TABLE I. Values of the strength parameter corresponding to the minima of the χ^2 curves in Fig. 2. The "free" and "angle-transformed" quantities are also shown for comparison.

α (MeV/c)	ξ	Reb_1	Imb_1	Reb_0	Imb_0
300	0.58	8.71	1.98	−4.06	−0.01
400	0.32	7.83	1.65	−3.76	−0.01
500	0.21	7.43	1.48	−3.62	−0.02
600	0.15	7.19	1.38	−3.53	−0.06
800	0.10	6.87	1.30	−3.43	−0.12
1000	0.05	6.61	1.19	−3.34	−0.14
3000	0.00	5.67	0.96	−3.09	−0.19
	Free	6.85	1.02	−1.01	0.79
	Angle transformed	7.56	0.91	−2.55	0.50

clearly contains sufficient sensitivity to observe even fairly small values of ξ. From the present analysis it seems clear that a value of ξ as large as 1.0 is definitely ruled out. While a maximum value of 0.7–0.8 is permitted, the preferred range is 0.2–0.6.

We have removed the Lorentz-Lorenz effect from the strength parameters and find that the best-known one (Reb_1) agrees with predictions while the others fall within reasonable ranges. These strengths can be of use as intermediate quantities to be directly predicted from more fundamental theories.

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Nucleon Knockout by Kaons

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The reaction (K^+,K^+p) is considered as a probe of the deep hole states in nuclei. Distorted-wave impulse-approximation calculations are presented for the knockout of a $1s_{1/2}$ proton from ^{40}Ca. We conclude that the experiment should be valuable.

The knockout of nucleons from nuclei by protons[1] and electrons[2] has proved to be a very useful tool in demonstrating the shell-model structure of both light nuclei and the surface of medium-weight nuclei. Binding energies, hole-state widths, form factors, and spectroscopic strengths have all been extracted with varying degrees of success using these reactions.

Recent proposals[3] have suggested knockout by K^+ mesons as a probe of nuclear structure. This is particularly relevant at the present time in conjunction with the possibility of upgrading current kaon beams.[4] In this Letter we present absolute distorted-wave calculations for knockout of nucleons by kaons in order to determine the value and feasibility of this experiment. We find that an upgrading of current beams by about a factor of 100 could make this experiment an important probe of deep hole states.

The deep hole states of medium-weight nuclei are particularly interesting[5] as they give us a direct view into the single-particle structure of nuclei. Theories of the lifetimes of these states can also give us insight into the dynamics of the interaction of quasiparticles with nuclear matter. Unfortunately, there are serious difficulties in studying deep hole states reliably using either the electron or proton probe. In knockout by electrons the difficulty arises from the weakness and long range of the electron-nuclear interaction. The resulting cross sections are small and fall off rapidly with increasing momentum transfer or energy loss. Background extractions and statistics (as well as what appears to be the broad natural width of the states) make extraction of clean information very difficult. In knockout by nucleons the difficulty arises from the strength of the proton-nucleon interaction. The distorting potentials for the projectile modify its wave function considerably and in a manner which is uncertain in the nuclear interior, exactly in the region where the deep-hole wave functions are concentrated.

What is needed is a probe which has a stronger interaction with nucleons than the electron does, yet one which interacts weakly enough so that distortion effects can be handled unambiguously. It has been suggested[3] that low-energy K^+ mesons ($0 < p_{1ab} \leq 800$ MeV/c) satisfy these requirements.

This suggestion is attractive on a number of counts. The K^+-N interaction leads to total cross sections[3] in this energy range of about 10 mb, which is about one quarter of the relevant nucleon-nucleon cross sections. The cross section is almost constant in energy, there being no resonances at all in the low-energy region. The lightest mass which can be exchanged is a ρ meson; so the range of the force is about 0.25 fm. This means that only S and P waves in the K^+-N system are relevant at the indicated energies and that the off-shell variation of the amplitude will be small.[6] The weakness of the interaction means that K^+ distortion effects should be reasonably small and should be calculable from first principles,[7] reducing the interior uncertainty of the probe wave function.

