[4]J. E. Bjorkholm and P. F. Liao, Phys. Rev. Lett. 33, 128 (1974).
[5]M. Matsuoka, Opt. Commun. 15, 84 (1975).
[6]M. D. Levenson, Phys. Rev. A 13, 2314 (1976).
[7]This backward wave is phase conjugate to the probe and has recently been used for image conjugation by D. M. Bloom and G. C. Bjorklund (to be published).
[8]M. Matsuoka, N. Nakatsuka, and J. Okada, Phys. Rev. A 12, 1062 (1975); T. R. Royt and C. H. Lee, Appl. Phys. Lett. 30, 332 (1977).
[9]See, for example, A. Lauberau, G. Wochner, and W. Kaiser, Phys. Rev. A 13, 2212 (1976), and references therein.
[10]M. Takatsuji, Phys. Rev. A 4, 804 (1971).
[11]D. Grischkowsky, M. M. T. Loy, and P. F. Liao, Phys. Rev. A 12, 2514 (1975).
[12]A. Abragam, *The Principles of Nuclear Magnetism* (Oxford Univ. Press, London, 1961), p. 33.
[13]A. Yariv, *Quantum Electronics* (Wiley, New York, 1967), p. 129.
[14]The observed behavior deviates from exponential dependence above ~5–10 Torr. This deviation is not understood.
[15]F. Biraben, B. Cagnac, and G. Grynberg, J. Phys. (Paris), Lett. 36, L41 (1975). In order to compare this measurement with ours, one must properly normalize the results for number density and temperature. In our experiment, rare-gas pressures are measured at $T=400°\mathrm{C}$, the temperature at which the measurements are made. In the experiment of Biraben, Cagnac, and Grynberg, the rare-gas pressures are measured at room temperature and then the cell is heated to 140°C to perform the measurements. The contribution to the linewidth due to rare-gas collisions is $\Delta\nu_p = 2n\sigma\overline{V}$, where n is the rare-gas density, $\overline{V}=(8kT/\pi\mu)^{1/2}$ is the mean relative velocity, μ is the reduced mass, and σ is the collision cross section. Note that both n and $\overline{V}$ can depend on the temperature T. Assuming only that the collision cross section is independent of velocity, the results of Biraben, Cagnac, and Grynberg would imply a pressure-broadening coefficient of 17.8 ± 3 MHz/Torr for our experimental conditions ($T\simeq400°\mathrm{C}$).
[16]P. F. Liao and J. E. Bjorkholm, Phys. Rev. Lett. 34, 1 (1975).

Computer Simulation of Field Reversal in Mirror Machines

J. A. Byers
Department of Physics, Lawrence Livermore Laboratory, Livermore, California 94550
(Received 24 August 1977)

Computer simulation of neutral injection into a mirror machine shows the existence of stable two-dimensional (r,z) field-reversed mirror configurations. The code follows ion orbits in self-consistent magnetic fields; all electron currents have been ignored. For small systems only a few gyroradii in extent, it is only necessary to focus the neutral beam off axis. Stability to the Alfvén ion cyclotron mode is due in part to finite-length effects. Some steady states for large systems have also been observed.

The concept of reversing the magnetic field lines within a magnetic mirror machine is a relatively new one,[1] although it is a well-known concept for high-β plasmas. The advent of high-power neutral beams has made it possible to consider high-β plasmas and possibly reversed magnetic fields within standard mirror machines. The closed field lines should greatly improve particle containment and thus improve its efficiency as a fusion device. There is currently a major experimental attempt at producing a reversed-field mirror plasma, in the 2*XIIB* device at Lawrence Livermore Laboratory.

In this paper I present the first results of a numerical simulation of field-reversed mirror (FRM) configurations. In the past the original Superlayer code, an electromagnetic code with relativistic electrons, developed for the Astron program, predicted reasonably well both the failure to achieve field reversal by multiple injection of electrons in the Astron experiment and the success achieved at Cornell University by injecting a single, high-current pulse.[2-4] Revised to simulate neutral injection, this code is being used to study buildup to field reversal in a mirror machine and to determine properties of the steady state. The FRM concept, particularly when the ion gyroradius, ρ_i, is comparable to the plasma length, L, and radius, a, is exceedingly difficult to describe with analytic methods because of the complexity of the ion orbits in magnetic fields that reverse direction in lengths comparable to ρ_i. In contrast, computer simulation of such effects is fairly straightforward.

