Study of Quark Structure Functions

F. T. Dao,[a] E. Flaminio,[b] Kwan-Wu Lai, W. Metcalf,[c] and Ling-Lie Wang
Brookhaven National Laboratory, Upton, New York 11973
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The quark structure functions of the proton are determined through a combined analysis of the reactions $pN \to l\bar{l}X$ and $eN \to eX$. The valence-quark structure function of the pion is also given by analyzing the $\pi N \to \mu\bar{\mu}X$ data measured by the Branson *et al.*

The simple picture that hadrons are made of quarks has been shown to be very fruitful in describing both hadron and lepton interactions. The quark structure functions were mostly obtained by analyzing lepton-hadron inelastic reactions. However, the procedure suffers from the fact that the sea-quark distributions have not been well determined, especially in the large-x region ($x > 0.1$). They are rather small quantities obtained by taking the difference of two structure functions νW_2 and W_3 of neutrino and antineutrino experiments with rather large errors.[1] In addition, the quark structure functions for mesons are difficult to obtain this way.

As the experimental measurements of dilepton production in hadron interactions become more refined, they provide a very useful additional means to analyze the quark structure functions via the Drell-Yan model.[2] This method, of course, depends upon the consistency of the two-way approach. As we shall show later, in some kinematic region the differential cross sections for dilepton production in pp are sensitive to the valence-quark distributions as in ep deep inelastic scattering. Thus they provide a detailed consistency check of the Drell-Yan model. On the other hand, once the method is shown to work, measurements of the same distributions for the reactions $\pi N \to l\bar{l}X$, in similar kinematic regions, can be used to obtain the pion valence-quark distributions. In other kinematic regions the lepton-pair distributions are sensitive to structure functions like the sea-quark distributions that are hard to obtain in lepton reactions. Thus further information on the structure functions can be obtained this way. We report the results of such an analysis.

The differential cross section for lepton-pair production in the Drell-Yan model is

$$m^3\, d\sigma^{B,T}/dm\, dx_F = \tfrac{8}{3}\pi\alpha^2(x_F^2 + 4m^2/s)^{-1/2}\,\tfrac{1}{3}\{\tfrac{4}{9}[u^B(x_1)\bar{u}^T(x_2) + \bar{u}^B(x_1)u^T(x_2)] + \tfrac{1}{9}[d^B(x_1)\bar{d}^T(x_2) + \bar{d}^B(x_1)d^T(x_2)] + \tfrac{1}{9}[s^B(x_1)\bar{s}^T(x_2) + \bar{s}^B(x_1)s^T(x_2)]\}, \quad (1)$$

where

$$x_1 = \tfrac{1}{2}[(x_F^2 + 4m^2/s)^{1/2} + x_F], \quad x_2 = \tfrac{1}{2}[(x_F^2 + 4m^2/s)^{1/2} - x_F], \quad (2)$$

m is the lepton-pair mass, $x_F = 2p_z^*/\sqrt{s}$, p_z^* is the longitudinal momentum of the lepton pair in the c.m. system of the particles B and T, and p_z^* is positive along the beam direction. The superscript B (T) stands for the beam (target) particle in the reaction. The factor of $\frac{1}{3}$ right in front of the curly bracket in Eq. (1) comes from the fact that the structure functions u, d, and s are already summed over three colors. Moreover, the structure function νW_2 in ep inelastic scattering is expressed in terms of the same distribution functions by

$$\nu W_2^{ep}(x) = \tfrac{4}{9}u^p(x) + \tfrac{1}{9}d^p(x) + \tfrac{4}{9}\bar{u}^p(x) + \tfrac{1}{9}\bar{d}^p(x) + \tfrac{1}{9}s^p(x) + \tfrac{1}{9}\bar{s}^p(x), \quad (3)$$

where u and d contain both the valence and the sea quarks, and we assume all quarks in the sea to be the same,[3] i.e., $u^p(x) = u_v^p(x) + u_s^p(x)$, $d^p(x) = d_v^p(x) + d_s^p(x)$, $\bar{u}^p(x) = \bar{d}^p(x) = u_s^p(x) = d_s^p(x) = s^p(x) = \bar{s}^p(x)$. Note that Eq. (1) is quadratic in the quark distributions, while Eq. (3) is linear; thus the color scheme for quarks can be tested.

