138 (1975); J. J. Thompson, C. E. Max, and K. G. Estabrook, Phys. Rev. Lett. 35, 663 (1975); B. Bezzerides, D. F. DuBois, D. W. Forslund, and E. L. Lindman, Phys. Rev. Lett. 38, 495 (1977).

[4]J. P. Friedberg, R. W. Mitchell, R. L. Morse, and L. F. Rudinski, Phys. Rev. Lett. 28, 795 (1972); P. Koch and J. Albritton, Phys. Rev. Lett. 32, 1420 (1974); K. G. Estabrook, E. J. Valeo, and W. L. Kruer, Phys. Fluids 18, 1151 (1975).

[5]I. B. Bernstein, Phys. Fluids 20, 577 (1977).

[6]J. Delettrez and E. B. Goldman, Laboratory for Laser Energetics, University of Rochester, Report No. LLE-36, 1976 (unpublished).

[7]E. J. Valeo and I. B. Bernstein, Phys. Fluids 19, 1348 (1975).

[8]The effective terms in the kinetic equation are $\vec{v}\cdot\nabla f = S$, so that for $S \propto \delta(r - r_{\rm cr})$, $f \sim S/v$ arises and therefore $S \propto v e^{-v}$ yields $f \propto e^{-v}$.

Drift-Wave Turbulence Effects on Magnetic Structure and Plasma Transport in Tokamaks

J. D. Callen
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 22 July 1977)

The small magnetic perturbations accompanying drift waves are shown to produce microscopic, fluctuating magnetic island structures and to enhance radial electron heat transport in tokamaks. The "magnetic-flutter"–induced electron heat-conduction coefficient is found to be $\chi_e^{\delta} \simeq \frac{3}{16}(\nu+\gamma)\delta^2$, where δ is the magnetic island width, ν is the electron-ion collision frequency, and γ is the drift-wave growth rate, or inverse island correlation time.

Small, helically resonant, magnetic perturbations can cause significant distortions of the magnetic surfaces in tokamaks. It has previously gone unrecognized that since the high-mode-number, drift-wave–type turbulence, which has been observed experimentally,[1] is probably not purely electrostatic,[2] it is, in fact, accompanied by magnetic perturbations that can induce densely packed magnetic island structures and thereby strongly affect radial plasma transport in tokamaks.

Stochastic magnetic turbulence effects were found to be important in determining radial electron heat transport in a fluid regime in the Zeta device.[3] Rechester and Rosenbluth[4] have advanced a model of radial electron heat transport due to ergodic magnetic field lines in a collisional plasma. Here, we investigate magnetic turbulence effects in tokamaks by replacing the stochastic turbulence assumption with drift-wave–induced magnetic perturbations, with their attendant spatial and time scale orderings. This new mechanism for radial electron heat transport in tokamaks may well resolve the apparent discrepancy[1] between the observed heat transport and that estimated quasilinearly from the fluctuation spectrum.

Small radial magnetic perturbations can easily modify radial plasma transport because plasma transport is much faster along magnetic field lines than perpendicular to them. To the extent that particles "see" a stochastic radial motion of the magnetic field lines, the effective radial heat transport is

$$\chi_{\rm eff} \sim \chi_\perp + (\tilde{B}_r/B)^2 \chi_\parallel, \qquad (1)$$

where $\chi_\perp$ and $\chi_\parallel$ are the heat transport coefficients perpendicular and parrallel to the magnetic field, $\tilde{B}_r$ is the (stochastic[4]) radial component of the magnetic perturbation, and B is the strength of the confining magnetic field. In a collisional plasma, $\chi_\parallel/\chi_\perp \sim \lambda^2/\rho^2$ where λ is the mean free path and ρ is the particle gyroradius. Thus, magnetic perturbation effects may be significant if $\tilde{B}_r/B > (\chi_\perp/\chi_\parallel)^{1/2} \sim \rho/\lambda$, which is very small in tokamaks (e.g., $\rho/\lambda \sim 10^{-6}$ for electrons).

