Pure Electron Plasma, Liquid, and Crystal

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We speculate on the possibility of liquefying and crystallizing a magnetically confined pure electron plasma.

Recent experiments have involved the magnetic confinement of an unneutralized electron gas of sufficient density to be called a plasma—a pure electron plasma.[1] It is interesting to consider what will happen if such a plasma is cooled to a very low temperature, say, to a few degrees kelvin or less. One thing that cannot happen is recombination, since there are negligibly few ions in the confinement region. For electron thermal energy less than the Coulomb interaction energy between neighboring electrons (i.e., $kT < e^2 n^{1/3}$), the electrons will be strongly correlated. For sufficiently low temperature, the electron-electron correlation function will exhibit oscillations characteristic of a liquid, that is, a pure electron liquid. For even lower temperature, one expects the liquid to experience a phase transition and become a pure electron crystal. At low temperature, one also expects quantum effects to be important in the electron dynamics. In this Letter we speculate on the possibility of realizing such conditions in the laboratory.

The confinement geometry for the experiments of Ref. 1 is basically cylindrical. Radial confinement of the plasma is provided by a uniform static magnetic field, $\vec{B}$, in the axial direction. The cylindrical wall is divided into three sections with the plasma residing in the grounded central section. The end sections are biased sufficiently negatively that axial confinement of the plasma is guaranteed.[2] The method for injecting the electrons and capturing them is described in Ref. 1.

The radial confinement must persist long enough for the plasma to be cooled, and it must persist in the face of strong electron-electron interactions. The key to understanding this confinement is the total canonical angular momentum of the electrons, $P_\theta = \sum_j [mv_{\theta_j} r_j - (e/c) A_\theta(r_j) r_j]$. Here, (r, θ, z) are cylindrical coordinates, and the vector potential is given by $A_\theta(r) = Br/2$, neglecting the diamagnetic field. One may show that the diamagnetic field is small compared to B, provided all electron velocities are small compared to c[3]. The confinement is easiest to understand for the case where B is sufficiently large that we may set $P_\theta \simeq (-eB/2c)\sum_j r_j^2$. To the extent that P_θ is conserved, there is a constraint on the allowed radial positions of the electrons (i.e., $\sum_j r^2 = \text{const}$) and only a small fraction of the electrons can reach a wall which is at a radius significantly larger than the initial radius of the plasma. Of course, P_θ is conserved by electrostatic interactions between the electrons, no matter how complicated and nonlinear these interactions may be. Examples of effects which do not conserve P_θ are electron-neutral collisions, finite wall resistance, radiation, and construction errors that are not cylindrically symmetric.

Assuming P_θ is conserved and the electrons remain confined, electron-electron interactions will eventually bring the electrons into thermal equilibrium with each other. Under these conditions, the distribution for a canonical ensemble is given by $\rho = Z^{-1}\exp[-(H - \omega P_\theta)/kT]$, where H is the Hamiltonian for the electrons.[4] The partition function $Z = Z(N, \omega, T)$ is determined by normalization to unity of the phase-space integral of the distribution, and the parameters T and ω are determined by the energy and angular momentum in the system.[5] In terms of velocity and position variables, the distribution takes the form

$$\rho(\vec{x}_1, \vec{v}_1, \dots, \vec{x}_N, \vec{v}_N) = (1/Z)\exp\{(-1/kT)[\textstyle\sum_j mv_j^2/2 + \mathcal{U}(\vec{x}_1, \dots \vec{x}_N) - \omega\sum_j (mv_{\theta_j} r_j - eBr_j^2/2c)]\}, \quad (1)$$

where $\mathcal{U}(\vec{x}_1, \dots, \vec{x}_N)$ is the energy required to assemble the electrons in the configuration $(\vec{x}_1, \dots, \vec{x}_N)$ and we have retained only electrostatic interactions between the electrons. Rearranging terms in the exponential allows one to rewrite the distribution as

$$\rho(\vec{x}_1, \vec{v}_1, \dots, \vec{x}_N, \vec{v}_N) = (1/Z)\exp\{(-1/kT)[\textstyle\sum_j m(\vec{v}_j - \vec{r}_j\omega\hat{\theta}_j)^2/2 + \mathcal{U}(\vec{x}_1, \dots, \vec{x}_N) + m\omega(\Omega - \omega)\sum_j r_j^2/2]\}, \quad (2)$$

where $\Omega = eB/mc$. The velocity dependence of each electron is simply a Maxwellian superposed on a rigid rotation of frequency ω. For sufficiently large B, the last term in the bracket insures that the probability of finding an electron at large r_j is exponentially small. Equivalently, one may say that the mean electron density is exponentially small at large r. Of course, the cylindrical wall is assumed to be outside the radius where the density becomes small. The term $\mathfrak{U}(\vec{x}_1, \ldots, \vec{x}_N)$ makes the density small near the ends, where the end cylinders are biased strongly negative.