There are two kinematic regions where Eq. (1) becomes especially simple. To be specific, we consider first the proton-proton or proton-nucleus interactions. In the case of m^2/s being small relative to x_F, from Eq. (2), $x_1 \approx x_F$, $x_2 \approx 0$, and Eq. (1) becomes

$$m^3\, d\sigma/dm\, dx_F \approx f s^p(x_2)\, 2\nu W_2^{ep}(x_1), \quad (4)$$

where $f = 8\pi\alpha^2/[9(x_F^2 + 4m^2/s)^{1/2}]$, and the approximation is good if x_1 is large enough ($\gtrsim 0.2$) so that

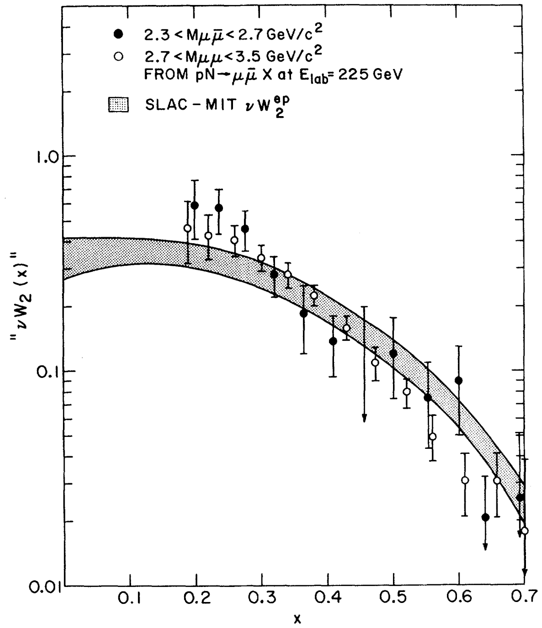

FIG. 1. The "νW_2" obtained using Eq. (4) with the data of Ref. 4. The data points are lowered by a factor of 3 (see text).

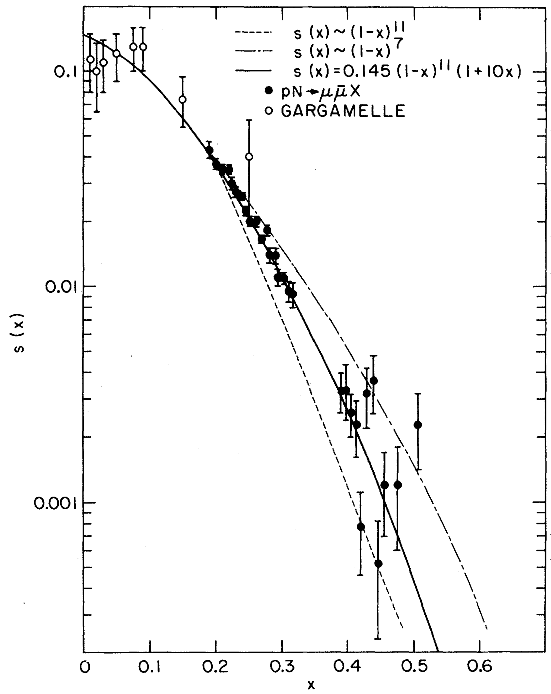

FIG. 2. The sea-quark distribution obtained using Eq. (5) with the data of Ref. 6. The open points are from ν, $\bar{\nu}$ reactions. The solid curve is a fit by the sea-quark distribution.

