has been used for many years in a series of elegant atomic physics experiments. See, for example, R. S. Van Dyke, Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. Lett. 38, 310 (1977).

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[5]To be specific, $\langle P_\theta \rangle = kT\partial(\ln Z)/\partial\omega$ and $\langle H \rangle = \omega\langle P_\theta\rangle + kT^2\partial(\ln Z)/\partial T$. Here the distinction between the exact value and the mean value of the angular momentum and energy is blurred because we use a canonical ensemble, rather than a microcanonical ensemble. For a large number of electrons, the two ensembles are equivalent, except that the canonical ensemble allows small fluctuations of H and P_θ about their mean values.

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[13]Electromagnetic modes carry off angular momentum as well as energy, and one might worry that a mode at frequency ω would produce an outward radial drift of the plasma and tap the electrostatic energy. To prevent this, the system should be operated as a waveguide beyond cutoff at frequency ω. In other words, one should choose the radius of the cylindrical wall to be such that $\omega < c/R < \Omega$.

Effects of Flow on Density Profiles in Laser-Irradiated Plasmas

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When the plasma outflow velocity relative to the critical surface is supersonic, compressional density profiles can form in the critical region. These compressions involve dissipative processes like those in collisionless shocks; associated plasma instabilities and reflected ions may inhibit energy transport and enhance laser-light absorption.

The manner in which laser-radiation pressure modifies plasma density profiles is important to laser-light absorption, because the expected mix of absorption processes and transport phenomena depends sensitively on density profiles near the critical surface. In this Letter we show analytically that plasmas which enter the critical region supersonically can exhibit compressional density profiles, having a nonmonotonic dependence of density on distance from the target surface. Supersonic compressions in the critical region necessarily involve dissipation properties like those in collisionless shocks, and the plasma instabilities responsible for the dissipation can affect laser-light absorption and energy transport.

In contrast, plasmas which enter the critical surface subsonically exhibit the familar density step[1] there. For some near-sonic flows, no steady profile exists. Our analysis offers new insights into recent computer hydrodynamics calculations in the sonic and supersonic regimes.

Jump conditions across the critical surface.—These may be obtained by integrating steady-state equations of mass and momentum conservation, $\nabla\cdot(\rho\vec{v})=0$, $\rho\vec{v}\cdot\nabla\vec{v} = -\nabla p - \nabla\cdot\overleftrightarrow{\Pi}_r$, from a point $\vec{x}_1$ on one side of the critical density to a point $\vec{x}_2$ on the laser side (see Fig. 1). The laser-radiation pressure tensor is[2]

$$\overleftrightarrow{\Pi}_r = \overleftrightarrow{I}\langle E^2 + B^2\rangle/8\pi - \langle \epsilon_r \vec{E}\vec{E} + \vec{B}\vec{B}\rangle/4\pi, \qquad (1)$$

where $\epsilon_r \equiv \mathrm{Re}[1-\omega_p^2/\omega(\omega+i\nu)]$, the laser frequency is ω, and the collision frequency is ν. For light normally incident on a one-dimensional plasma, the normal component of $\overleftrightarrow{\Pi}_r$ is $\langle E^2+B^2\rangle/8\pi$. If $\vec{x}_1$ and $\vec{x}_2$ are close together, we need not specify the overall geometry. For spherical plasmas we require $|\vec{x}_1 - \vec{x}_2| \ll r$.

We assume that the flow is approximately steady for the short time $|\vec{x}_1 - \vec{x}_2|/c_1$ required to cross the critical region. This is well justified for current experiments which typically have $|\vec{x}_1 - \vec{x}_2| \approx 1-2\ \mu\mathrm{m}$, $c_1 \approx 3\times10^7$ cm/sec, so that

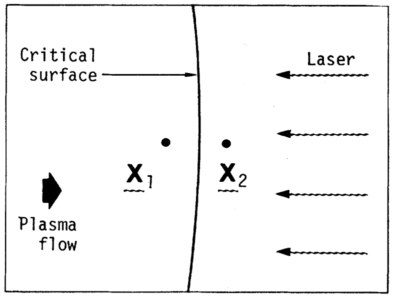

FIG. 1. Geometry for jump conditions. Laser light is incident from the right. Point $\vec{x}_1$ is to the left of critical surface; $\vec{x}_2$ is to the right.