The magnetic component of drift waves is caused by the fluctuating current produced by the difference between the wave-induced perpendicular drifts ($\vec{V}_\perp$) of the ions and electrons.[5] Since the perturbed current must be divergence-free for these low-frequency oscillations, the component of the perturbed current parallel to the magnetic field is given by $\tilde{J}_\parallel = (ik_\parallel)\nabla\cdot[ne(\vec{V}_{\perp i} - \vec{V}_{\perp e})]$. With the polarization and finite ion gyroradius drifts taken into account, the perturbed current induces, through Ampere's law, a perturbed magnetic field $\tilde{B}_1$ such that

$$\tilde{B}_\perp/B \simeq i[\omega/k_\parallel(x)V_A](k_\perp\hat{\rho}_i)(V_s/V_A) \times(1+T_i\omega_*/T_e\omega)(e\tilde{\varphi}/T_e), \qquad (2)$$

where $\tilde{\varphi}$ is the perturbed potential, $\omega_* \equiv k_\theta(cT_e/eB)\,d\ln n/dr$ and $k_\parallel(x) \equiv k_\theta x/L_s$, with $x \equiv r - r_s$ being the radial distance away from a rational surface, k_θ is the poloidal-mode number, $L_s^{-1} \equiv (r/Rq)q^{-1}dq/dr$, $V_A \equiv B/(4\pi n m_i)^{1/2}$, $V_s \equiv (T_e/m_i)^{1/2}$, and $\hat{\rho}_i = V_s/\Omega_i$. Note the following (1) $\tilde{B}_\perp$ bends or twists the magnetic field lines but does not compress them; (2) since $V_s/V_A = (\frac{1}{2}\beta_e)^{1/2}$, where β_e is the ratio of electron pressure to magnetic energy density, $\tilde{B}_\perp \sim \sqrt{\beta_e}$; and (3) this expression for $\tilde{B}_\perp$ is valid only for $k_\theta \gg \partial/\partial x$ and $\omega < k_\parallel(x)V_A$ [i.e., $x > x_A$, where $x_A \equiv \omega L_s/k_\theta V_A \simeq \hat{\rho}_i(L_s/r_n)(V_s/V_A)$]. The equation governing the full x dependence of $\tilde{B}_r$ is derived and discussed by Catto *et al.*[6] For typical parameters in present tokamak experiments ($k_\perp\hat{\rho}_i \lesssim 1$, $V_s/V_A \lesssim 1/20$, $T_i \lesssim T_e$, and $e\tilde{\varphi}/T_e \lesssim 10^{-2}$), one has $\tilde{B}_\perp/B \lesssim 10^{-3}(\omega/k_\parallel V_A) < 10^{-3}$, which is easily large enough to affect Eq. (1).

These magnetic perturbations can change the magnetic topology of a tokamak by forming thin, high-order magnetic islands[7] at rational surfaces. The radial variations of the perturbed potential $\tilde{\varphi}$ and the magnetic field are shown in Figs. 1(a) and 1(b). In Fig. 1(c) I illustrate the magnetic islands formed by adding this magnetic perturbation to the helical component of the equilibrium magnetic field near a rational surface,[8] namely

$$B_\eta \simeq -[B_\theta - (r/R)(n/m)B_\varphi] \simeq x B_\theta q^{-1}dq/dr,$$

