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Detectable Gravitational Radiation from Stellar Collapse

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The problem of astrophysical sources of detectable gravitational radiation is considered from the stellar-evolution viewpoint. Calculations are presented which indicate that the final stages of evolution may well be dominated by rapidly rotating, collapsing cores which develop nonaxisymmetric configurations. Such events emit large amounts of gravitational radiation which should be detectable in the near future.

Of the scenarios proposed for generation of detectable gravitational radiation, the collapse of rotating stellar cores appears most promising in terms of expected event frequency and intensity. The expected event rate is in the range 10^{-1} to 10^{-2}/year-galaxy so any reasonable detector should be sensitive to events well beyond our galaxy. The flux of gravitational waves radiated in each event depends on the gravitational wave efficiency ϵ ($=E_{rad}/2Mc^2$) which in turn depends critically on the characteristics of the collapse. For axisymmetric collapse, calculations[1] yield $\epsilon \lesssim 10^{-4}$, a value much too low to allow practical detection of events outside our galaxy. In the case of nonaxisymmetric collapse, where the rate of change of mass quadrapole moment is directly due to rotation, as well as to collapse, $\epsilon \sim 10^{-2}$ may be achieved.[2] Which type of collapse occurs depends on the detailed distribution of angular momentum within the star prior to collapse. This can established from stellar-evolution calculations starting from earlier stages where the rotation can be determined from observations of normal stars. We have developed a code to evolve differentially rotating stars[3] with time-dependent redistribution of angular momentum.[4] Currently the code cannot follow the hydrodynamic collapse; so the results are only strictly valid for the precollapse stages. However, on the basis of physically plausible extrapolations of our precollapse models, we can obtain a picture of the collapse which should be qualitatively correct. In this Letter, we examine the implications of this picture for the problem of gravitational wave detection.

The calculations we describe here apply to a $10M_\odot$ star. As an initial condition, we (conservatively) assume that the star begins the main sequence stage rotating as a solid body with an angular velocity $\omega = 6\times10^{-5}$ s^{-1}, a typical surface value for such stars. During the evolution, differential rotation develops due to expansion of the envelope and contraction of the core. This in turn leads to gas-dynamical instabilities which redistribute angular momentum among the layers of the star. Such instabilities are taken into account in the calculations, as described in detail elsewhere.[4] Note that the assumption of solid-body rotation in convective regions (as opposed to the possibility of uniform specific angular momentum) was employed. While the problem of rotation in convective regions does introduce an uncertainty into the calculations, our assumption gives a lower limit on the rotation of the core and, therefore, constitutes a conservative approach. The only possibly significant redistribution mechanism not included in the calculations involves the effects of magnetic fields. However, a number of arguments indicate that magnetic fields do not play a significant role in angular momentum redistribution. If the magnetic fields of neutron stars are due to compression of precollapse fields, the measured surface field strengths[5] imply

$$\beta_m = \frac{\text{magnetic field energy}}{\text{gravitational binding energy}} < 10^{-10}$$

during precollapse stages of evolution. The corresponding rotational parameter in the cores of our evolved models is

$$\beta_r = \frac{\text{rotational kinetic energy}}{\text{gravitational binding energy}} \gtrsim 10^{-2},$$

indicating that magnetic fields are not important. Of course, it is possible that neutron-star fields are not indicative of precollapse fields. In particular, it has been argued[6] that differential rotation will wind and amplify magnetic fields until $\beta_m/\beta_r \gtrsim 1$, as in the numerical experiments of LeBlanc and Wilson.[7] However, these calculations assumed a perfectly conducting fluid, so that field lines were pinned to fluid elements. In a real fluid of finite conductivity, slippage and field-line reconnection will limit the amplification process. Numerical experiments by Weiss[8] show that a differentially rotating fluid of finite conductivity will expel magnetic fields, thus decoupling the rapidly rotating core from the slowly rotating envelope. Expulsion, rather than amplification, is consistent with the dynamically *weak* magnetic fields of pulsars where $\beta_m/\beta_r \ll 1$.