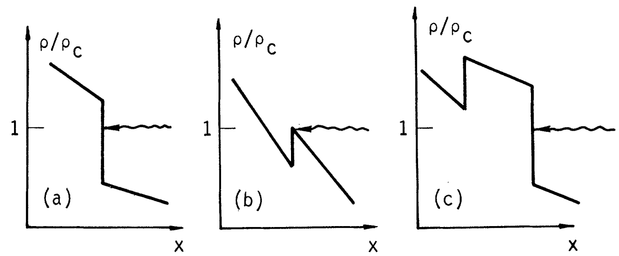

FIG. 2. Schematic density profiles for (a) D front, $M_1 \leq M_D < 1$; (b) R front, $M_1 \geq M_R > 1$; (c) Shock ($M > 1$) plus D front.

$|\vec{x}_1 - \vec{x}_2|/c_1$ is several picoseconds, whereas typical laser risetimes are greater than 10–20 psec.

We choose $\vec{x}_1$ so that $\overleftrightarrow{\Pi}_r(\vec{x}_1) = 0$, and define $\Pi \equiv \hat{n}\cdot\overleftrightarrow{\Pi}_r(\vec{x}_2)\cdot\hat{n}$, where $\hat{n}$ is the local normal to the critical surface. The jump conditions in a frame moving with the critical surface are then

$$\rho_1 v_1 = \rho_2 v_2, \tag{2}$$

$$p_1 + \rho_1 v_1^2 = p_2 + \rho_2 v_2^2 + \Pi. \tag{3}$$

Our procedure is to solve Eqs. (2) and (3) for M_2/M_1, as a function of Π and $M_1 \equiv v_1(p_1/\rho_1)^{-1/2}$. We replace the energy equation by the assumption that the region between $\vec{x}_1$ and $\vec{x}_2$ is isothermal: $c_2 \equiv (p_2/\rho_2)^{1/2} = c_1$. Our results are readily generalized to the case $c_2 > c_1$, however. Isothermality is usually well satisfied because the range of a thermal electron is typically longer than $|\vec{x}_1 - \vec{x}_2|$: The range is $\sim 20\ \mu\mathrm{m} \times [n_e/(10^{21}\ \mathrm{cm}^{-3})]^{-1} \times (T_e/1\ \mathrm{keV})^2$, whereas $|\vec{x}_1 - \vec{x}_2|$ is at most a few times c/ω_{pe} which is a few times $(0.17\ \mu\mathrm{m})[n_e/(10^{21}\ \mathrm{cm}^{-3}]^{-1/2}$. (The critical density for 1.06-μm light is $10^{21}\ \mathrm{cm}^{-3}$.) Note that Eqs. (3) and (4) remain valid in the collisionless case, provided that the particle distribution functions are approximately isotropic. The solution to Eqs. (2) and (3) is

$$\frac{M_2}{M_1} = \frac{\rho_1}{\rho_2} = \frac{1 + M_1^2 - (\Pi/\rho_1 c_1^2) \pm [(1 + M_1^2 - \Pi/\rho_1 c_1^2)^2 - 4M_1^2]^{1/2}}{2M_1^2}. \tag{4}$$

From the requirement that M_2/M_1 be real, we obtain the region where steady-state solutions exist: Either

$$M_1 \geq M_R \equiv 1 + (\Pi/\rho_1 c_1^2)^{1/2} \tag{5a}$$

or

$$M_1 \leq M_D \equiv 1 - (\Pi/\rho_1 c_1^2)^{1/2}. \tag{5b}$$

We call radiation-pressure fronts satisfying (5a) "R type," and those satisfying (5b) "D type." (This nomenclature is borrowed from the theory of ionization fronts.[3] R and D fronts are analogous to detonations and deflagrations, respectively, in chemical combustion.[4]) Note that for D fronts the requirement that the flow velocity not change sign across x_c implies $\Pi \leq \rho_1 c_1^2$ so that $M_D \geq 0$. There are no steady solutions with $M_D < M < M_R$.

"Critical" fronts[3] are those for which the equality sign holds in (5). They have $M_2 = 1$ and are analogous to Chapman-Jouguet deflagrations and detonations.[4] Just as in the case of ionization fronts, there is no *a priori* reason to assume that radiation-pressure fronts are "critical."