some of the $s^p(x_1)$ contributions can be neglected. Equation (4) can be applied to both pp and p-$[\frac{1}{2}(p+n)]$ interactions. Therefore, in this kinematic region, the distribution in x_F gives a direct check of consistency of the Drell-Yan model with the quark model in ep inelastic scattering. Unfortunately no measurements of this type are available for dilepton mass large enough to fully justify the application of the Drell-Yan model. However, we shall use the presently available data to perform such a comparison. The "νW_2" obtained from the measurements by Branson *et al.*[4] of $pN \to \mu\bar{\mu}X$ at $E_{\rm lab} = 225$ GeV is presented in Fig. (1). The J/ψ contributions have been subtracted using the fit given in Ref. 4. This "νW_2" is compared to the νW_2^{ep} measured in ep inelastic scattering.[5] We see that the agreement in shape is reasonable. With use of $s(x_2) = 0.15$, our "νW_2" is about a factor of 3 too big in magnitude compared to νW_2^{ep}. (Note that color symmetry is used here for the quarks.) However, we are encouraged by the fact that the x_F distributions do not change very much as the mass of the dilepton changes; therefore we hope that such agreement in shape will hold in larger mass regions where the model is expected to work better. In any event, we see the importance and relevance of the measurements of x_F distributions.

Another interesting kinematic region is at $x_F = 0$ but varying m^2/s. Equation (1) simply becomes

$$m^3 \frac{d\sigma^{p,(p+n)/2}}{dm\, dx_F}\bigg|_{x_F=0} \approx f s^p(x)[\nu W_2^{ep}(x) + \nu W_2^{e,(p+n)/2}(x)], \qquad (5)$$

where $x = (m^2/s)^{1/2}$ and the approximation is good for large x ($\gtrsim 0.2$). Given the experimentally measured νW_2^{ep}, $\nu W_2^{e,(p+n)/2}$, and the dilepton distributions at $x_F = 0$, we can obtain[4] $s^p(x)$. Recently very good measurements of $pN \to \mu\bar{\mu}X$ at $x_F = 0$, $E_{\rm lab} = 400$ GeV, and $m_{\mu\bar{\mu}}$ varying from 5 to 14 GeV/c^2 have been made at Fermilab.[6] The sea-quark distributions thus obtained are shown in Fig. 2. We see that the sea-quark distribution is rather steep, $\sim(1-x)^9$, in the large-x region. The data points for the sea-quark distribution obtained in ν and $\bar{\nu}$ inelastic scattering are shown in the same figure. They are in agreement with the sea-quark distribution obtained from lepton pair production data. Notice also that the magnitude of the $pp \to \mu\bar{\mu}X$ cross section is consistent with the quarks having three colors.

Provided with this new sea-quark distribution and the improved νW_2 measurement in ep and en inelastic scattering of Atwood,[5] we have reanalyzed the up- and down-valence-quark distributions for the proton. The following constraints are used for the valence-quark distributions: $u_v^p(x) \propto \sqrt{x}$ and $d_v^p(x) \propto \sqrt{x}$ as $x \to 0$, $\int_0^1 x^{-1} u_v^p(x)\,dx = 2$, and $\int_0^1 x^{-1} d_v^p(x)\,dx = 1$. The former is given by the argument that the valence-quark distributions are controlled by secondary Regge poles as $x \to 0$; thus they go to zero roughly like $\sqrt{x}$. The latter two simply give the normalization that there are two up quarks and one down quark in the proton. These constraints play a very important role in our fit to obtain the quark distributions in the small-x region ($x < 0.2$), because the division between the sea and valence quark is rather arbitrary and the data are not very precise. The results are[7]

$$u_v^p(x) = 2.99\sqrt{x}(1-x)^4(1+5.99x-2.63\sqrt{x}),$$
$$d_v^p(x) = 1.02\sqrt{x}(1-x)^5(1.0+5.75x), \qquad (6)$$
$$s^p(x) = 0.145(1-x)^{11}(1+10x).$$

We consider these to be first-order estimates of the quark distributions. There can be scaling-violation effects, e.g., the effects from asymptotically free gauge theory. The large scaling violation observed in $\mu p \to \mu X$ measured at Fermilab[8] can also be attributed to the production of new quark states[9] like charm. All these effects contribute mostly to the region of $x < 0.2$. While these effects will not change very much of our analysis of the data given by Ref. 6, it is very important to compare the structure functions for $x < 0.2$ from both the $ep \to eX$ and $pp \to l\bar{l}X$ experiments when they are available.