where $\eta \equiv n\varphi - m\theta$ is the helical angle variable at

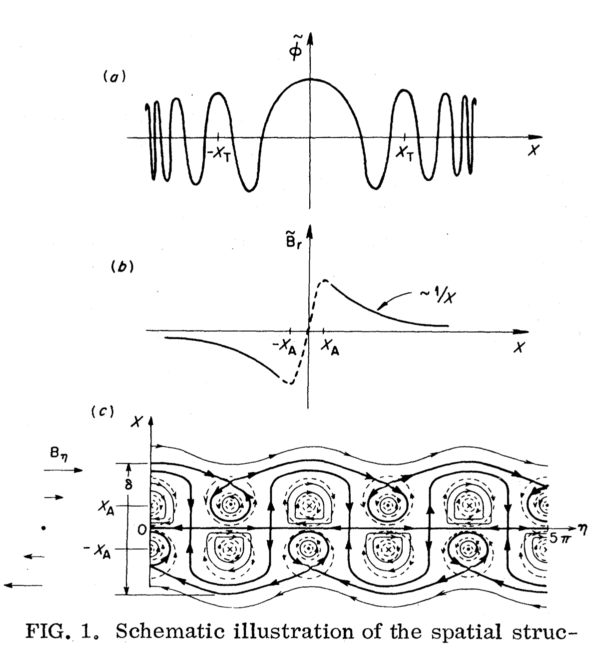

FIG. 1. Schematic illustration of the spatial structure of (a) the perturbed potential $\tilde{\varphi}$; (b) the magnetic field perturbation $\tilde{B}_r$; and (c) the magnetic island structure formed by the combination of B_r and the helical component of the equilibrium magnetic field $B_\eta \simeq B_\theta x q'/q$.

a given rational surface at which $q(r_s) = m/n$; here, (φ, θ) and (n, m) are, respectively, the toroidal and poloidal angle coordinates and mode numbers. With $\tilde{B}_r \simeq B(x_A/x)$ for large x, the full width δ of the induced magnetic island structure and the distance $(2\pi RN)$ that a magnetic field line travels in tracing out a particular magnetic island are specified by[7]

$$\delta \simeq 2[(\tilde{B}/B_\theta)(2x_A r_s/nq')]^{1/3}, \quad N \sim q/n\delta q', \qquad (3)$$

where N is the toroidal winding number for circumnavigation of a magnetic island. For typical parameters in present tokamaks [$\tilde{B}/B_\theta \sim (B/B_\theta) \times(\tilde{B}/B) \sim 1/50$, $x_A \sim 0.1$ cm, $r_s \sim 10$ cm, $q' \sim 0.1$ cm^{-1}, $q \sim 2$, and $n \sim 50$], we obtain $\delta \sim 0.4$ cm and $N \gtrsim 1$. If the magnetic perturbation were nearly uniform in space, such as occurs in tearing modes or externally induced perturbations, then in Eq. (3) the $\frac{1}{3}$ power becomes $\frac{1}{2}$, the x_A is replaced by 2, and N becomes a factor of 4 larger.[7]

The effects of the drift-wave magnetics are quite different on ions and electrons. In one drift-wave period, since $\omega \gg v_i/2\pi RN$, ions are not aware of the magnetic island structure, but instead see a stochastic $\vec{\tilde{B}}_\perp$. The quasilinear effect of the stochastic $\vec{\tilde{B}}_\perp$ is negligible since it is a factor of β smaller than the usual $\vec{E} \times \vec{B}$ electrostatic quasilinear effect of drift waves on ion transport processes.

In contrast to ions, electrons are aware of the full magnetic island structure since they circumnavigate an island many times in a wave or growth period, i.e., $\omega \ll v_e/2\pi RN$. All Fourier components of the mode structure that are nonresonant on the rational surface contribute stochastically to the usual quasilinear response of the electrons. However, the electrons do not respond stochastically to the resonant component of the mode structure, which causes the magnetic-island formation.

The resonant electron distribution function in the presence of the islands must be calculated from a drift-kinetic equation. Magnetic perturbations come in both directly through the $\partial\tilde{A}_\parallel/\partial t$ term and indirectly through the unit vector along $\vec{B}$, which becomes $\vec{n} \equiv \vec{B}/B \simeq \vec{n}_0 + \vec{\tilde{B}}_\perp/B$, where $\vec{n}_0$ is the unit vector along the equilibrium magnetic field. Taking into account these magnetic perturbation effects, perturbing the lowest-order (in finite Larmor radius) drift-kinetic equation about a local Maxwellian distribution $\{f \equiv f_m + \tilde{f}$, with $f_m = n_e(2\pi T/m)^{-3/2}\exp[-(E - q\Phi)/T]$, and $E \equiv mv^2/2 + q\Phi\}$, and assuming that the lowest-order $\tilde{f}$ is an adiabatic response ($\tilde{f} \equiv -q\tilde{\varphi}f_m/T + g$), one obtains[2]

$$\left(\frac{\partial}{\partial t}+v_{\parallel}\vec{n}_0\cdot\nabla - C\right)g$$
$$= -i(\omega-\omega_*{}^T)\frac{q\tilde{\varphi}f_m}{T} - v_{\parallel}\left(\frac{\tilde{B}_r}{B}\right)\left(1-\frac{\omega}{\omega_*{}^T}\right)\frac{\partial f_m}{\partial r}. \quad (4)$$