A simple way to understand the density profile is to interpret the last term in the brackets of Eq. (2) as the potential energy of the electrons due to a hypothetical cylinder of uniform positive charge. The electrons match their density to that of the positive charge,

$$4\pi ne = -\nabla^2[(-1/e)m\omega(\Omega-\omega)(r^2/2)] = (2m/e)\omega(\Omega-\omega), \quad (3)$$

neutralizing it out to some radius where the supply of electrons is exhausted. At that radius the electron density falls off abruptly, assuming the Debye length is much smaller than the column radius.[6] Near the negatively biased end cylinders the density also falls abruptly; so the overall picture is of a uniform density plasma bounded by some surface of revolution where the density falls abruptly. The condition on the magnitude of the density given by Eq. (3) can be rewritten as the well-known condition[7] for the dynamical equilibrium of a uniform-density rigidly rotating unneutralized plasma column [i.e., $\omega_p^2 = 2\omega(\Omega-\omega)$]. The Brillouin limit[7] on the density [i.e., $\omega_p \leq \Omega/\sqrt{2}$] results from choosing ω to maximize the density for a given field.

The interpretation of the last term in the bracket of Eq. (2) as the potential energy due to a hypothetical cylinder of uniform positive charge also can be used as the basis for a comparison between our system of electrons in a rather thoroughly studied theoretical model.[8-14] In this model, electrons, or more generally, charged particles of a single species interact electrostatically and are immersed in a uniform neutralizing background charge. The model has been studied because of its application to correlation effects in such diverse systems as plasmas, metals, neutron stars, dielectric solutions, and colloidal suspensions. It is clear that replacement of the magnetic field in our confinement device by a cylinder of uniform positive charge would leave the distribution in Eq. (2) unchanged, except for the rigid-body rotation apparent in the velocity variables. This rotation does not enter the partition function; so the thermodynamic properties for the magnetically confined electrons are the same as those for the electrons in a cylinder of uniform positive charge. The spatial correlation functions are also the same for the two systems.

The theoretical studies done for electrons in a uniform positive charge usually assume the system is infinite in extent. For this case $\mathfrak{U}(\vec{x}_1, \ldots, \vec{x}_N)$ takes the simple form

$$\sum_{i<j} e^2/|\vec{x}_1 - \vec{x}_j|.$$

By scaling lengths in terms of a, where $\frac{4}{3}\pi na^3 = 1$, one can see that correlation effects depend only on the parameter $\Gamma = e^2/akT$. For reference, we note that the plasma expansion parameter $g \equiv 1/n\lambda_D^3$ is given by $g = 4\pi\sqrt{3}\Gamma^{3/2}$. The expansion schemes associated with weak correlation require $\Gamma \ll 1$.[9] As Γ approaches unity, these schemes break down, and physics associated with strong correlation becomes important. Monte Carlo calculations show that the pair correlation function begins to exhibit oscillations characteristic of a liquid for $\Gamma \simeq 2$ and that a liquid-solid phase transition occurs for $\Gamma \simeq 155$.[11,12]

All of the preceding analysis has assumed that the electrons obey classical mechanics, but as the temperature is reduced the system can enter a regime where kT is less than $\hbar\Omega$ and the discreteness of the Landau levels becomes important. Also, the electron spins tend to become aligned with the magnetic field. At somewhat lower temperatures, kT is also less than $\hbar\omega_p$, and quantum effects become important for the collective dynamics of the electrons. In the quantum-mechanical case, one cannot introduce velocity variables and remove the magnetic field from the partition function. Nevertheless, arguments similar to those presented by Wigner[8] for the unmagnetized system lead one to speculate that the electrons may arrange themselves as a lattice when $e^2/a \gg \hbar\omega_p$. This inequality is more familiar in the form $r_s \gg 1$, where $r_s = a/a_B$ and $a_B = \hbar^2/me^2$ is the Bohr radius. Just how extreme the inequality must be for a stable lattice to exist is apparently a difficult theoretical question, and estimates in the literature for an unmagnetized system vary over a wide range.[10,12] A detailed analysis for the magnetized system would be quite useful.