Since our results on the proton structure functions obtained in $pN \to \mu\bar{\mu}X$ are in reasonable agreement in shape with those obtained in $eN \to eX$, we now apply a similar method to $\pi N \to l\bar{l}X$ to obtain an estimate for the pion structure functions.

Using Eq. (1) and assuming the valence-quark and valence-antiquark distributions in the pion being given by the same function $q^\pi(x_1)$, and ignoring some of the sea-quark contributions appropriate in the kinematic region of $m^2/s \ll x_F$, $x_1 \approx x_F$, and $x_2 \approx 0$, we obtain

$$m^3 \frac{d\sigma^{(\pi^+ + \pi^-)/2,\,(p+n)/2}}{dm\,dx_F} \approx f\,\tfrac{1}{2}\,\nu W_2^{e,(p+n)/2}(x_2)\,q^\pi(x_1). \qquad (7)$$

Given $\nu W_2(x_2)$, we can learn about $q^\pi(x_1)$. It is given in Fig. 3 using the measurements (Ref. 4) of $\pi^\pm N \to \mu\bar{\mu}X$ at $E_{lab} = 225$ GeV with J/ψ subtracted. Here again the q_v^π obtained has too high a nor-

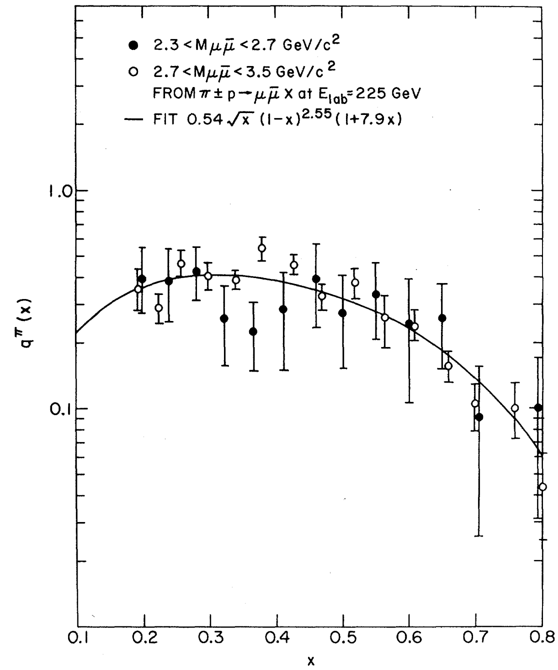

FIG. 3. The valence-quark distribution of the pion obtained using Eq. (7) with the data of Ref. 4. The data points are lowered by a factor of 3.5 (see text).

malization (about a factor of 3 in the case of quarks having three colors, the same as in the case of "νW_2"); however, we think that the x variation can well reflect that of the π structure function because the x variation does not depend strongly upon the dilepton mass. Of course, this should be tested in the future. We have also analyzed the data on $\pi^+ N \to \mu\bar{\mu}X$ at 150 GeV/c.[10] The x dependence obtained is consistent with that at 225 GeV. We determine the normalization of $q_v^\pi(x)$ by requiring that $q_v^\pi(x) \propto \sqrt{x}$ as $x \to 0$ and that $\int_0^1 x^{-1} q_v^\pi(x)\,dx = 1$. We obtain

$$q_v^\pi(x) = 0.54\sqrt{x}(1-x)^{2.55}(1+7.9x), \qquad (8)$$

which is plotted in Fig. 3 compared to the data points with the magnitude adjusted. The pion valence-quark distribution is substantially flatter[11] than that of the proton as given in Eq. (6).