Here, I have assumed that $\tilde{f}$, $\tilde{\varphi}$, $\tilde{B}_r \sim \exp[i(n\varphi - m\theta - \omega t)]$, used the fact that $\tilde{B}_r \simeq ik_\theta \tilde{A}_\parallel$, and in the $\vec{E}\times\vec{B}$ rest frame where $\Phi=0$, defined

$$\omega_*{}^T \equiv \omega_* d\ln f_m/d\ln n.$$

The usual quasilinear particle and energy transport fluxes in the plasma can be obtained from Eq. (4) for the $m/n \neq q(r_s)$ mode components. The $\tilde{B}_r{}^2$ contribution to Eq. (1) is obtained by assuming that the collision operator in Eq. (4) gives the dominant contribution to the left-hand side of Eq. (4) (i.e., $C\sim v \gg \omega, \omega_b$, etc.), and then computing the contribution to g and the heat flux due to the inhomogeneous $\tilde{B}_r$ term on the right-hand side of Eq. (4).

Here, we wish to determine the additional singular contribution (g_δ) to the perturbed electron distribution function due to the presence of the drift-wave–induced magnetic islands. Since the definition of a magnetic field line is $dl/B = dx/\tilde{B}_r$, we can identify $v_\parallel \tilde{B}_r/B = dx/dt$ as the radial velocity of a particle moving along a magnetic field line within the magnetic island structure. Thus, g_δ is obtained by integrating the $\tilde{B}_r$ term in Eq. (4) over the characteristics of the left-hand side [including the nonlinear term $v_\parallel(\tilde{B}_r/B)\partial g/\partial r$] of the equation (for $\mathrm{Im}\omega>0$):

$$g_\delta \equiv -\sum_\omega e^{-i\omega t}\left(1-\frac{\omega}{\omega_*{}^T}\right)\left(\frac{\partial f_m}{\partial r}\right)$$
$$\times\int_{-\infty}^{t} dt' \left(\frac{dx}{dt}\right)'\exp[-i(\omega+i\nu)(t'-t)], \quad (5)$$

where the primes denote integration along the (perturbed) electron trajectories and $\tilde{B}_r \equiv \sum_\omega \hat{B}_r \times e^{-i\omega t}$ has been decomposed into its temporally and spatially varying parts such that $dx/dt = v_\parallel \hat{B}_r/B$. For simplicity we have taken an energy-independent Krook collisional model.

Since the radial velocity dx/dt must be periodic in t with a period of $\tau_\delta \equiv 2\pi/\omega_\delta = 2\pi RN/v_\parallel$, we assume

$$x = x_0 + \tfrac{1}{4}\Delta\sin\omega_\delta t, \quad (6)$$

where $\frac{1}{4}\Delta$ is the spatial half-width of a given contour within a magnetic island [cf. Fig. 1(c) and Ref. 7]. Performing the time-history integration in Eq. (5) along this trajectory, we obtain

$$g_\delta \simeq -\sum_\omega e^{-i\omega t}(1-\omega/\omega_*{}^T)(\partial f_m/\partial r)(\tfrac{1}{4}\Delta)$$
$$\times\{\sin\omega_\delta t - i[(\omega+i\nu)/\omega_\delta]\cos\omega_\delta t + \cdots\}, \quad (7)$$

in which we have made an expansion in $(\omega+i\nu)/\omega_\delta \ll 1$. From Eq. (6) we see that the $\sin\omega_\delta t$ term in Eq. (7) represents a flattening of the total electron distribution function $(f_m+\tilde{f})$ around x_0 due to the magnetic island. This is, however, a reversible process, as evidenced by the $e^{-i\omega t}$ coefficient. The $i[(\omega+i\nu)/\omega_\delta]\cos\omega_\delta t$ correction term to the flattening represents the irreversible processes of wave growth and collisional changes in the island-averaging period τ_δ through diffusion in $v_\parallel$, which will ultimately lead us to obtain net transport in the radial direction from this perturbation.