In order to see whether this suggestion is reasonable, given the current state of kaon beams, in this Letter we calculate the absolute angular correlation for the reaction $^{40}Ca(K^+, K^+p)^{39}K$ leading to the $1s_{1/2}$ proton hole state. The energy of the incident K^+ is taken to be 300 MeV ($p_{1ab} \approx 620$ MeV/c). Calculations with the distorted-wave impulse approximation (DWIA) for the knockout are performed using the code AAB described by Koshel.[8]

The K-nucleus distorted waves are found by solving a Klein-Gordon equation. Since S and P waves suffice to describe the K^+-N interaction and since the force range is small, we take the optical potential to be of Kisslinger type.[3,9] This gives an optical potential of the form

$$-2EV(r) = b_0\rho(r) - b_1\nabla\cdot\rho(r)\nabla\,, \qquad (1)$$

where b_0 and b_1 are K^+-N amplitudes[3] and ρ is the nuclear density. The density is taken to be a Fermi shape[10] with a radius of 3.51 fm and a diffuseness of 0.563 fm. It is normalized so that the volume integral of ρ is equal to the total number of nucleons. The K-N amplitudes are taken from the phase-shift analysis of Martin.[11] The proton optical potential is taken from the study of van Oers.[12]

The calculations are performed for the angular correlation in the coplanar geometry with the outgoing K^+ and p making equal angles with the beam in the lab. The proton energy is kept constant at 89 MeV.

The K^+-N transition amplitude appearing inside the DWIA integral should in principle be a fully off-shell T matrix.[8,13] Since the force is so short ranged, the variation of the amplitude as a function of its momenta should be slow. This justifies factoring the amplitude out of the integral and using the on-shell T matrix at the final relative energy.[13] This gives an amplitude at a relative K-N energy of

$$T_{rel} = 0.65\,T_{1ab} - Q\,, \qquad (2)$$

where T_{1ab} is the lab kinetic energy of the K^+ and Q is the separation energy of the proton. The factor 0.65 arises from the kinematic transformations.

The near symmetry of the kinematics used implies that the K-N amplitude appearing will be for a scattering angle in the c.m. near 90°. This means that the P-wave amplitude will contribute little to the transition amplitude for our particular kinematic final states. Therefore, only the S-wave amplitude is retained. Again, the Martin amplitudes are used.

The $1s_{1/2}$ proton bound state is taken to have a binding energy[2] of 58.5 MeV. The wave function is taken to be the one found in a Woods-Saxon well whose depth is adjusted to give the right separation energy. The geometric parameters of the well are those of Nakamura *et al.*[2]

Three calculations are performed in order to see the effects of the various distortions. The full distorted-wave calculation includes distortions on all three external legs. A second calculation sets the K^+ optical potentials equal to zero but retains the distortion of the external proton leg. Finally, a plane-wave calculation is done. Here all the distortions are turned off. In all of the calculations, the spectroscopic strength C^2S is set equal to unity. Here C is the usual isospin-conserving Clebsch-Gordan coefficient and S is the spectroscopic factor.[8] (For the closed $1s_{1/2}$ proton shell, C^2S should actually have a value around 2.)

The results are shown in Fig. 1. The peak cross section for the full DWIA calculation occurs near 45° and has a value of 1.4 μb/sr$^2\cdot$MeV. If the kaon distortions are turned off, the peak value is only increased by a factor of 1.9. If the proton distortion is also turned off, the peak value increases further by a factor of 3.2 giving a plane- to distorted-wave ratio of 6.0. This shows that the major distortion is due to the escaping proton and that the overall reduction from distortions is not sufficient to cast doubt on the validity of the assumed reaction mechanism. The corresponding reduction factor[14] for a $(p, 2p)$ reaction at a comparable incident momentum ($E \sim 200$ MeV) at the coplanar symmetric quasifree peak is about 28. The absolute magnitude of this $(p, 2p)$ cross section is about 10 times larger than that for the reaction (K^+, K^+p). Thus the larger cross section is obtained at the cost of a large increase in distortion effects and a corresponding increase in the uncertainty of the extraction of structure information.