The pion sea-quark distribution $s^\pi(x)$ can also in principle be determined by making use of data on $\pi^\pm N$ scattering. However, because subtractions between data points are involved, we are unable to obtain a meaningful $s^\pi(x)$ with the accuracy of the present data.

Summarizing, we have shown that "νW_2" computed using the Drell-Yan model from the longitudinal-momentum distributions of the continuum dilepton production in pN interactions at $E_{\rm lab} = 225$ GeV[4] is in reasonable agreement in shape with νW_2^{ep}. The origin of the factor-of-3 discrepancy between theory and the data of Ref. 4 can be due to at least one of the three reasons: (1) low-mass effect, (2) failure of color symmetry, (3) systematic error in the data. We are not too concerned with possibility (1), because, as stated above, the x distribution does not change too much as the dilepton mass changes; with possibilities (2) and (3) only time can tell. Using the mass distribution of dilepton production in pN interactions at $E_{\rm lab}=400$ GeV,[6] we have determined the sea-quark distribution for $x \geqslant 0.2$; it behaves like $(1-x)^9$ for large x. The valence-quark distributions of the pion are also obtained from the longitudinal-momentum distributions of the dileptons produced in $\pi^{\pm}N$ reactions at $E_{\rm lab} = 225$ GeV.[4] They are substantially flatter than those for the proton.

Such studies ought to be further carried out as more dilepton production data, at various energies and especially at high dilepton masses, become available in the future. It is foreseeable that the sea-quark distribution of the pion and the strange-quark distribution of the kaon can be determined this way.

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[a] Permanent address: Tufts University, Medford, Mass. 02155.

[b] Permanent address: Istituto di Fisica dell'Università di Pisa, and Istituto Nazionale de Fisica Nucleare, Pisa, Italy.

[c] Permanent address: University of Rochester, Rochester, N. Y. 14627.

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[4] J. G. Branson *et al.*, Phys. Rev. Lett. 38, 1331, 1334 (1977).

[5] The structure functions νW_2^{ep} and νW_2^{en} (plotted in the variable x') used are from W. T. Atwood, SLAC Report No. 185, 1975 (unpublished); R. E. Taylor, in *Proceedings of the European Physical Society International Conference on High Energy Physics, Palermo, Italy, 1975*, edited by A. Zichichi (Editrice Compositori, Bologna, 1976).

[6] S. W. Herb *et al.*, Phys. Rev. Lett. 39, 252 (1977). The same method of analyzing the proton sea-quark distributions was used in R. F. Peierls, T. L. Trueman, and L.-L. Wang, Phys. Rev. D 16, 1397 (1977). We would like to thank Professor L. M. Lederman for communicating to us that similar analysis for the sea quark is being carried out by him and his co-workers. He also informed us that their data are under careful analysis taking into account Fermi motion and the detailed $p_\perp$ distribution of the μ pair. Thus, our sea-quark distribution may be subject to some changes. However, that should not have noticeable effect on the valence-quark distributions given in Eq. (6).

[7] The data used for νW_2^{ep} and νW_2^{en} are the same as given in Ref. 5. For comparison with previously determined quark structure functions of the proton, see Ref. 6 and the references given therein.

[8] H. L. Anderson *et al.*, Phys. Rev. Lett. 38, 1450 (1977).

[9] For example, J. M. Wang and L.-L. Wang [Phys. Lett. 61B, 377 (1976)] discussed such a mechanism.

[10] K. J. Anderson *et al.*, Phys. Rev. Lett. 36, 237 (1976).

[11] Pion structure functions have been obtained from the fusion model for J/ψ production by A. Donnachie and P. V. Landshoff, "J/ψ Production from Hadron Beams," to be published.