In the following I discuss the numerical algorithm along with a general discussion of the code's

attributes and defects. I then present the data to support the following conclusions: (i) Apparently stable, steady-state high-β configurations are obtained for a variety of injection conditions.
(ii) Stability to the Alfvén ion cyclotron (AIC) mode[5] is due, in part, to finite-length effects.
(iii) Given sufficient injection current, field reversal occurs with a minimum of attention to the details of the beam focusing. This is particularly true for small systems only a few gyroradii in extent, $L/\rho_i \lesssim 2$; for larger systems some form of programmed injection may be necessary.
(iv) There are also some results showing stable r, z equilibria for large size systems, $L/\rho_i \gg 1$. Complete stability will require examination of modes with azimuthal variation, which this Letter does not discuss.

The new code, Superlayer II, is a two-dimensional (r, z) Darwin model, designed to examine low-frequency ($\omega \lesssim \omega_{ci}$) phenomena. The code's main function is to calculate the self magnetic field generated by the ions and the ion orbits in this field as a function of time. The code allows five phase-space dimensions in the ion motion. The code ignores all electron currents, space charge, and electrostatic fields. It also allows only one component of the vector potential, A_θ. At this level of description the code is identical to a previous code of Dickman, Morse, and Nielson.[6] The details of the algorithm differ, however. Also this code accurately models neutral injection into a plasma, including ionization, charge exchange on the beam, and electron drag (a fixed electron temperature, T_e, is assumed). The equations solved are (in MKS units)

$$-\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\right]-\frac{\partial^2 A_\theta}{\partial z^2}=\mu_0 J_\theta, \quad A_r=A_z=0, \tag{1}$$

$$E_\theta=-\partial A_\theta/\partial t, \quad E_r=E_z=0, \tag{2}$$

$$d\vec{v}_i/dt=(e/m)(\vec{E}+\vec{v}\times\vec{B}). \tag{3}$$

In the following, superscripts denote finite-difference time levels. Starting with positions $(r, z)^{n+1}$ and velocities $(v_r, v_z, v_\theta)^{n+1/2}$ the code can obtain $(A_\theta, B_r, B_z)^{n+1}$, since with the use of conservation of canonical momentum the current $J_\theta{}^{n+1}$ is known in terms of $A_\theta{}^{n+1}$. Equation (1) becomes a more complex equation,

$$\left\{-\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(rA_\theta)\right]-\frac{\partial^2 A_\theta}{\partial z^2}+\frac{\omega_{pi}{}^2}{c^2}A_\theta\right\}^{n+1}=\sum_l K_l{}^{n+1}, \tag{4}$$

where the K_l for each particle is a known quantity depending only on position. We use boundary conditions $A_\theta=0$ at conducting walls in both r and z. The equation is finite-differenced and solved iteratively.

In order to be free from problems of inaccuracy at the singularity at $r=0$ in cylindrical coordinates, the particle mover dispenses with the use of conservation of angular momentum and, instead, uses the Boris technique, which uses a rectilinear coordinate system for the velocity push and then rotates the velocity angle to account for the cylindrical coordinates.[7] This procedure requires explicit use of E_θ, which is known from a time difference of A_θ. The mover is in a predictor-corrector form.

The code's main defects at present are azimuthal symmetry, lack of electron currents or space-charge effects, and accelerated time scales. Imposing symmetry suppresses possible modes of instability. The absence of electron currents and electrostatic fields is more fundamental. For small levels of self-field, one analytic model[8] predicts cancellation of electron return currents on an Alfvén transit time. Even for large currents space-charge neutralization by line tying on open lines would appear to effectively suppress electron currents; yet at field nulls or on closed lines this process does not operate and electron currents should then appear. Codes including electron physics effects are under development both at Lawrence Livermore Laboratory[9] and at Cornell University.[10] The use of accelerated time scales refers to a rather severe approximation: The relevant time scales, τ, such as the drag time or filling time, are shortened considerably, typically by factors of order of 10–100. The product $I\tau$ is kept consistent with the experiment (where I is the injection current and τ is the electron drag time), and an attempt is made to keep τ long compared to an axial bounce period.

Even with these assumptions, the code should be able to examine the r-z equilibrium, i.e., the balance between the self-consistent magnetic field forces, which depend on the ion currents, and the ion pressure. The code includes effects of the mirror-tearing mode[6,11] and the Alfvén ion cyclotron mode,[5] an electromagnetic mode with $k_\perp = 0$.

