Structure of Yrast Traps

T. Døssing. K. Neergård, K. Matsuyanagi, and Hsi-Chen Chang[(a)]
The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark

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An "island" of isomeric states with high multiplicities found in a recent experiment are interpreted as "yrast traps" in deformed nuclei rotating around a symmetry axis. According to our interpretation, a group of trap states for neutron numbers around 82 and angular momenta below 40 is connected with the nuclear shell structure for weakly deformed potentials. A second group of traps situated in a more narrow region around neutron number 82 and with angular momentum around 50 is attributed to the occurence of shell structure for a ratio of axes 2:3.

Recently a search for high-spin isomeric states ("yrast traps") was reported[1] by a joint group from the Niels Bohr Institute and Gesellschaft für Schwerionenforschung. Several isomers with multiplicities between 8 and 20 were found, all belonging to nuclei situated in a specific part of the investigated area of the N-Z plane around the neutron number 82.

According to a suggestion[2] by Bohr and Mottelson, yrast traps may appear when, for certain values of the angular momentum, the single-particle potential becomes axially symmetric with respect to the spin direction. The reason is that, with this symmetry of the potential, there is no organization of the individual levels of the yrast region into rotational bands parallel to the yrast line. The electromagnetic transitions which carry off angular momentum have single-particle character. Under such circumstances a given configuration of the independent-particle system may be unable to decay through a transition with low multipolarity and in this way becomes a trap.

Numerical calculations based on this idea were carried out[3,4] by the Lund-Warsaw group. The present theoretical "search" for trap states is more extensive as to the set of nuclei and deformations considered and covers the experimentally investigated region.[1] We find as a result of this search new classes of traps connected with strongly oblate and weakly prolate shapes, which were not predicted in the earlier studies. We also attempt to get a deeper qualitative insight into the general structure of the trap configurations.

For most even nuclei in the experimentally investigated area we have calculated by the Strutinsky method[3,5,6] the energy at various angular momenta as a function of the deformation parameters β and γ. Our deformation space and parameters are the same as in Ref. 6. In particular, a Woods-Saxon single-particle potential was used.

From the results of this calculation we have extracted the difference between the minimal energy with restriction to shapes that rotate around the symmetry axis and the minimal energy for the total deformation space. The shapes with rotation around the symmetry axis included in the deformation space extend from weakly prolate shapes (ratio of axes 1.08:1) through the spherical shape to very strongly oblate shapes (ratio of axes 1:2.5). The energy differences calculated in this way are shown in Fig. 1 on a N-Z diagram for a number of angular momenta. In the white areas of the diagram the equilibrium shape corresponds to rotation around the symmetry axis; hence, for these values of N, Z, and I, traps are possible. We define a trap state in the same way as in Ref. 3, namely, as a state which cannot decay through an $E1$, $M1$, $E2$, or $M2$ single-particle transition. In addition we require that it be an yrast state. For all combinations of N, Z, and I within the white areas of Fig. 1 we search for independent-particle states which satisfy these conditions. The angular momenta of theoretical trap states found in this way are shown as encircled numbers in Fig. 1.

The theoretical traps fall into several groups with different deformation. For $I<40$ and neutron numbers 80, 84–88, and 114, the deformation is small ($\beta<0.15$). The shape is prolate for $N=80$ and 114, and oblate for $N=84$–88. The traps calculated by the Lund-Warsaw group[3,4] all belong to the small oblate deformation. For neutron numbers 82–84 we predict a number of traps belonging to a strongly oblate shape with ratio of axes approximately equal to 2:3 ($\beta\approx0.4$). This deformation comes into play only for $I>40$ and all theoretical traps with $I>40$ belong to it. The groups mentioned so far (except for $N=114$) build up an "island" of theoretical traps with $N=80$–88, $Z=62$–70, the contour of which approximately corresponds to that of the empirical "island."[1] Our model gives traps also in the adjacent odd-A nu-