Straightforward algebra using Eq. (4) shows that R fronts represent compressions ($\rho_1 < \rho_2$; $M_1 > M_2$), whereas D fronts are rarefactions ($\rho_1 > \rho_2$; $M_1 < M_2$). Figure 2 indicates this distinction schematically. Thus D fronts represent the "density step" familar from plasma simulations.[1] Plasma flows subsonically into x_c and then accelerates. The total pressure outside the critical surface ($p_2 + \rho_2 v_2^2 + \Pi$) is balanced predominantly by the thermal pressure p_1.

In contrast, R fronts have not yet received attention in the laser-plasma literature. Here plasma flows supersonically into the critical region. The upstream pressure thus cannot adjust to the added inward laser momentum Π at $\vec{x}_c$. When the flow reaches the critical surface, its ram pressure $\rho_1 v_1^2$ must nearly balance the total pressure ($p_2 + \rho_2 v_2^2 + \Pi$) in the underdense region. Since the upstream flow cannot accomodate to the laser-

TABLE I. Properties of D and R fronts.

	D FRONTS	R FRONTS
	$M_1 \leq M_D = 1-(\Pi/\rho_1 c_1^2)^{1/2}$	$M_1 \geq M_R = 1+(\Pi/\rho_1 c_1^2)^{1/2}$
	Rarefactions ($\rho_2<\rho_1$)	Compressions ($\rho_2>\rho_1$)
CRITICAL	$M_1 = M_D$, $M_2 = 1$	$M_1 = M_R$, $M_2 = 1$
WEAK	$1>(\rho_2/\rho_1)>M_D$, $M_2<1$	$M_R>(\rho_2/\rho_1)>1$, $M_2>1$
STRONG	$M_D>(\rho_2/\rho_1)>0$, $M_2>1$	$\infty > (\rho_2/\rho_1)>M_R$, $M_2<1$

radiation pressure, matter piles up in the critical-density region, rising somewhat above ρ_c as shown in Fig. 2(b).

Table I summarizes the above results, and lists some additional properties of D and R fronts which may be derived from Eq. (4). The "weak" and "strong" solutions correspond to small and large density jumps respectively.

Fronts with M_1 in the range $M_D < M_1 < M_R$ will be fundamentally unsteady near ρ_c. In addition, we speculate that the "strong R" fronts described in Table I are unstable. If one imagines a perturbation which lowers the peak density slightly below critical [see Fig. 2(b)], laser light will penetrate to a new critical surface well inside the original one. But at the new surface the Mach number $M_1 > 1$ is too large to maintain the preexisting D-type profile. Hence the unbalanced outward force pushes the critical region back toward its original position, where the cycle can repeat.

Shock plus D fronts.—For supersonic flows we suggest an alternative configuration which seems considerably more stable than an R front. As shown in Fig. 2(c), a simple shock can stand upstream of the critical surface, bringing the flow speed from supersonic to subsonic. Because of the shock, the flow at ρ_c may be subsonic also, allowing an ordinary D front to exist. Since ρ_c is now permitted to lie quite far down on the D-type density step, a small perturbation to the peak density in Fig. 2(c) will not be likely to affect the critical-surface position, and the overall configuration should be stable. Such collisionless shocks have been seen in particle-in-cell computer simulations,[5] for supersonic relative velocities of the plasma and critical region. (Shock-plus-D ionization fronts are known to exist in astrophysical contexts.[3])

For such a shock configuration to exist, the flow between the shock and critical surface must adjust so that the jump conditions across these two discontinuities are consistent with each other. In spherical geometry this is particularly attractive, since mass conservation ($\rho v r^2 =$ const.) allows flexibility in flow parameters between the shock and the critical surface. As a simple example of a shock-plus-D-front configuration, consider a case where the post-shock Mach number is $M_0 \leq 1$ and the region between the shock and ρ_c is isothermal. Then in spherical flow the Mach number M_1 just on the high-density side of x_c satisfies[6]

$$M_1^2 - \ln M_1^2 = M_0^2 - \ln M_0^2 + 4\ln(r_c/r_s), \tag{6}$$

where r_c and r_s are the critical and shock radii, respectively. For $M_0 < 1$, the Mach number decreases and the density increases between r_s and r_c. In order to have a D front, M_1 must also satisfy Eq. (5b): $M_1 \leq 1-(\Pi/\rho_1 c_1^2)^{1/2}$. For a given $\Pi/\rho_1 c_1^2$, a value of M_1 consistent with (5b) can be found simply by choosing the appropriate radius ratio r_c/r_s in Eq. (6). Analogous arguments show that a shock-plus-R-front structure is not allowed, if the flow is isothermal between r_s and r_c.