As of this writing most of the computer runs have concentrated on parameters similar to those in the present 2XIIB experiment ($L/\rho_i \approx 2$). The code properly simulates neutral injection where newly injected particles are deposited along the entire beam path, leading to an extended plasma of size $a/\rho_i \approx 2$. The code operates either with pulsed injection or with continuous injection balanced by loss of hot ions due to electron drag. In either case, after a transient buildup period, the code predicts that reversal is possible provided the beams are focused off axis; high $\beta_\perp > 1$ is obtainable regardless of the focusing. This condition, however, is accompanied by a slow but steady axial loss of particles (this did not occur with ringlike injection). This loss is persistent, but its cause is not understood. However, the loss rate is sufficiently slow that it may not prevent buildup in a system with realistic time constants: The loss rate is of the same order as the actual real-time loss rate of hot particles due to electron drag:

$$\nu_{\text{loss}}/\omega_{ci} \approx 10^{-4} \text{ and } \nu_{\text{drag}}/\omega_{ci} \approx 10^{-4}$$

for

$$n = 10^{14}\ \text{cm}^{-3}, \quad T_e = 100\ \text{eV}.$$

For 2X-like parameters ($E = 15$ keV, $B_{\text{vac}} = 7$ kg), the code predicts a stable, strongly reversed steady state for $I \approx 600$ A. Shown in Fig. 1 are typical field lines in an r-z cross section indicating a central core of closed field lines. The external-field-mirror ratio over the occupied region is only 1.05; that is, the self-field alone confines the axial pressure. The plasma has expanded radially to a radius $r = a$, somewhat beyond the injection radius r_0, and most of the plasma samples both open and closed field lines. Smaller orbit systems should confine the plasma more on closed field lines.

The above result requires only sufficient current and focusing the beam off axis a distance $y \lesssim \rho_i$. If the beam is focused on axis, the code shows that field reversal does not occur. Rather than forming a uniformly directed current layer, on-axis injection tends to form dipole current layers, which arise from the greater radial expansion caused by on-axis injection, coupled with the natural tendency for cancellation of currents due to overlapping orbits.

Stability (to θ symmetric modes) is self-evident since the system develops a steady state. Marginal stability to the tearing mode, a mode with zero real frequency, is identical to the condition for an equilibrium in a finite-length system. Stability to the AIC mode, a mode with real frequency $\omega \approx \omega_{ci}$, is more subtle. For the above case we observe temperature anisotropy $T_\perp/T_\parallel = 5.0$. In this case charge exchange off the neutral beam is a strong factor in keeping the energy distribution more peaked to the injection angle, i.e., perpendicular to B. Even when the code is run without charge exchange the observed anisotropy is $T_\perp/T_\parallel \approx 2.5$. Both cases are far from isotropy and are unstable to the AIC mode according to the simple infinite-medium criterion.

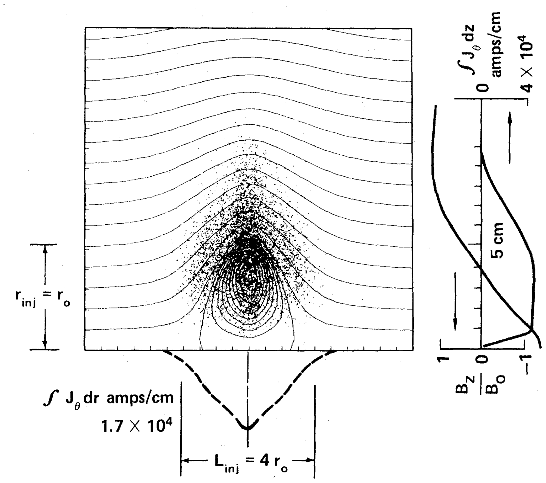

FIG. 1. An r-z cross section showing particle positions and field lines for a field-reversed case. The injection region is a uniform rectangle of extent r_0 in radius and $4r_0$ in the axial direction. Also shown are radial and and axial profiles $B_z(r)$, $\int J_\theta\, dz$, and $\int J_\theta\, dr$.