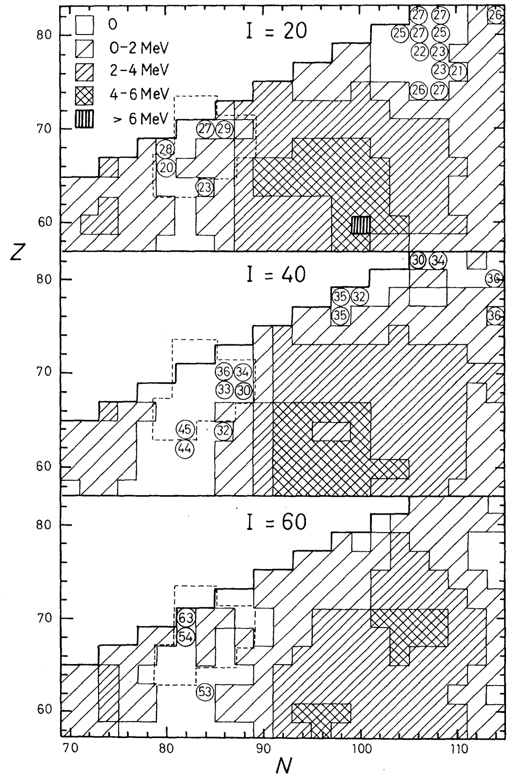

FIG. 1. Contour diagram of the minimal energy with restriction to shape that rotate around the symmetry axis measured relative to the minimal energy in the whole β-γ plane. This energy is shown for even-even nuclei and for the three values 20, 40, and 60 of the angular momentum. The encircled numbers are the spins of theoretical traps, grouped into the inervals $20 \leq I \leq 29$, $30 \leq I \leq 49$, and $I \geq 50$. The dashed line shows the approximate contour of the empirical "island" of isomers (Ref. 1) assuming that four neutrons were evaporated.

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For neutron numbers 98–110 the deformation is oblate with $\beta \approx 0.25$. Some of the nuclei belonging to this group were investigated in the experiment, but no isomers were found. A possible reason is that when the hexadecapole degree of freedom is taken into account most of these nuclei may be found to have at the spin values considered a shape deviating from oblate symmetry. In fact, in calculations[7] for $I = 0$ the hexadecapole degree of freedom is decisive for producing prolate shapes of the W–Os nuclei.

The main pattern in the occurence of trap states can be understood in terms of simple systematic features of the single-particle spectrum. As a first example we consider the case of weakly oblate shapes ($N = 84$–88, $Z = 64$–70).

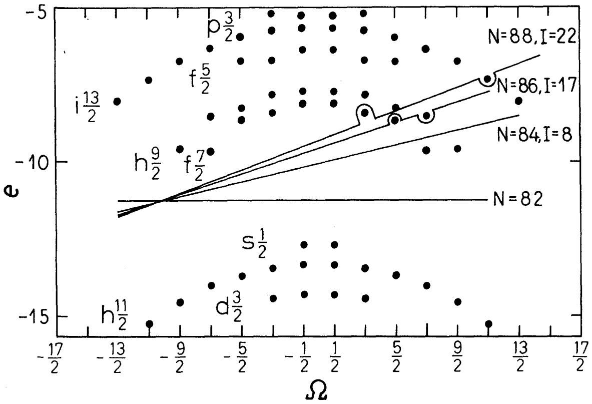

FIG. 2. Single-neutron energies e for the oblate deformation $\beta = 0.1$ vs the angular momentum Ω with respect to the symmetry axis. The quantum numbers given are those of the spherical level towards which each state converges in the limit of $\beta \rightarrow 0$. The neutron Fermi surfaces of the trap configurations in the region $N = 84$–88, $Z = 64$–70 are shown.

The single-particle energies for neutrons in the oblate potential with deformation $\beta = 0.1$ are plotted in Fig. 2 versus the angular-momentum projection. (Note that by definition Ω is the projection on the direction of the total angular momentum, and therefore the sum of single-particle Ω is equal to I.) In a corresponding plot for the spherical potential the dots would lie on horizontal lines labeled by the quantum numbers (nlj). Turning on an oblate deformation these lines are bent into a bell shape. Still, for the small deformation $\beta = 0.1$ a gap remains at $N = 82$.

Configurations with neutron numbers $N = 84$–86, 88 and increasing angular momentum can be obtained by filling particles into levels above the $N = 82$ gap. None of the configurations shown can decay with a single-particle transition without changing the angular momentum by at least three units. Hence they are trap configurations. The neutron configurations formed in this way are involved in the trap states for $N = 84$–88, $Z = 64$–70. They are composed of particles in high-Ω members of the subshells $f_{7/2}$, $h_{9/2}$, and $i_{13/2}$.