Evolutionary models were computed from the initial main sequence to carbon ignition, a time span covering of 98% of the time to core collapse. Details of these calculations are given elsewhere.[4] During this period, the primary effect of gas-dynamical redistribution is to decrease the total angular momentum of the core by 40%. The rotational characteristics of the last model are displayed in Fig. 1. Hydrodynamical calculations of the late stages of evolution of nonrotating stars[9] indicate that the evolution from carbon

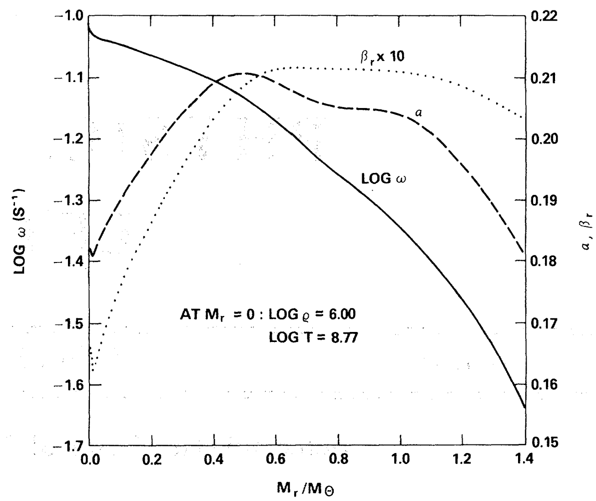

FIG. 1. Rotational parameters as a function of the Lagrangian coordinate for the core of a $10_\odot$ star at carbon ignition. The lines for α and $\log\omega$ indicate local values. The dotted line shows β_r (as defined in the text) integrated from the center to M_r.

ignition to collapse may be reasonably approximated by homologous contraction of the core. Because of thermal neutrino emission, the time scales for these stages are much shorter than the calculated time scales for gas-dynamical redistribution. For local conservation of angular momentum, the parameters describing the effects of rotation scale with core density ρ as

$$\alpha = \omega/\omega_{\rm cr} = \alpha_0(\rho/\rho_0)^{1/6}$$

and

$$\beta_r = \beta_{r0}(\rho/\rho_0)^{1/3},$$

where $\omega_{\rm cr}$ is the critical (Keplerian) angular velocity and subscripts 0 refer to values at the last detailed model. Both of these parameters have critical values and the core evolution is largely determined by which critical value is reached first. At $\alpha = 1$, contraction is halted until the excess angular momentum can be dissipated (by turbulence or equatorial mass shedding). This would imply axisymmetric collapse. At $\beta_r = 0.26$, the core is dynamically unstable to nonaxisymmetric instabilities leading to fission.[10,11] This will not halt the collapse but it will obviously alter the evolution and lead to nonaxisymmetric collapse. The degree of differential rotation within the core determines which scenario occurs. Apply the scaling laws for α and β_r to our $10M_\odot$ star ($\alpha_0 = 0.21$, $\beta_{r0} = 0.021$), we find that, at

$\log\rho = 9.28$ (at the center), $\alpha = 0.75$ and $\beta_r = 0.26$, indicating that *the core will fission.* We note that the fractional increase in core density from the last detailed model to the critical point is less than 1% of the increase over our computed model sequence so the extrapolation is reasonable.

From the nonrotating models, we estimate that $\beta_r = 0.26$ will be reached within one week (nominally, within one day) of core collapse. This time scale determines the consequences of reaching the critical point. Fission will occur on a dynamical time scale and the nonaxisymmetric configuration will be fully developed by the time of collapse. At constant angular momentum and density, the parameter α decreases as a result of fission. The decrease in α favors contraction and may actually trigger collapse. The rapidly rotating, nonaxisymmetric core will generate shock waves in the surrounding envelope, dissipating rotational energy in the manner described by Sakurai.[12] From Sakurai's analysis, we find that the time scale for transport of angular momentum by this mechanism is, at least, two orders of magnitude greater than the time to core collapse. During the collapse, most of the gravitational energy is radiated in the form of neutrinos, which can also remove some of the angular momentum. However, less than 10% of the angular momentum of the core will be lost in this manner.[13] Thus, the core will approach neutron-star densities with most of its angular momentum intact. At such densities, relativistic effects are important and gravitational wave radiation will be the dominant mode of angular momentum loss.[14]

