Jets from Quantum Chromodynamics

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(Received 26 July 1977)

The properties of hadronic jets in e^+e^- annihilation are examined in quantum chromodynamics, without using the assumptions of the parton model. We find that two-jet events dominate the cross section at high energy, and have the experimentally observed angular distribution. Estimates are given for the jet angular radius and its energy dependence. We argue that the detailed results of perturbation theory for production of arbitrary numbers of quarks and gluons can be reinterpreted in quantum chromodynamics as predictions for the production of jets.

The observation[1] of hadronic jets in e^+e^- annihilation provides one of the most striking confirmations of the parton picture.[2] In particular, the distribution of events in the angle θ between the jet axis and the e^+-e^- beam line is observed to be very close to the form $1+\cos^2\theta$ that would be expected for the production of a pair of relativistic charged pointlike particles of spin $\frac{1}{2}$. We shall argue here that the existence, angular distribution, and some aspects of the structure of these jets follow as consequences of the perturbation expansion[3] of quantum chromodynamics[4] (QCD), without assuming the parton picture (in particular, the transverse-momentum cutoff) in advance. Thus, the observed features of jets provide evidence for an underlying asymptotically free gauge field theory with elementary spin-$\frac{1}{2}$ quarks. We also wish here to demonstrate a general approach, which may be applicable to a wide range of high-energy phenomena.

Our procedure is to define a partial cross section for jet production, which in asymptotically free theories like QCD can be calculated perturbatively at high energy. By ordinary dimensional analysis, any sort of total or partial cross section in QCD can be written in the form

$$\sigma=E^{-2}f(m/E, g_E, x)\,, \qquad (1)$$

where E is the energy; x stands for all other dimensionless variables characterizing the final state; m stands for all mass variables; and g_E is the gauge coupling constant, defined at a renormalization point with four-momenta of order E. [We express the cross section in terms of g_E, rather than a coupling g_κ defined at a renormalization point with momenta of arbitrary scale κ, in order to avoid factors of $\ln(E/\kappa)$. Physical quantities are of course independent of the choice of renormalization point.] Even in asymptotically free theories, where $g_E \to 0$ as $E\to\infty$, it is generally not possible to calculate the cross section perturbatively for large E, because the cross section will exhibit singularities for $m/E\to 0$. It is of course for this reason that asymptotic freedom has as a rule been used directly to justify perturbative calculations of Green's functions and Wilson coefficient functions, rather than cross sections themselves.

However, by performing various sums over states, it is possible to define a wide range of cross sections which are free of $m\to 0$ singularities. To learn what they are, we observe that quantum field theories of massless particles have always been found (in the absence of superrenormalizable couplings) to be physically sensible, i.e., that any cross section which would actually be measurable in such a massless theory is free of infrared divergences in each order of perturbation theory.[5] Hence in the real world with $m\neq 0$, any sort of partial cross section which would be measurable for $m=0$ is expected to be free of singularities in m as $m\to 0$, and can therefore be calculated perturbatively[3] in QCD for $E\to\infty$.

For instance, the cross section for production of a definite number of particles does have singularities for $m\to 0$, because for $m=0$ we could not expect to be able to tell the difference between one particle or several particles moving in the same direction. At the opposite extreme, the total cross section for $e^+e^-\to$ hadrons would clearly be measurable even for zero quark mass, and hence must be free of singularities in m (to lowest order in α) for $m\to 0$. Indeed, although the original application of asymptotic freedom to this process was by way of the vacuum-polarization Green's function at Euclidean momentum,[6] it is easier to justify the use of QCD perturbation theory[3] here directly, by working with the cross section itself.

To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\epsilon,\delta)$ for e^+e^- hadron production events, in which all but a fraction $\epsilon\ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta\ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2\ll\Omega\ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measurable for $m=0$, because the only quarks or gluons which are likely to be diffracted or radiated away from a calorimeter at θ have very long wavelength, and so carry negligible energy. Thus σ should be free of mass singularities for $m\to 0$, and calculable by a perturbation expression in g_E for $E\to\infty$.