In order to have a trap state of the total nuclear system both the neutrons and the protons must be in trap configurations. The proton configurations for $Z = 64$–70 in the weakly oblate potential are built from particles in high-Ω $h_{11/2}$ states and holes in low-Ω $d_{5/2}$ states. Quite generally, when holes are involved in a trap configuration for an oblate shape they must have low Ω. (This rule may be inferred from a study of the neutron spectum in Fig. 2.)

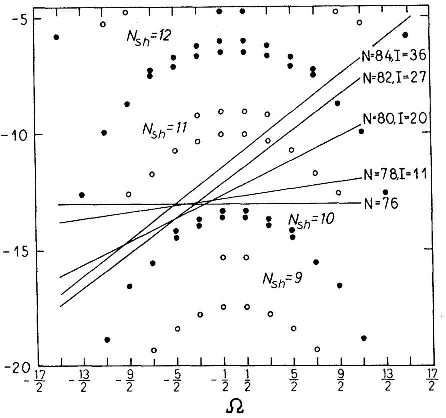

FIG. 3. Single-neutron energies for the oblate deformation $\beta = 0.40$. The closed points belong to even values of the shell quantum number N_{sh} and the open points to odd N_{sh}. The Fermi surfaces discussed in the text are shown.

For a prolate potential we can make a similar discussion exchanging the words "particle" and "hole." Prolate configurations involving high-Ω members of the $h_{11/2}$ and $i_{13/2}$ shells enter the traps with $N = 80$ and $N = 114$. Generally, however, the dominance of orbits with low j in the upper end of the major shells makes the prolate single-particle spectra less suitable for building traps.

The neutron configurations discussed above for the small oblate deformation have a structure similar to that of known isomers above ^{208}Pb composed of particles in aligned spherical orbits with large j. The special stability of these aligned configurations is usually discussed in terms of the properties of the residual interaction.[8]

It is worth noting that in fact the density distribution of such a configuration is oblate with symmetry axis along the direction of the total angular momentum. If holes are added to the aligned configuration the particle-hole interaction favors hole states having low density along the "equator" thus contributing to the oblateness of the total density. In the present model the special stability of this type of configurations results from the lowering of the energies of the high-Ω particles and low-Ω holes in an oblate potential. The relation between the two approaches will be further discussed in a separate publication.[9]

The trap states for $N = 82$ and 84, $Z = 62-70$, and $I \geq 44$ exhibit another single-particle structure of particular interest. These nuclei aquire at $I \approx 50$ a strongly oblate shape with a ratio of axes close to 2 : 3. The occurence of this deformation appears to be connected with the shell structure associated with the quantum number[10] $N_{\mathrm{sh}} = 3n_z + 2n_\perp$, where n_z and $n_\perp$ are the usual asymptotic quantum numbers describing the number of oscillations parallel and perpendicular to the symmetry axis. As shown in Fig. 3, the neutron number $N = 76$ completes the $N_{\mathrm{sh}} = 10$ shell, and above $N = 76$ we get trap configurations with $N = 78$, 80, 82, and 84, similar to those described above for the small oblate deformation. Among these, only the $N = 82$ and 84 configurations have a sufficient neutron spin to come into play at $I \approx 50$. They can, for $Z = 62-70$, combine with proton configurations analogous to those discussed above for the weakly oblate case being built from a few particles in high-Ω states with $N_{\mathrm{sh}} = 11$ and 12, and several holes in low-Ω states with $N_{\mathrm{sh}} = 10$.

The systematics of the 2 : 3 shell structure points to the possible existence of similar traps connected with the completion of other shells with even value of N_{sh}, for example, for nucleon numbers above 48 ($N_{\mathrm{sh}} = 8$) or 114 ($N_{\mathrm{sh}} = 12$).

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[a] Present and permanent address: Atomic Energy Institute, Academy of Science, Peking, China.

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[8] See, for example, J. Blomquist *et al.*, Phys. Rev. Lett. 38, 534 (1977), and references therein.

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