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Quantum Chromodynamics Test for Jets

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A new quantity, the maximum of the directed momentum, is proposed which measures jetlikeness in e^+e^- annihilation. This quantity is computed in the quark-gluon model by using renormalization-group improved perturbation theory.

Hadronic jets have been observed in electron-positron annihilation experiments at energies above 5 or 6 GeV.[1] The jet angular distribution is nicely predicted by the spin-$\frac{1}{2}$ parton model.[2] Very recently, Sterman and Weinberg[3] have shown that the jet structure follows from a trustworthy perturbative calculation in a pure field theory, quantum chromodynamics (QCD), without assuming any phenomenology such as a transverse-momentum cutoff.

In this paper a new quantity, the maximum directed momentum, or d, is defined which has two distinct virtues. First it can be used as a direct measure of the jetlikeness of an event. Second, it can be computed in QCD in the spirit of the Sterman-Weinberg work.[3] Thus the measurement of d would provide a nice test of QCD.

To define d go into the center-of-mass frame where each final-state particle has momentum $\vec{p}_a$ with $a=1$ to n (n is the number of final-state particles). For an arbitrary unit vector $\hat{r}$ consider

$$d(\hat{r}) \equiv \sum_a \vec{p}_a \cdot \hat{r}\, \theta(\vec{p}_a \cdot \hat{r}), \tag{1}$$

where θ is the unit step function. $d(\hat{r})$ tells how much momentum is directed along $\hat{r}$ since only those momenta whose $\hat{r}$ component is positive contribute to the sum. Then d is defined as

$$d \equiv \frac{\max_{\hat{r}} d(\hat{r})}{\sum_a |\vec{p}_a|}. \tag{2}$$

Thus d is found by maximizing $d(\hat{r})$ over all directions and dividing by a normalization factor. In the case of pure jets, i.e., all the momentum coming in two back-to-back, zero-opening-angle streams, one gets $d=\frac{1}{2}$. If the final state is an isotropic distribution of an infinite number of particles (the "opposite" of jets) then $d=\frac{1}{4}$. The extent to which d is close to $\frac{1}{2}$ is the extent to which the event is jetlike. This will be demonstrated again later.

The quantity d can be reliably computed, order by order perturbatively, in QCD. The argument follows that of Sterman and Weinberg.[3] Suppose we have a massive theory and do all renormalizations at some four-momenta of order E, the total energy. By use of standard renormalization-group techniques, g_E, the running coupling constant, is also defined relative to that point. In an asymptotically free theory like QCD,[4] g_E tends to zero as E grows. Even so, most quantities cannot be reliably computed because there will in general be terms like a power of $\ln(m/E)$ which crop up in higher orders (m is some typical mass). These terms ruin the perturbation expansion and they become singular as $m \to 0$.

There is a way out based on a physical assumption yet to be contradicted. Those quantities (partial cross sections, averages, etc.) which in the massless case are physically sensible, i.e., measurable in principle, will have a perturbative expansion free of $m \to 0$ singularities. So if one can unambiguously measure a quantity in the $m=0$ case it will have a reliable perturbative expansion.

The main problem with massless particles is that they are kinematically allowed to split into many massless parallel particles. If this happens the directed momentum $d(\hat{r})$ and its maximum d will both remain unchanged. They are sensible quantities in the massless case. A quantity like sphericity[5] which is quadratic in the momenta will change if a massless particle splits.

There are two types of processes contributing to the average age $\langle d\rangle$ of d, to order g_E^2. The first is $e^+e^-\to\gamma\to q\bar{q}$. To order g_E^2 this includes a vertex-correction interference term which has an infrared divergence. For this process d is always $\frac{1}{2}$. The second type is the gluon bremsstrahlung process: $e^+e^-\to\gamma\to q\bar{q}g$. The differential cross section is[6]

$$d\sigma=\Big(4\sum_{\text{flavors}}Q^2\Big)\frac{e^4g_E^2}{24\pi^3E^2}\frac{1}{\frac{1}{2}-\omega_1}\frac{1}{\frac{1}{2}-\omega_2}\times(\omega_1^2+\omega_2^2)d\omega_1d\omega_2, \qquad (3)$$

where ω_1 and ω_2 are the dimensionless quark and antiquark energies and the 4 is an SU(3) factor. This differential cross section becomes singular in the region where the gluon energy goes to 0. In this bremsstrahlung case the value of d is the maximum of the quark, antiquark, or gluon energies. As the gluon energy goes to 0 the quark-antiquark pair come out back to back, each with energy $E/2$. The value of d is $\frac{1}{2}$. In the calculation of $\langle d\rangle$ the infrared divergence in the first process cancels the divergence from the second the same way they are canceled in the total cross-section calculation. There are no leftover singularities, as we expected. The value of $\langle d\rangle$ is[7]

$$\langle d\rangle=\tfrac{1}{2}-ag_E^2/6\pi^2, \qquad (4)$$

where

$$a=\tfrac{1}{2}\int_1^2(dy/y)\ln(1+y)-\tfrac{1}{144}-\tfrac{3}{32}\ln 3. \qquad (5)$$

The integral cannot be done in terms of known elementary functions[8] and a is approximately 0.197.

We see that as $g_E\to 0$, $\langle d\rangle\to\frac{1}{2}$ so that at very high energies we may expect pure jets. If an event has very many particles, uniformly distributed in two back-to-back cones with opening half-angles α, then $d=(1+\cos\alpha)/4$. So α is a measure of the narrowness of the jet. Equating this with $\langle d\rangle$ from Eq. (4) we get

$$\cos\alpha=1-2ag_E^2/3\pi^2. \qquad (6)$$

Using the standard QCD formula[4] $g_E^2=24\pi^2/25\ln(E/\Lambda)$ with $\Lambda=0.5$ GeV at the current experimental energy of $E=7.4$ GeV, we get $\alpha\simeq 17°$. The decrease in α as E grows will be very slow.

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[4]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[5]J. Ellis, M. K. Gaillard, and G. G. Ross, Nucl. Phys. B111, 253 (1976). Interestingly, sphericity is finite to order g_E^2 but there are no assurances about higher orders.

[6]This agrees with another interesting paper on jets [A. T. DeGrand, Y. J. Ng, and S.-H. H. Tye SLAC Report No. SLAC-PUB-1950, 1977 (to be published)], but it disagrees with Ref. 5.

[7]In practice it is much easier to compute $\langle d-\frac{1}{2}\rangle$ since this is 0 for the first process and is finite for the second.

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