We have calculated $\sigma(E,\theta,\Omega,\epsilon,\delta)$ to order g_E^2. It proved algebraically convenient to set the quark masses equal to zero from the beginning, but to use a finite gluon mass $\mu\ll\epsilon E$ as an infrared cutoff in intermediate stages of the calculation. To order g_E^2, σ receives contributions from three distinct kinds of final state[7]: (a) One jet may consist of a quark or antiquark plus a hard (energy $\geqslant\epsilon E$) gluon, the other jet of just an antiquark or quark; (b) there may be a quark in one jet, an antiquark in the other, and a soft (energy $\leqslant\epsilon E$) gluon which may or may not be in one of the jets; (c) there may be just a quark and antiquark, one in each of the jets. Working to order g_E^2, we evaluate the contributions of (a) and (b) using only tree graphs, while for (c) we include the tree graph and its interference with one-loop graphs. The respective contributions to σ are then

$$\sigma_a=(d\sigma/d\Omega)_0\,\Omega(g_E^2/3\pi^2)[-3\ln(E\delta/\mu)-2\ln^2 2\epsilon-4\ln(E\delta/\mu)\ln(2\epsilon)+\tfrac{17}{4}-\pi^2/3], \tag{2}$$

$$\sigma_b=(d\sigma/d\Omega)_0\,\Omega(g_E^2/3\pi^2)[2\ln^2(2\epsilon E/\mu)-\pi^2/6], \tag{3}$$

$$\sigma_c=(d\sigma/d\Omega)_0\,\Omega\{1+(g_E^2/3\pi^2)[-2\ln^2(E/\mu)+3\ln(E/\mu)-\tfrac{7}{4}+\pi^2/6]\}, \tag{4}$$

where $(d\sigma/d\Omega)_0$ is the cross section for $e^+e^-\to q\bar{q}$ in Born approximation:

$$\left(\frac{d\sigma}{d\Omega}\right)_0=\frac{\alpha^2}{4E^2}(1+\cos^2\theta)\sum_{\text{flavors}}3Q^2. \tag{5}$$

As expected, each separate contribution is singular for $\mu\to 0$, but cancellations[8] occur in the sum, and the final result is free of mass singularities:

$$\sigma(E,\theta,\Omega,\epsilon,\delta)=(d\sigma/d\Omega)_0\,\Omega[1-(g_E^2/3\pi^2)(3\ln\delta+4\ln\delta\ln 2\epsilon+\pi^2/3-\tfrac{5}{2})]. \tag{6}$$

This formula immediately demonstrates the dominance of two-jet final states at very high energy where $g_E^2/3\pi^2$ is small. By summing Eq. (6) over a set of cones of solid angle Ω that fill the 4π steradians around the e^+e^- collision, and comparing the result with the QCD expression[6] $(1+g_E^2/4\pi^2)\sigma_0$ for the total cross section, we see that the fraction of all events which have all but a fraction ϵ of their energy in some pair of opposite cones of half-angle δ is

$$f=1-(g_E^2/3\pi^2)(3\ln\delta+4\ln\delta\ln 2\epsilon+\pi^2/3-\tfrac{7}{4}). \tag{7}$$

If $g_E^2/3\pi^2\ll 1$, then even if we take ϵ and δ to be moderately small, the two-jet probability f will be close to unity. To be quantitative, suppose we define a jet angular radius $\delta(E)$, by requiring that 70% of all events have at least 80% of their energy emitted within two cones of half-angle $\delta(E)$. Setting $f=0.7$ and $\epsilon=0.2$ in Eq. (7), and using the asymptotic QCD formula[9] $g_E^2=24\pi^2/25\ln(E/\Lambda)$ with $\Lambda=500$ MeV, we find that $\delta(E)$ is about 13° at the energy $E=7.4$ GeV of current experiments,[1] and decreases as $E^{-0.25}$ at higher energies. In contrast, with a fixed transverse-momentum cutoff $P_\perp$, we would expect a jet angular radius $\varphi(E)$ which would decrease much faster, like $1/E$ or $(\ln E)/E$. At relatively low energy $\varphi(E)$ will be greater than $\delta(E)$, so that our calculation of the jet radius is probably invalidated by the nonperturbative effects[3] associated with $P_\perp$. However, at sufficiently high energy $\delta(E)$ becomes greater than $\varphi(E)$, and perturbation theory becomes valid for angular radii down to $\delta(E)$. The angle $\delta(E)$ then defines the outermost angular distance from the jet axis at which any appreciable hadron energy is to be found. Even at such high energies, it is possible that the fixed-transverse-momentum jet of the parton model will survive deep within the cone of half-angle $\delta(E)$, beyond the reach of perturbative methods, but the angle $\varphi(E)$ becomes so small for sufficiently high energy that the jet angular distribution is in any case constrained to have the $1+\cos^2\theta$ form of QCD perturbation theory.

