Apparent Higher-Order Z_1 Effects Due to Asymmetric Energy Straggling

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Recently published data for the stopping power of fully stripped channeled ions are interpreted in terms of the Vavilov theory of energy straggling. The qualitative dependence of $(dE/dx)_p/Z_1^2$ on charge and velocity can be explained by the dependence of the asymmetry of the energy-loss probability density on these variables through the parameter ξ/ϵ_{max}.

There has been a great deal of interest recently in the problem of higher-order Z_1 corrections to the electronic stopping-power formula. Datz *et al.*[1] give a brief summary of theoretical and experimental work which has been done in the last several years. This work has been concerned primarily with the passage of slowly moving ($\beta < 0.1$) particles with small electric charges ($Z_1 \leq 3$) through amorphous absorbing media. The theory of Lindhard,[2] which includes Z_1^3 corrections due to close- and distant-collision polarization effects as well as the Z_1^4 correction from the Bloch[3] formula, seems to agree well with experimental results involving oppositely charged pions[4] as well as those for absolute stopping powers of H, He, and Li ions.[5]

For $Z_1 > 3$ at these low velocities it is difficult to separate the effects due to electron capture and loss from those due to higher-order Born or Bloch corrections. Datz *et al.*[1] evaded this problem by investigating the stopping power of channeled ions with atomic numbers from 1 to 9 (excluding 4) at $\beta = 0.065$ and 0.087. By using Au{111} channels they were able to measure the energy loss of fully stripped ions which did not undergo charge-exchange collisions in the 4850-Å path-length channels. Their results can be summarized as follows: For $\beta = 0.065$, the quantity S/Z_1^2 (where S is the extrapolated leading edge or the 10% height of the energy-loss distribution) increases by ~7% in going from $Z_1 = 1$ to $Z_1 = 5$ and remains constant for $5 \leq Z_1 \leq 9$; for $\beta = 0.087$, this same quantity increases by a factor of ~1.7 in going from $Z_1 = 1$ to $Z_1 = 5$ (no values for $Z_1 > 5$ were reported for this velocity). Datz *et al.*[1] conclude that no existing theory explains even the qualitative behavior of these results. It is the purpose of this Letter to indicate a mechanism which, although incapable in its most elementary form of providing detailed quantitative agreement with experiment, does satisfactorily explain the qualitative observations of Ref. 1.

It is well known that the Bethe-Bloch formula yields the mean energy loss when multiplied by the thickness of a thin absorber. This quantity is generally different from the most probable energy loss, which is usually the relevant experimental quantity. The relationship between the mean and most probable energy loss is provided by the Vavilov theory[6] for thin absorbers and the Tschalär theory[7] for thick absorbers. For those measurements of stopping power for which $\Delta E = E_i - E_f = (dE/dx)\Delta x$ is a good approximation, the Vavilov theory is applicable. The key parameter in this theory is G, which is given by

$$G = \frac{2\pi Z_1^2 e^4}{mv^2} N\Delta x/\epsilon_{max} \equiv \xi/\epsilon_{max},$$

where $-e$ is the electron charge, m is the electron mass, N is the electron volume density, $v = \beta c$ is the ion velocity, Δx is the absorber thickness, and $\epsilon_{max} = 2mv^2/(1-\beta^2)$ is the kinematic limit on the energy transfer. When $G \gg 1$, the Gaussian energy-straggling theory of Bohr[8] applies; and when $G < 0.01$, the Landau theory[9] applies. The mean and most probable energy losses coincide in the Gaussian limit and are considerably different in the Landau regime [for example, the most probable energy-loss rate for a relativistic cosmic-ray muon is 1.5 MeV/(g/cm^2) in a 0.3-cm plastic scintillator, which is roughly 75% of the mean energy-loss rate]. Since $G \propto Z_1^2$, a given value of β and Δx do not uniquely specify the quantity $(dE/dx)_p/Z_1^2$, even in the first Born approximation of Bethe [$(dE/dx)_p$ denotes the most probable loss as defined by $(\Delta E)_p = (dE/dx)_p\Delta x$]. This fact is of considerable consequence for cosmic-ray experiments concerned with the heavy component. Simple scaling of ground-level muon calibration results by Z_1^2 can lead to 20% errors of gain and threshold settings.

The thickness of the absorber used by Datz *et al.*[1] is sufficiently small so that $G \sim 1$ for $Z_1 = 1$ and for N given by the total Au electron density.