It is instructive to collect some of these criteria on a single figure. In Fig. 1, the ordinate is $\log_{10}(T)$, where T is measured in degrees kelvin.

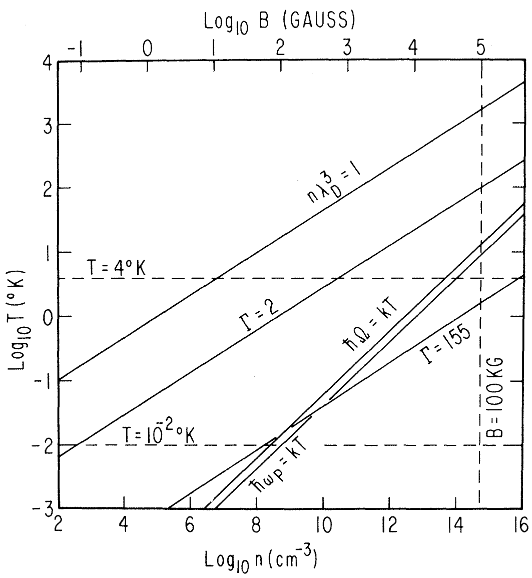

FIG. 1. Parameter space for the pure electron substance.

The abscissa along the bottom of the figure is $\log_{10}(n)$, where n is measured in electrons per cubic centimeter. The abscissa along the top of the figure is $\log_{10}(B)$, where B is measured in gauss. The two horizontal scales are related by the Brillouin condition, $\omega_p = \Omega/\sqrt{2}$. In other words, the value of B which appears above a particular value of n is the lowest field for which that density can be confined. The various stages of increasing correlation are indicated by lines at $n\lambda_D^3 = 1$, $\Gamma = 2$ and $\Gamma = 155$. The onset of quantum behavior is indicated by lines at $\hbar\Omega = kT$ and $\hbar\omega_p = kT$. The smallest value of r_s on the graph is $r_s \simeq 10^3$ corresponding to $n = 10^{16}$ cm^{-3}. The dashed lines are rough indications of technical limits. A field of 100 kG is readily available with superconducting coils. A heat reservoir may be reduced to 4°K with liquid helium and to 10^{-2} °K with a dilution refrigerator.

One way to cool the electrons themselves is to arrange for fluctuations in electron charge density to induce charge onto a conductor, with the induced charge flowing to the conductor through a cooled resistor. The resistor is then a low-temperature heat reservoir in thermal contact with the electrons. To the extent that the conductor is cylindrically symmetrical and perfectly conducting, its coupling to electrostatic fluctuations can remove energy but not angular momentum from the electrons. A similar technique has been used to cool a small number of electrons in a Penning trap to a few degrees kelvin.[2] Spontaneous cyclotron radiation also can be an effective cooling mechanism, for sufficiently large magnetic field. Of course, the radiating electron must be above the lowest Landau level. Also, the cyclotron frequency must be above the cutoff frequency for the waveguide formed by the cylindrical wall,[13] and the the waveguide must be terminated in cooled resistive end sections.

The cooling mechanisms are in competition with various heating mechanisms. Processes which remove angular momentum from the electrons may free some of the electrostatic energy and kinetic energy of rotation to heat the electrons. Considering the good vacuum achievable in cryogenic systems, we estimate that heating due to electron-neutral collisions can be overcome by the cooling mechanisms, provided that the radius of the electron column is sufficiently small. If finite wall resistance proves to be a problem, one can go to superconducting walls. It is difficult to estimate theoretically the heating rate due to small asymmetric-construction errors, but extrapolation from the parameters of existing experiments suggests that these effects may be made small enough to be unimportant. However, the reader should be cautioned that the extrapolation covers several orders of magnitude and is not particularly convincing.

Preliminary calculations indicate it would be feasible to measure the expected electron correlations using laser scattering, provided there are at least 10^{10} electrons in the system.

In conclusion, we note that the part of parameter space in Fig. 1 bounded by the dashed lines at $B = 100$ kG and $T = 10^{-2}$ °K contains regions where the electrons should be in a liquid state and regions where they should be in a crystalline state. Also, there are regions where the dynamics should be classical and regions where it should be quantum mechanical. It is not obviously impossible to study these states of matter experimentally.

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[1]J. S. deGrassie and J. H. Malmberg, Phys. Rev. Lett. 39, 1077 (1977); see also J. H. Malmberg and J. S. deGrassie, Phys. Rev. Lett. 35, 577 (1975).