Our definition of two-jet events has an obvious generalization to arbitrary numbers of jets. To order g_E^2, the fraction f of hadronic e^+e^- events that (by definition) are not of the two-jet type consists entirely of three-jet events.[10] However, in order to determine the angular radius of the jets in three-jet events, it would be necessary to carry our calculations to order g_E^4, where four-jet events are beginning to enter. Continuing in this way, it should be possible to test the detailed predictions of QCD for production of any numbers of quarks and gluons, but always reinterpreting these particles in terms of jets.[11]

We can also conclude from Eq. (6) that the two-jet events have just the same $1+\cos^2\theta$ angular distribution as in the Born approximation for $e^+e^- \to q\bar{q}$. This result is expected to persist for massless quarks to all orders in g_E^2, because for $e^+e^- \to q\bar{q}$ the conservation of current and chirality limit the matrix element $\langle q\bar{q}|J^\mu|0\rangle$ to just a γ^μ term, while the effect of adding more gluons or quarks to these jets is merely to convert $\ln(E/\mu)$ factors to factors of $\ln\epsilon$ or $\ln\delta$. The dominant corrections to this angular dependence come from the finite quark mass and from the ambiguity between three-jet and two-jet events; the former gives an angular distribution $1+a\cos^2\theta$, with $1-a=4\langle m^2\rangle/E^2$.

It might be thought that the partial jet cross section $\sigma(E, \theta, \Omega, \epsilon, \delta)$ should be measurable for massless theories, and hence free of mass singularities in the limit of zero mass, even if we specify the charge in each jet. If this were the case (and if there are no failures of perturbation theory[3] in QCD when jet charges are measured) then our calculation would not account for real jets with integer total charge, since to order g_E^2 it is only possible to produce jets of third-integral charge.[12] However, direct calculation to order g_E^4 shows that the cross section for final states with a definite value for the charge emitted in a given solid angle will have singularities in the limit that the quark mass vanishes. As far as we can tell, the reason that cross sections for the emission of massless particles with a definite total charge into a definite solid angle cannot be measured is that any attempt to stop these particles would result in the emission of soft charged massless particles in all directions.[13] Fortunately, as long as we define jets in terms of energy but not charge, we must sum over final states in which soft quarks are emitted in arbitrary directions, and the mass singularities are expected to cancel.

The methods of this paper can be applied to any field theory, not just QCD. However only in an asymptotically free field theory like QCD can we deduce the simple behavior which seems to be observed experimentally: a total cross section dominated at high energy by two-jet events, with an angular distribution characteristic of the lowest-order production of elementary particles.

This work grew out of extensive discussions of one of us (G.S.) at the University of Illinois with Shau-Jin Chang and Jeremiah Sullivan, and out of the stimulus provided to the other (S.W.) from a seminar given at Stanford Linear Accelerator Center by Nathan Weiss. In addition, we would like to thank James Bjorken, Howard Georgi, Gail Hanson, Tom Kinoshita, Benjamin Lee, T. D. Lee, Michael Nauenberg, David Politzer, Helen Quinn, John Stack, Roberto Suaya, Frank Wilczek, and Edward Witten for helpful comments. One of us (S.W.) wishes also to thank the Physics Department of Stanford University for their kind hospitality. This work was supported in part by the National Science Foundation Grants No. PHY-76-15328 and No. PHY 75-20427.

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[1]G. Hanson *et al.*, Phys. Rev. Lett. 35, 1609 (1975); R. F. Schwitters, in *Proceedings of the International Symposium on Leptons and Photon Interactions at High Energy, Stanford, California, 1975*, edited by W. T. Kirk (Stanford Linear Accelerator Center, Stanford, Calif., 1975), p. 5; G. Hanson, SLAC Report No. SLAC-PUB-1814, September 1976 (unpublished).

[2]For early theoretical predictions of jets in parton models, see S. D. Drell, D. J. Levy, and T. M. Yan, Phys. Rev. 187, 2159 (1969), and Phys. Rev. D 1, 1617 (1970); N. Cabibbo, G. Parisi, and M. Testa, Lett. Nuovo Cimento 4, 35 (1970); J. D. Bjorken and S. D. Brodsky, Phys. Rev. D 1, 1416 (1970); R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, New York, 1972), p. 166.