We have evidence that small axial scale lengths can add strong stabilizing effects. Figure 2 shows results from a linearized particle code[12] that models instabilities about a given equilibrium. Here we compare results from an infinite-medium model and a finite-geometry model, for linearly polarized AIC modes. The latter is one-dimensional a slab (z variation only) and includes an external quadratic potential well which creates a localization in z of the plasma. We find complete stability for a case with $T_\perp/T_\parallel = 10.0$, $L/\rho_i \approx 1.5$, whereas the infinite-medium result gives a growth rate $\gamma/\omega_{ci} \approx 0.16$. However, this stabilizing effect rapidly disappears as the plasma length is increased. This result is consistent with the idea that stability occurs when the

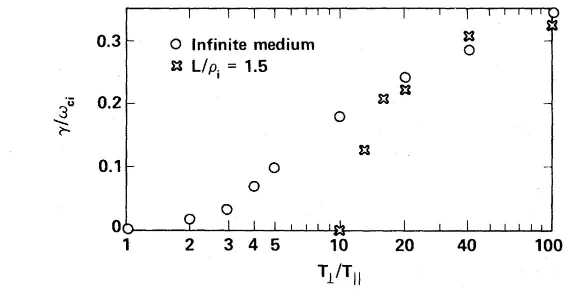

FIG. 2. Linear growth rates for the Alfvén ion cyclotron mode. Comparison of infinite medium and finite geometry.

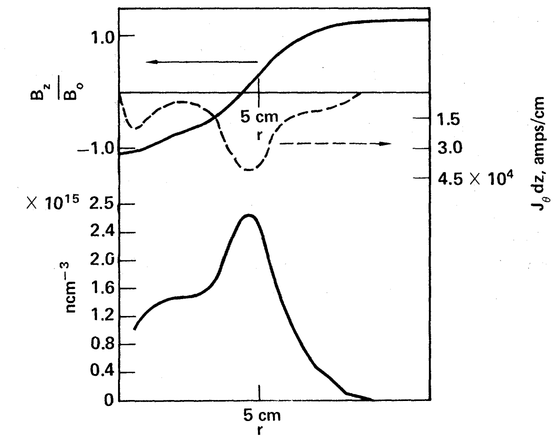

FIG. 3. Radial profiles of J_θ and B_z for a several-gyro-orbit system, $a/\rho_i = 4$. The current peaks at the field null. The density also peaks at the field null and has a shape similar to that of J_θ.

plasma length becomes shorter than one-half of the minimum unstable wavelength or $Lk_{\max} < \pi$; the theory for $T_\perp/T_\parallel = 10$ shows $k_{\max} \approx 2$. Also, the simulation result near marginal stability shows a mode shape with only a one-half wavelength. Consistent with this result, most of the Superlayer code runs to date have been restricted to systems with only a small number of gyroradii ($a/\rho_i, L/\rho_i < 2$), and a major question is the persistence of these results for large L/ρ_i, a/ρ_i systems.

Longer axial systems can result if the injection region is extended. Very long systems are subject to breakup due to collisionless tearing modes, but, even here, we have examples where the plasma finally finds a steady state with the individual plasma bunches coalesced into one. A ringlike plasma originally injected over a length of $64r_0$ tears apart into multiple bunches and finally coalesces into a single bunch with mean length $L \approx 20r_0$. Coalescence is a well-known phenomenon, having been observed for a variety of models.[6,13] But in this case there is a new important result: Only a minimal amount of axial loss was observed.

One example of a system moderately large in the radial direction, $a/\rho_i \approx 4$, is shown in Fig. 3. A reversed-field steady state occurs with the current peaked at the field null. In this case the plasma length was again comparable to the radius, $L \approx a$. For large systems the question of existence of an equilibrium becomes more obviously distinct from the question of how to achieve or build up to that equilibrium. For small systems we found that it was only necessary to focus the beam above axis to achieve field reversal. In contrast, long axial systems may want to tear apart, at least initially. Large radial systems tend to form dipole current layers. Some form of programmed injection may prove necessary in the general case.

It may not be necessary to have an extremely large system. To have an economically viable reactor, initial studies indicate that L/ρ_i, a/ρ_i $\approx 5-10$ may be sufficient.[14] Our present work suggests that moderate-size two-dimensional, r-z equilibria do exist. The persistence of these configurations in dependent on the stability to azimuthal modes.

I would like to thank the following people for scientific advice and technical assistance: B. I. Cohen, W. C. Condit, R. Ferch, T. K. Fowler, A. Friedman, A. A. Mirin, L. D. Pearlstein, S. Rompel, D. Schnabel, and D. Watson. This work was performed under the auspices of the U. S. Energy Research and Development Administration under Contract No. W-7405-Eng-48.

[1]The possibility of stable reversed-field mirror machines was first suggested by T. K. Fowler, private communication.

[2]M. Brettschneider, J. Killeen, and A. A. Mirin, J. Comput. Phys. 11, 360 (1973).