Hence, it is imperative to estimate quantitatively the effects of distribution asymmetry. Since center-channeled ions suffer both close and distant collisions only with the 6s and 5d electrons and distant collisions with the 5p, 5s, and 4f electrons[1] (for Au), it is not appropriate to apply directly the Vavilov theory to an amorphous gold absorber. However, the Vavilov theory yields an expression for the difference of the mean and most probable energy losses, $\Delta E - (\Delta E)_p$, which depends on the electron density of the medium but is independent of the atomic binding. This is easily seen to be due to the fact that the distant-collision energy loss is usually very tightly peaked compared to the close-collision energy loss, i.e., the shape of the energy-loss distribution depends primarily on the close collisions while the position of the most probable loss depends on both the close and distant collisions. It is therefore reasonable that $\Delta E - (\Delta E)_p$ can be obtained for center-channeled ions by applying the Vavilov theory to an absorber with an electron density $\frac{11}{79}N$ (there are eleven 6s and 5d electrons for Au). This is done quite easily in the regime $G \gtrsim 1$ by using the graph provided by Sellers and Hanser.[10] If it is assumed that there are no higher-order corrections to the Bethe-Bloch formula, the value ΔE which pertains to the experiment of Ref. 1 can be obtained by multiplying the saturated 10%-height ordinate of Fig. 3 and Fig. 4 of Ref. 1 by $Z_1^2 \Delta x$. For $\beta = 0.065$, this corresponds to (13 keV)Z_1^2; and for $\beta = 0.087$, (9.0 keV)Z_1^2 (assuming that the large-velocity data saturate for $Z_1 \geqslant 5$). These underestimate the mean energy loss by ~10% (see Fig. 1 of Ref. 1 wherein the 10%-height energy loss is ~0.9 of the peak loss for 2-MeV/amu F^{7+} channeled ions). Correcting for this, I find $\Delta E = (14.3\text{ keV})Z_1^2$ for $\beta = 0.065$ and $\Delta E = (9.9\text{ keV})Z_1^2$ for $\beta = 0.087$. With this information and the graph of Ref. 10, I can calculate $(dE/dx)_p/Z_1^2 = 1 - [\Delta E - (\Delta E)_p]/\Delta E$ in arbitrary units (for a given β). The 10%-height values for fully stripped ions of Ref. 1, normalized to unity at $Z_1 = 5$, are indicated in Fig. 1 of this work along with the calculated function $1 - [\Delta E - (\Delta E)_p]/\Delta E$ for $\beta = 0.065$. It is seen that quite reasonable agreement obtains if we interpret the measurements of Ref. 1 to correspond to most probably energy losses. This is not unreasonable since the extrapolated leading edge of the above-mentioned muon Landau distribution is roughly the same fraction of the peak as the corresponding value for the F^{7+} channeled-ion spectrum of Ref. 1. For $\beta = 0.087$, the agreement is not nearly as good. The Vavilov theory predicts a $Z_1 = 1$ to $Z_1 = 5$ most probable energy-loss ratio of 86% while the experimental value is 60%. However, the Vavilov theory does predict that the effect becomes more pronounced with increasing β, in qualitative agreement with experiment. The numerical discrepancy may be due to systematics involved with the deconvolution procedure used in Ref. 1.

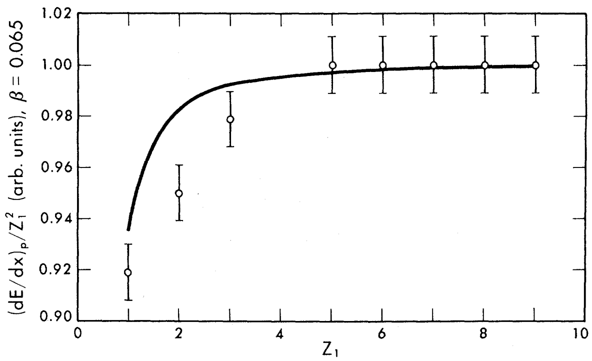

FIG. 1. Ratio of most probable energy loss to Z_1^2 as a function of Z_1. The curve is calculated from the Vavilov theory, assuming the absence of higher-order corrections. The data are from the 10%-height values from Ref. 1. The absorber is an Au{111} channel with a length of 4850 Å and the incident particle velocity is $\beta = 0.065$.