[2]A related confinement geometry, the Penning trap,

has been used for many years in a series of elegant atomic physics experiments. See, for example, R. S. Van Dyke, Jr., P. B. Schwinberg, and H. G. Dehmelt, Phys. Rev. Lett. 38, 310 (1977).

[3]Of course, the density cannot exceed the Brillouin limit (i.e., $nmc^2 < B^2/8\pi$), to be derived shortly.

[4]L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Addison-Wesley, Reading, Mass., 1974), p. 11.

[5]To be specific, $\langle P_\theta \rangle = kT\partial(\ln Z)/\partial\omega$ and $\langle H \rangle = \omega\langle P_\theta\rangle + kT^2\partial(\ln Z)/\partial T$. Here the distinction between the exact value and the mean value of the angular momentum and energy is blurred because we use a canonical ensemble, rather than a microcanonical ensemble. For a large number of electrons, the two ensembles are equivalent, except that the canonical ensemble allows small fluctuations of H and P_θ about their mean values.

[6]R. C. Davidson, *Theory of Nonneutral Plasmas* (Benjamin, Reading, Mass., 1974), p. 107.

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[11]S. G. Brush, H. L. Sahlin, and E. Teller, J. Chem. Phys. 45, 2102 (1966).

[12]E. L. Pollock and J. P. Hansen, Phys. Rev. A 8, 3110 (1973).

[13]Electromagnetic modes carry off angular momentum as well as energy, and one might worry that a mode at frequency ω would produce an outward radial drift of the plasma and tap the electrostatic energy. To prevent this, the system should be operated as a waveguide beyond cutoff at frequency ω. In other words, one should choose the radius of the cylindrical wall to be such that $\omega < c/R < \Omega$.

Effects of Flow on Density Profiles in Laser-Irradiated Plasmas

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When the plasma outflow velocity relative to the critical surface is supersonic, compressional density profiles can form in the critical region. These compressions involve dissipative processes like those in collisionless shocks; associated plasma instabilities and reflected ions may inhibit energy transport and enhance laser-light absorption.

The manner in which laser-radiation pressure modifies plasma density profiles is important to laser-light absorption, because the expected mix of absorption processes and transport phenomena depends sensitively on density profiles near the critical surface. In this Letter we show analytically that plasmas which enter the critical region supersonically can exhibit compressional density profiles, having a nonmonotonic dependence of density on distance from the target surface. Supersonic compressions in the critical region necessarily involve dissipation properties like those in collisionless shocks, and the plasma instabilities responsible for the dissipation can affect laser-light absorption and energy transport.

In contrast, plasmas which enter the critical surface subsonically exhibit the familar density step[1] there. For some near-sonic flows, no steady profile exists. Our analysis offers new insights into recent computer hydrodynamics calculations in the sonic and supersonic regimes.

Jump conditions across the critical surface.—These may be obtained by integrating steady-state equations of mass and momentum conservation, $\nabla\cdot(\rho\vec{v}) = 0$, $\rho\vec{v}\cdot\nabla\vec{v} = -\nabla p - \nabla\cdot\overleftrightarrow{\Pi}_r$, from a point $\vec{x}_1$ on one side of the critical density to a point $\vec{x}_2$ on the laser side (see Fig. 1). The laser-radiation pressure tensor is[2]

$$\overleftrightarrow{\Pi}_r = \overleftrightarrow{I}\langle E^2 + B^2\rangle/8\pi - \langle\epsilon_r\vec{E}\vec{E} + \vec{B}\vec{B}\rangle/4\pi, \tag{1}$$

where $\epsilon_r \equiv \mathrm{Re}[1 - \omega_p^2/\omega(\omega + i\nu)]$, the laser frequency is ω, and the collision frequency is ν. For light normally incident on a one-dimensional plasma, the normal component of $\overleftrightarrow{\Pi}_r$ is $\langle E^2 + B^2\rangle/8\pi$. If $\vec{x}_1$ and $\vec{x}_2$ are close together, we need not specify the overall geometry. For spherical plasmas we require $|\vec{x}_1 - \vec{x}_2| \ll r$.

We assume that the flow is approximately steady for the short time $|\vec{x}_1 - \vec{x}_2|/c_1$ required to cross the critical region. This is well justified for current experiments which typically have $|\vec{x}_1 - \vec{x}_2| \approx 1-2$ μm, $c_1 \approx 3\times10^7$ cm/sec, so that