[3]J. A. Byers, J. P. Holdren, J. Killeen, A. B. Langdon, A. A. Mirin, M. E. Rensink, and C. G. Tull, Phys. Fluids 17, 2061 (1974).

[4]J. Killeen, Nucl. Fusion 16, 841 (1976).

[5]R. C. Davidson and J. M. Ogden, Phys. Fluids 18, 1045 (1975).

[6]D. O. Dickman, R. L. Morse, C. W. Nielson, Phys. Fluids 12, 1708 (1969).

[7]J. P. Boris, in *Proceedings of the Fourth Annual*

Conference on Numerical Simulation of Plasmas, Naval Research Laboratory, Washington, D.C., 1970, edited by J. P. Boris and R. A. Shanny (U.S. GPO, Washington, D.C., 1971).

[8]H. L. Berk and L. D. Pearlstein, Phys. Fluids 19, 1831 (1976).

[9]J. A. Byers, B. I. Cohen, W. C. Condit, and J. D. Hanson, UCRL Report No. UCRL-79437 (to be published).

[10]A. Friedman, R. L. Ferch, R. N. Sudan, and A. T. Dobrott, Cornell University Report No. LPS 190 (unpublished).

[11]H. P. Furth, Nucl. Fusion, Suppl. Pt. 1, 169 (1962).

[12]J. A. Byers, in *Proceedings of the Fourth Annual Conference on Numerical Simulation of Plasmas*, Naval Research Laboratory, Washington, D.C., 1970, edited by J. P. Boris and R. A. Shanny (U.S. GPO, Washington, D.C., 1971).

[13]J. M. Finn and P. K. Kaw, Phys. Fluids 20, 72 (1977).

[14]W. C. Condit, T. K. Fowler, and R. F. Post, UCRL Report No. UCRL-52008, 1976 (unpublished).

Low-Temperature Properties of a Superconducting Disordered Metal

John E. Graebner, Brage Golding, R. J. Schutz, F. S. L. Hsu, and H. S. Chen
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 14 October 1977)

Specific heat C_p and thermal conductivity κ measurements between 0.1 and 10 K on the superconducting ($T_c = 2.53$ K) and structurally disordered metal $Zr_{0.7}Pd_{0.3}$ exhibit an approximately linear term in C_p and a $T^{1.9}$ dependence of κ below T_c. The magnitudes of these terms are close to those found for insulating glasses, thereby suggesting that disorder-induced localized excitations exist at similar densities in very different classes of disordered solids.

Metallic glasses present a unique opportunity for understanding how the basic low-temperature thermodynamic properties and phonon and electron transport of metals are affected by structural disorder. For example, although there is direct evidence[1] for a preferential softening of transverse phonons in the disordered state relative to the crystal, leading to an enhanced phonon specific heat at low temperatures, there is no way to predict with any certainty the corresponding changes of the electronic specific heat. Recent measurements of thermal conductivity,[2] sound velocity,[3,4] and resonant acoustic absorption[4] in metallic glasses indirectly suggest that there may exist extra localized excitations at energies below 1 K similar to those found in insulating glasses.[5-7] Direct evidence in disordered metals for these excitations, usually described as two-level configurational or tunneling systems,[8,9] would support the intrinsic nature of these states in the amorphous phase,[10] particularly since metallic glasses, unlike insulators, possess rather closely packed structures with mass densities only a few percent lower than those in their crystalline phases.

In the present work, we have measured the specific heat C_p of a bulk disordered metallic alloy,[11,12] a-$Zr_{0.7}Pd_{0.3}$, between 0.1 and 10 K. Since this material is *superconducting below 2.53 K*, it is possible to evaluate the individual contributions to C_p in this temperature region. We find that the observed specific heat *cannot* be accounted for solely by the phonon and electron contributions. An additional contribution, approximately linear in T, is observed, whose magnitude is strikingly similar to that observed below 1 K for insulating glasses. In addition, we observe a thermal conductivity in the superconducting state whose temperature dependence is $T^{1.9}$, also remarkably similar to that found in insulating glasses. We suggest that there are no fundamental differences between the disorder-induced excitations found in this disordered metal and those found in insulating glasses.

The sample of $Zr_{0.7}Pd_{0.3}$ was prepared in bulk form from the melt[13] as a ribbon of cross section 0.085×0.0032 cm. X-ray diffraction measurements yielded results similar to those found previously.[11] A 1.2 m length of ribbon (0.259 g) was wound into a spiral of 1 cm o.d. and glued with a minimum amount of GE 7031 varnish onto the sapphire-plate sample holder of a calorim-