The induced particle transport flux, which adds to the neoclassical and quasilinear particle fluxes, is calculated from the velocity-space moment of the drift-kinetic equation: $\Gamma_r{}^\delta \equiv \langle \int d^3v(v_\parallel \tilde{B}_r/B)g_\delta\rangle$, where $\langle\ \rangle$ represents an island and wave-period average. Again splitting $\tilde{B}_r$ into its spatially and temporally varying parts and noting that $v_\parallel \hat{B}_r/B = dx/dt$, we find

$$\Gamma_r{}^\delta \simeq -\sum_{\omega=\omega_{mn}} \tfrac{1}{2}(\tfrac{1}{4}\Delta)^2(\nu+\gamma)n_e$$
$$\times\left(\frac{d\ln n_e}{dr} - \frac{e}{T}\frac{d\Phi}{dr} - \frac{\omega e B}{ck_\theta T_e}\right), \quad (8)$$

in which ω_{mn} are the set of rational-surface drift frequencies occurring within the plasma. Since the ion particle flux is the much smaller quasilinear one, $\Gamma_r{}^\delta$ would cause a nonambipolar radial flow of electrons. This causes a radial potential to build up rapidly ($\Delta t \ll \tau_\delta$) and make the particle flux ambipolar. From Eq. (8), a sufficient condition for ambipolar diffusion is

$$(e/T_e)d\Phi/dr = (1-\omega_{mn}/\omega_*)d\ln n_e/dr. \quad (9)$$

Thus, in the rest frame of the plasma, except for high-energy runaways,[9] electrons are electrostatically confined in the radial direction by this ambipolar potential and the electron particle transport is only the ion quasilinear value. Nonetheless, the radial electron heat flux can be enhanced significantly.

So far, we have calculated g_δ within only a single closed magnetic island of width $\frac{1}{2}\Delta < \frac{1}{2}\delta$ [cf: Fig. 1(c) and Ref. 7]. However, since the separatrices, which are isotherms, connect the island structures on opposite sides of the rational surface, the electron temperature profile will be quickly ($t<\tau_\delta$) flattened over the entire magnetic island structure. Thus, $\frac{1}{4}\Delta$ in Eq. (7) should be replaced by $\frac{1}{2}\delta$. When this and Eq. (9) are taken into account, $\frac{1}{4}\Delta(\partial f_m/\partial r)$ in Eq. (7) is replaced by $\frac{1}{2}\delta \times(E/T-\frac{3}{2})(f_m d\ln T/dr)$. Then, calculating the island-induced electron heat flux in the same manner as the particle flux, we obtain $Q_r^{\ \delta}\simeq -n_e\chi_e^{\ \delta}\times dT_e/dr$, where

$$\chi_e^{\ \delta}\simeq\tfrac{3}{4}\sum_{\omega=\omega_{mn}}(\nu+\gamma)(\tfrac{1}{2}\delta)^2. \qquad (10)$$

The physical implication of this "weak-turbulence" result is that electron heat diffuses radially with a step size of half the magnetic island width ($\frac{1}{2}\delta$) at a characteristic rate given by the sum of the drift-wave growth rate and the (90°) collisional scattering rate. In a fully developed turbulent state, the γ would apparently be replaced by the "birth and death rate" for the various magnetic islands. Net transport occurs only if the radial heat transport is irreversible through an entire magnetic-island life cycle. In an abstract sense, Eq. (10) would be replaced by $\chi_e^{\ \delta}\sim\int d\tau(dx/dt)_t \times(dx/dt)_{t+\tau}$. In order for there to be radial electron heat transport over the entire plasma radius, the various magnetic islands represented by the ω_{mn} in Eq. (10) must be closely packed, or perhaps overlap. Note, however, that magnetic braiding is not required for heat transport in this model. Since the spatial separation of rational surfaces having the same n is only $\Delta x\sim 1/nq'$ ~ 0.2 cm for typical parameters, such proximity is quite probable for the various m modes having the same n, and is virtually certain for the doubly denumerable infinity of admissible combinations of m and n.

The drift-wave growth rate in Eq. (10) can be that due to one or all of the destabilizing effects of trapped electrons, finite ion Larmor radius, plasma current, etc. Our treatment of the effects of drift-wave magnetics would also apply to the hypothesized high-mode-number drift tearing modes,[10] with δ and N modified as indicated after Eq. (3) for these $\tilde{B}_r\sim$ const modes.