In the above simplified theory it has been assumed that the distant-collision energy loss is sharply peaked about its mean value. This is strictly incorrect and, in fact, the Blunck-Leisegang[11,12] term in the energy-straggling formula, which represents the width of the distant-collision energy-loss distribution, can become appreciable at the small velocitites considered above. The modified energy-straggling distribution is obtained by a convolution of a Gaussian distribution having the Blunck-Leisegang standard deviation with the asymmetric Vavilov distribution. Such a convolution results in only a second-order shift for the value of $\Delta E - (\Delta E)_p$. As an extreme example consider the above-mentioned muon Landau distribution. If this is convolved with a ten-photoelectron statistical distribution [which has a ~100% FWHM (full width at half-maximum) compared to a 20% FWHM for the Landau distribution] the effective most probable energy loss shifts from 1.5 MeV/(g/cm^2) to 1.6 MeV/(g/cm^2), which is still quite different from the mean value of 2 MeV/(g/cm^2). Hence inclusion of distant-collision energy straggling will modify the above treatment to a very small extent.

In any case, it seems that consideration of energy straggling is necessary for a proper evaluation of the higher-order corrections proposed in the literature. In view of the somewhat unrealistic models used thus far for the calculations (i.e., representing atoms as classical or quantum-mechanical harmonic oscillators or as a plasma) any final evaluation must be made experimentally. Since the higher-order corrections of Lindhard[2] are small compared to those due to the asymmetry of the energy-loss distribution for the channeled-particle experimental conditions of Ref. 1, it is clear that it is extremely important to separate the two contributions for these kinds of experiments. This consideration is not so crucial for nonchanneled experiments where one is not constrained to use very thin crystals. In this regard, it may be noted that the random-direction data of Ref. 1 are in reasonable agreement with Lindhard's calculations[2] for those charges for which electron capture and loss effects are not a problem. This is not to be unexpected since the distribution asymmetry effect for the random orientation is much smaller than for the channeled orientation due to the increased number of interacting electrons. Finally, it should be mentioned that the experimental technique of Ref. 5 is not subject to asymmetric-distribution corrections insofar as it is based on a measurement of the mean energy loss (the heating of a thin foil by a penetrating beam of particles is measured which corresponds to a measurement of the sum of the energy losses of the individual particles).

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[1]S. Datz, J. Gomez del Campo, P. F. Dittner, P. D. Miller, and J. A. Biggerstaff, Phys. Rev. Lett. 38, 1145 (1977).

[2]J. Lindhard, Nucl. Instrum. Methods 132, 1 (1976).

[3]F. Bloch, Ann. Phys. (Leipzig) 16, 285 (1933).

[4]W. H. Barkas, N. J. Dyer, and H. H. Heckman, Phys. Rev. Lett. 11, 26 (1963).

[5]H. H. Andersen, J. F. Bak, H. Knudsen, P. Møller Petersen, and B. R. Nielsen, Nucl. Instrum. Methods 140, 537 (1977).

[6]P. V. Vavilov, Zh. Eksp. Teor. Fiz. 32, 940 (1957) [Sov. Phys. JETP 5, 749 (1957)].

[7]C. Tschalär, Nucl. Instrum. Methods 64, 237 (1968).

[8]N. Bohr, Philos. Mag. 30, 581 (1915).

[9]L. Landau, J. Phys. (U.S.S.R.) 8, 201 (1944).

[10]B. Sellers and F. A. Hanser, Nucl. Instrum. Methods 104, 233 (1972). The definition of G given by these authors [their Eq. (4)] is incorrect [compare with Eq. (5) of Ref. 6 or with the definition given in this Letter]. This does not affect the validity of the graph in Ref. 10 which was used to calculate the results of this Letter.

[11]O. Blunck and S. Leisegang, Z. Phys. 128, 500 (1950).

[12]U. Fano, Ann. Rev. Nucl. Sci. 13, 1 (1963).

Transitions between Rydberg Levels of Helium Induced by Electron Collisions

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Electronic transfer rates from levels with principal quantum number p from 8 to 14 have been measured in an afterglow helium plasma at electron temperatures from 390 to 2000°K. While the total transfer rates are in satisfactory agreement with both classical and quantum treatments, transitions with $|\Delta p| > 1$ are found to have a substantial probability; this favors a classical treatment in this energy range.

Rate coefficients for electron-induced transfers between high Rydberg states of atoms are of great interest in many astrophysical situations, as well as in laboratory plasma physics. Up to now, however, most information on this subject has been theoretical; experimental evidence was indirectly deduced from comparisons between computed and observed excited-state populations either in an unperturbed plasma[1] or in a plasma perturbed by photoionization of highly excited levels.[2]

We report here preliminary results of direct measurements of excitation transfers between high Rydberg states of helium induced by collisions with free electrons. The experimental system has been described in detail elsewhere[3]; briefly, high Rydberg ^3P sublevels are reached by direct photoexcitation from the He (2^3S) metastable atoms present in sufficient concentration in a high-purity, room-temperature helium afterglow at 1.5 Torr.[4] Electron density and tempera-