles) observed for a phase-slip center in a tin strip; $R/\lambda = 2dR/dX$.

sult is obtained with the assumption that only inelastic scattering affects $\lambda_Q$. Also the indicated errors do not reflect variations in $\tau_Q$ which may occur for differently prepared films. The value obtained does agree, however, with the value of $(1.0 \pm 0.2) \times 10^{-10}$ sec obtained by Clarke and Patterson in their injection experiments on tin films.[10]

The data represented as rectangles in Fig. 4 are measurements of the differential resistance made in the usual way with a small ac current superimposed on the bias current. The resistance scale is related to the length scale by the factor $R/\lambda = 2dR/dX$. Only a portion of the characteristic for an isolated phase-slip center is accessible experimentally because of the nucleation of other phase-slip centers and because, at low temperatures, the characteristic is hysteretic [observable hysteresis occurs for $(T_c - T)/T_c \gtrsim 0.05$ in Fig. 4]. The high-temperature data in Fig. 4 definitely overestimate $R_D$ (the resistance for large $I/I_c$) because these data sample $dV/dI$ only for small values of $I/I_c$ where the $I$-$V$ characteristic is nonlinear for reasons intrinsic to the phase-slip process. The low-temperature data are significantly affected by simple heating in the sample and also overestimate $R_D$. They are obtained from severely hysteretic characteristics, showing marked upward curvature. Only in the center of the temperature range covered (solid rectangles) was $dV/dI$ constant (within the sensitivity of the measurement) over a finite current range; in this temperature range, $R_D$ varies an amount comparable to the variation in $\lambda_Q$ but is still larger than $2\lambda_Q dR/dX$ by about 20%. We conclude that at least the dominant contribution to $R_D$ may be attributed to the quasiparticle current in the nonequilibrium region and this contribution reflects the temperature dependence of $\lambda_Q$. We have shown all of the differential resistance data to illustrate the sources of the difficulty in obtaining more convincing evidence for the temperature dependence of $R_D$, at least in our experiments. We also observed clear evidence of a more fundamental problem which would mask any real divergences of $R_D$ near $T_c$ even for samples having more farorable characteristics: For both Sn and In, variation of 10–20 mK in the local value of $T_c$1 occurred on a scale $\lesssim \lambda_Q$. Such variations would obscure a real divergence in $R_D$ at high temperatures as heating does at low temperatures.

In summary, direct measurements of $\overline{\mu}_P$ and $\overline{\mu}_Q$ near phase-slip centers substantially confirm the description of the nonequilibrium region by Skocpol, Beasley, and Tinkham. A substantial branch imbalance exists over the length, $\lambda_Q$, which has the temperature dependence expected from quasiparticle-injection experiments and is approximately proportional to the phase-slip resistance.

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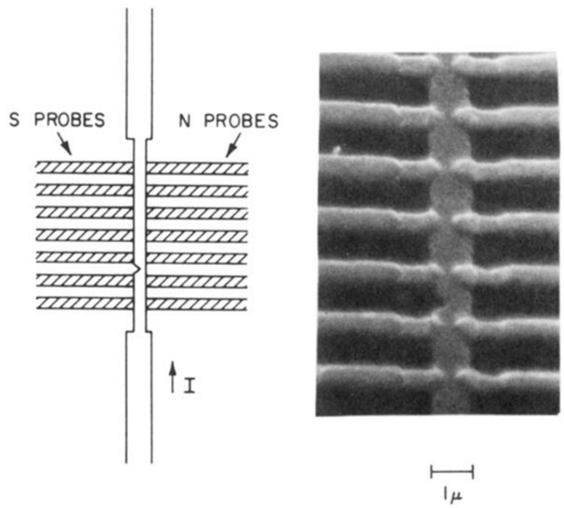


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