Since the electron heat transport due to the magnetic island formation scales as $\chi_e^{\ \delta}\sim\delta^2\sim(e\tilde{\varphi}/T_e)^{2/3}$, whereas the typical quasilinear estimate gives $\chi_e^{\ QL}\sim(e\tilde{\varphi}/T_e)^2$, at low fluctuation levels the magnetic effects are likely to dominate. For the fluctuations observed[1] in ATC and TFR, we estimate $\nu\sim\gamma\sim 2\times10^5$ sec^{-1}, $k_\perp\rho_i\lesssim 1$, $\gamma/\omega_*\sim\frac{1}{3}$, $V_s\sim 3\times10^7$ cm/sec, $(d\ln n_e/dr)^{-1}\sim 10$ cm, $e\tilde{\varphi}/T_e \sim 3\times10^{-3}$, and obtain $\delta\sim 0.4$ cm; hence, $\chi_e^{\ \delta}\sim 10^4$ cm^2/sec vs $\chi_e^{\ QL}\sim 10^3$ cm^2/sec. Thus, we find $\chi_e^{\ \delta}\gg\chi_e^{\ QL}, D^{QL}$, etc. The electron energy confinement time estimated from $\tau_{Ee}\sim a^2/4\chi_e^{\ \delta}$ is also so reasonably close to that observed experimentally. Finally, we note that the temperature perturbations $\tilde{T}$ induced by the magnetic islands are given by $\tilde{T}\sim(\delta/T)\,dT/dr$. Since $\tilde{T}\sim(e\tilde{\varphi}/T)^{1/3}$ and $\tilde{n}\sim e\tilde{\varphi}/T$ with $\tilde{T}/T\gg\tilde{n}/n$ typically, the "magnetic flutter" effects may be operative in the PLT experiment where large temperature fluctuations were recently observed in the drift-wave frequency range with a broader spectrum than the density fluctuations.[11]

The author gratefully acknowledges the encouragement and many useful discussions with K. T. Tsang and B. V. Waddell. This research was sponsored by the U. S. Energy Research and Development Administration under contract with Union Carbide Corporation.

[1]E. Mazzucato, Phys. Rev. Lett. 36, 792 (1976); C. M. Surko and R. E. Slusher, Phys. Rev. Lett. 37, 1747 (1976); Equipe TFR, in *Proceedings of the Sixth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Berchtesgaden, West Germany, 1976* (Vienna, 1977), Vol. I, p. 35.

[2]W. M. Tang, C. S. Liu, M. N. Rosenbluth, P. J. Catto, and J. D. Callen, Nucl. Fusion 16, 191 (1976). See also P. J. Catto, A. M. El-Nadi, C. S. Liu, and M. N. Rosenbluth, Nucl. Fusion 14, 405 (1974).

[3]M. G. Rusbridge, Plasma Phys. 11, 35 (1969). The author is grateful to A. Gondhalekar for pointing out this reference.

[4]A. Rechester and M. N. Rosenbluth, to be published.

[5]B. B. Kadomtsev, in *Plasma Turbulence* (Academic, London, 1965), pp. 82–83.

[6]Catto, El-Nadia, Liu, and Rosenbluth, Ref. 2.

[7]J. D. Callen, G. G. Kelley, and B. V. Waddell, "Magnetic Island Characteristics in Tokamaks" (to be published).

[8]B. B. Kadomtsev and O. P. Pogutse, Zh. Eksp. Teor. Fiz. 65, 575 (1973) [Sov. Phys. JEPT 38, 283 (1974)].

[9]The radial diffusion coefficient governing runaway electrons should be about $\chi_e^{\ \delta}$, as long as their radial drift orbit excursions ($x_s\equiv qv_e/\Omega_e$) are small (i.e., $x_s \ll\delta$).

[10]J. F. Drake and Y. C. Lee, Phys. Rev. Lett. 39, 453 (1977); Liu Chen, P. H. Rutherford, and W. M. Tang, Phys. Rev. Lett. 39, 460 (1977).

[11]V. Arunasalam, R. Cano, J. C. Hosea, and E. Mazzucato, Phys. Rev. Lett. 39, 888 (1977).