[3]We will not attempt to deal here with the nonperturbative effects which in QCD presumably account for the trapping of quarks and gluons. Instead, we adopt the rule of thumb, that when colored particles are not

explicitly isolated, these effects become negligible at sufficiently high energy. [This is the case for instance if these effects behave like $\exp(-\text{const.}/g_E^2)$, where g_E is the gauge coupling defined at a renormalization point with momenta of order E.] We can offer no proof of this assumption, and we cannot predict in advance at what energy the nonperturbative effects become negligible, but we note that a rule of this sort has had to be assumed in every application of QCD to physical problems, including the calculation of the e^+e^- total cross section itself.

[4]H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973); D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973); S. Weinberg, Phys. Rev. Lett. 31, 494 (1973).

[5]We know of no rigorous proof of this principle in the form stated here. It is supported by arguments of T. Kinoshita, J. Math. Phys. (N.Y.) 3, 650 (1962), especially Appendix A; and T. D. Lee and M. Nauenberg, Phys. Rev. 133, B1549 (1964). Recent work indicates that no special problems arise in non-Abelian gauge theories: see Y.-P. Yao, Phys. Rev. Lett. 36, 653 (1976); T. Appelquist, J. Carazzone, H. Kluberg-Stern, and M. Roth, Phys. Rev. Lett. 36, 768 (1976); L. Tyburski, Phys. Rev. Lett. 37, 319 (1976); E. C. Poggio and H. R. Quinn, Phys. Rev. D 14, 578 (1976); G. Sterman, Phys. Rev. D 14, 2123 (1976); F. G. Krausz, Phys. Lett. 66B, 251 (1977). We hope to prove the cancellation of mass singularities described here to all orders in a future publication.

[6]T. Appelquist and H. Georgi, Phys. Rev. D 8, 4000 (1973); A. Zee, Phys. Rev. D 8, 4038 (1973). For a discussion closer in spirit to that of the present paper, see T. Appelquist and H. D. Politzer, Phys. Rev. D 12, 1404 (1975).

[7]We are dropping terms here which vanish for $\mu \to 0$, and in the remaining expression we drop finite terms of order ϵ or δ or Ω. In consequence, there is no contribution to order g_E^2 from final states with a soft quark or antiquark outside the jets, or with both quark and antiquark in the same jet.

[8]To the order studied here, this cancellation can be derived from the theorem of Lee and Nauenberg, Ref. 5 (see especially Appendix D and Sec. II, remark 3), using the fact that the only state which is degenerate with a specific physical quark-antiquark state and which can be produced from it by a single action of the interaction Hamiltonian consists of a quark, an antiquark, and a soft or collinear gluon.

[9]D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[10]A perturbative analysis of three-jet events in order g_E^2 is given by J. Ellis, M. K. Gaillard, and G. G. Ross, Nucl. Phys. B111, 253 (1976).

[11]It has been widely conjectured that QCD predictions for the production of quarks and gluons can be taken seriously at sufficiently high energy if reinterpreted in terms of jets. For instance, this is the guiding assumption of Ellis *et al.*, Ref. 10. Our aim here is to show that this hypothesis can actually be *derived* in QCD, by using the absence of mass singularities in suitably defined jet cross sections. The suggestion that the QCD result for the total two-jet probability can be derived in this way was first made in 1975 by one of us (G.S.) in an unpublished University of Illinois preprint. (This paper explicitly exhibited the cancellation of mass singularities to order g_E^2 in the jet probability, but did not give the correct results for the finite part.) Later, the other author (S.W.) independently suggested that QCD results for jet total probabilities can be justified in this manner, and extended this reasoning to jet distributions as well. The present paper is intended to incorporate this earlier unpublished work of both authors, but goes beyond it in various respects, including the calculations leading to Eqs. (2)–(7).

[12]For discussions of this point in the context of parton models, see R. P. Feynman, in *Neutrino '72-Proceedings*, edited by A. Frenkel and G. Marx (OMDK-Technoinform, Budapest, 1972), Vol. II, p. 75; G. R. Farrar and J. L. Rosner, Phys. Rev. D 7, 2747 (1973); R. N. Cahn and E. W. Colglazier, Phys. Rev. D 9, 2658 (1974); S. J. Brodsky and N. Weiss, SLAC Report No. SLAC-PUB-1926 (to be published).

[13]This was independently suggested to one of us (S.W.) by E. Witten.