The above analysis indicates that stellar collapse will generate large amounts of gravitational radiation. Detailed calculations of the expected fluxes are not yet available, so that rough estimates will have to suffice. For a Jacobi ellipsoid of neutron-star dimensions, the gravitational wave luminosity is, in the weak-field approximation, 3.6×10^{53} erg/s.[15] This is definitely a lower limit because such calculations do not include the direct effects of collapse and enhanced radiation due to fission and subsequent coalescence of the fragments. If we use this lower limit to estimate the intensity of the waves (the dimensionless amplitude $\langle h \rangle_{\rm rms}$) at Earth, we get the following results: (a) events within our Galaxy distance $D < 25$ kpc, event frequency (e.f.) $\sim 10^{-2}$ to 10^{-1}/year, $\langle h \rangle_{\rm rms} \gtrsim 10^{-19}$; (b) events within the local group of galaxies—$D < 10^6$ pc, e.f. $\sim 10^{-1}$ to 1/year, $\langle h \rangle_{\rm rms} \gtrsim 2 \times 10^{-21}$; (c) events out to the Virgo cluster of galaxies—$D \sim 2 \times 10^7$ pc, e.f. ~ 25 to 250/year, $\langle h \rangle_{\rm rms} \sim 10^{-22}$.

Actual fluxes may be larger by an order of magnitude or more. Existing detectors[16] are sensitive to $\langle h \rangle_{\rm rms} \sim 10^{-17}$; so an event in the local region of our Galaxy would be required for detection by these experiments. However, estimates[17] of sensitivities ($\langle h \rangle_{\rm rms} \sim 10^{-20}$ to 10^{-21}) achievable with resonant detectors operating at low temperatures indicate that events in the local group of galaxies could be monitored by detectors now under construction or in the planning stages. The radiation should be characterized by frequencies in the kilohertz range (where most of the current and planned detectors are sensitive) and by intensity fluctuations and frequency drifts on time scales of a fraction of a second. This signature should be easily recognizable, especially for an event detected by two or more experiments operating at different frequencies. Confirmed detection of even a single event would be a major experimental achievement, in addition to offering a unique look at the "guts" of stellar collapse.

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Diffraction Dissociation of High-Energy Protons on Hydrogen and Deuterium Targets

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We report results from a measurement of the inclusive processes $pp \to Xp$ and $pd \to Xd$ in the range $5 < M_x^2/s < 0.1$, $0.01 \lesssim |t| \lesssim 0.1$ $(\text{GeV}/c)^2$, and incident proton momenta of 65, 154, and 372 GeV/c. Both pp and pd data show an exponential t dependence and a dominant $1/M_x^2$ behavior for $M_x^2/s \lesssim 0.05$. By comparing pp and pd data we test factorization and, using the Glauber model, we measure the XN total cross section, $\sigma_{XN} = 43 \pm 10$ mb.

In an experiment designed to study inelastic, high-energy diffractive phenomena, we have obtained the invariant differential cross sections $d^2\sigma/dt\,d(M_x^2/s)$ for the inclusive reactions

$$pp \to Xp \tag{1}$$

and

$$pd \to Xd \tag{2}$$

by measuring the energy and angle of low-energy recoil protons and deuterons from a gas jet target situated in the main ring of the Fermi National Accelerator Laboratory. We report results for three values of incident momentum p_0 (65, 154, and 372 GeV/c) over an invariant-mass range of the unobserved system $5s < M_x^2 < 0.1s$ and and for small values of the square of the invariant-four-momentum transfer, $0.01 \lesssim |t| \lesssim 0.06$ $(\text{GeV}/c)^2$ for $pp \to Xp$ and $0.025 \lesssim |t| \lesssim 0.17$ $(\text{GeV}/c)^2$ for $pd \to Xd$.

At small momentum transfer of this experiment, the target particle (p or d) is expected to recoil coherently. The high incident energy allows the incoming proton to dissociate into high-mass states while keeping the minimum momentum transfer $|t_{\min}|^{1/2} \cong M_x^2/2p_{\text{lab}}$ within the coherence region. We may then test whether the cross sections for Reactions (1) and (2) scale to their respective elastic cross sections, according to the concept of factorization at the inelastic vertex. Moreover, by applying the Glauber model[1,2] we can deduce the total cross section, σ_{XN}, of the diffractive state with mass M_x and the quantum numbers of the proton interacting with a single nucleon.

For these reactions, the square of the missing mass is given quite accurately by

$$\frac{M_x^2 - m_p^2}{s} \simeq 1 - x \simeq \frac{|t|^{1/2}}{m_r}\left(\cos\theta - \frac{p_0 + m_r}{p_0}\,\frac{|t|^{1/2}}{2m_r}\right), \tag{3}$$

where x is the Feynman scaling variable defined as $p_{\parallel}/p_{\max}$ in the center-of-mass system, θ is the scattering angle of the recoil target particle relative to the incident proton direction, m_r is the mass of the recoil particle, and $s \simeq 2m_r p_0$ is