Thus supersonic flows can produce two forms of compressional structures: shocks just inside the critical surface, and R fronts right at the critical surface. As with detonations,[4] an R front may be shown to be the limiting case of a shock-plus-D-front configuration, as the distance between the shock and the D front goes to zero. Hence the two types of compressions are related. Both require dissipation, presumably in the form of plasma turbulence, in order to exist. Such dissipation may have important consequences. For example, ion wave turbulence produced in an ion acoustic shock can inhibit heat transport and enhance absorption processes in the critical region.

Conditions for supersonic flow relative to the critical surface.—In current short-pulse implosion experiments, the critical region moves outward relative to the target surface prior to the peak of the laser pulse, and moves inward after the peak.[7] Hence the second half of the laser pulse may favor supersonic flow, since the relative velocity between the plasma outflow and the inward moving critical surface may be large.

Future experiments will emphasize use of longer laser pulses and more adiabatic, ablative implosions. If the laser risetime is long compared to the fluid's transit time from the ablation surface to the critical surface, a global quasisteady state will be set up. Plasma flow characteristics are then determined by the inward heat flux from the critical surface to the ablation surface. Efficient inward heat flow produces rapid ablation, and thus high Mach numbers at ρ_c.

This general conclusion can be illustrated by the following idealized example. Consider a regime where the electron mean free path near ρ_c is longer than the temperature-gradient scale length, so that heat is carried by free-streaming electrons. A model in this situation which limits the heat flow speed to a fraction of $(kT_e/m_e)^{1/2}$ is[8] $\vec{q} = -5\varphi\rho c^3\nabla T/|\nabla T|$, where $\varphi \sim 1$ for normal heat flow and $\varphi \ll 1$ for strongly inhibited heat flow. In a global steady state, the heat equation inside $\vec{x}_c$ is $(\rho v/2)\,(v^2 + 5p/\rho) + q = \text{const.}/r^2$. The constant may be shown to be zero by evaluating it in the unablated region where $q = T = v = 0$. Then the heat equation yields[8] $M(1 + M^2/5) = 2\varphi$, where M is the Mach number relative to the critical surface. Thus, in this example the flow will be supersonic only if the flux limit φ exceeds 0.6 and subsonic otherwise. The first case should lead to a shock plus a D front and the second gives a D front alone.

Discussion.—Recent hydrodynamic computer studies[9] have found behavior typical of flows for which $M_1 > M_D$. Our work allows interpretation of these previously puzzling results. Brueckner,[9] and Virmont, Pellat, and Mora,[9] noted compressional density profiles resembling those predicted in Fig. 2(c), for the case of 1-μm laser light. In particular, Virmont, Pellat, and Mora[9] found that most characteristics of the compressional profile are stationary in a frame moving with the critical surface. In light of our analysis, we interpret this structure as a shock plus a D front.

Mulser and van Kessel[6] noted that density steps like that shown in Fig. 2(a) are seen only for subsonic flows. This agrees with our prediction for D fronts. Since there was no dissipation mechanism in their model, they would not have been able to see an R front or a shock plus D front in the supersonic case. Instead, a density plateau near ρ_c was formed. (We have not analyzed density plateaus in the present work, but Virmont, Pellat, and Mora[9] have shown that such plateaus are unstable to short-wavelength perturbations.)

Several provisos must be added to the results reported here. Our steady-state solutions may suffer from varying degrees of instability. Our jump conditions contain no suprathermal electrons. The conditions for R and D fronts have been phrased in term of Π, the normal component of the radiation stress tensor just outside the critical surface. The fields E and B which determine Π must, of course, be found self-consistently with the density profile outside the critical surface, in a full solution to the problem.

In summary, compressions and shocklike structures in the critical region are predicted to accompany supersonic outflow relative to the critical surface. Turbulence associated with these structures may affect energy transport and laser-light absorption in some regimes. Flows with near-sonic velocities have been shown to be fundamentally unsteady.

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