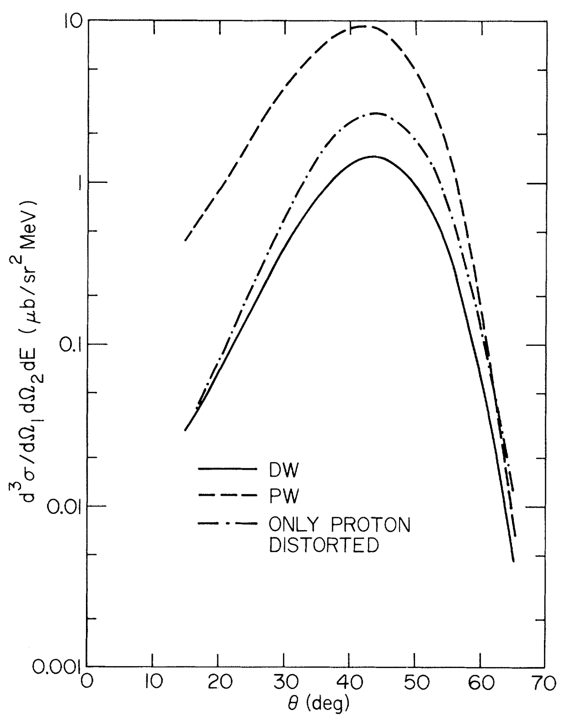

FIG. 1. Absolute cross sections for knockout of a $1s_{1/2}$ protron from ^{40}Ca by a 300-MeV kaon. Solid line, full DWIA (distorted-wave impulse approximation); dashed-dotted line, only proton distorted; dashed line, PWIA (plane-wave impulse approximation, no distortions). C^2S is taken to be 1.

The measured $(e, e'p)$ cross section for knockout of the $1s$ proton in ^{40}Ca by 500-MeV electrons is about[2] 10 nb/sr$^2\cdot$MeV. The distortion of the electron wave is not expected to produce any significant modifications while the proton distortion reduces the cross section by a factor[2] of 4.4. The distortion effects are comparable to that for the (K^+, K^+p) reaction but the cross section is 100 times smaller.

It thus appears that on theoretical grounds, the K^+ would be an excellent probe for the study of deep-lying hole states. We must, however, consider whether the experiment is feasible with existing accelerators. We may obtain an estimate of the counting rate as

$$N = N_0 L/\lambda, \tag{3}$$

where N_0 is the available flux, L the target thickness, and $\lambda = 1/\rho\sigma$ the mean free path; ρ is the density and σ the cross section to the desired slice of phase space. In order to keep the energy loss of the outgoing proton less than 1 MeV, we restrict our target thickness[15] to less than $\frac{1}{5}$ g/cm^2. We use our peak distorted-wave cross section and angular acceptance of 10 msr for each final particle and an energy acceptance of 10 MeV. For a ^{40}Ca target, this gives

$$N = N_0 \times 10^{-11}. \tag{4}$$

Current beams,[4] e.g., at the alternating-gradient synchrotron (AGS) at Brookhaven National Laboratory, deliver on the order of 10^5 kaons/sec. This would yield a count rate of about three counts a month in our angle-energy bin. This is certainly much too small a cross section to do realistic experiments. If, however, the K^+ production of the AGS is upgraded by a factor of 100 as has been proposed,[4] one would obtain on the order of one count an hour. This is close to being reasonable, since much of the desired phase space could be collected simultaneously.

We conclude that the K^+ meson would theoretically provide an excellent probe of the deep hole states via knockout. The effects of distortion should be much less than in $(p, 2p)$ reactions. The cross sections are larger than those observed in $(e, e'p)$ reactions, but the availability of high-flux electron beams tends to give the electron reaction the advantage at the present time. Fluxes of K^+ mesons of the order of 10^7–10^8 per second would make this experiment possible and would add a valuable tool to our current collection of probes of the nucleus.

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Charge-State Dependence of K-Shell–Vacancy Production Cross Sections in Slow Ion-Atom Collisions

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K-shell–vacancy production cross sections in Ne→ CH_4, N_2, O_2, Ne and N→ CH_4, N_2, O_2, Ne collisions are measured for incident charge states 1^+ to 4^+ and 1^+ to 3^+, respectively, in the projectile energy range 50 to 500 keV. The data indicate there is a real deviation between experiment and the calculations of rotational-coupling cross sections. The data suggest that the deviations are not due to a nonstatistical distribution of the $2p$ vacancies among the $2p\pi$ molecular orbital or a velocity-dependent coupling $N(v)$ as has been previously assumed.

In low-velocity ion-atom collisions the production of K-shell vacancies via rotational coupling of the $2p\sigma$-$2p\pi$ molecular orbitals (MO), transiently formed during the collision, has been an area of extensive investigation. From the beginning of these studies one of the best evidences for the molecular picture of these collisions was the dependence of the inner-shell–vacancy production on the charge state of the incident projectile. For example, in Ne→Ne collisions the cross section was predicted and observed to double in going from 1^+ to 2^+ projectiles.[1] This was in agreement with the MO picture since the number of $2p\pi$ vacancies, and hence the K-shell-vacancy production cross section, was predicted to be proportional to the projectile charge state. A series of calculations[2] of K-shell–vacancy production via rotational coupling of the $2p\sigma$-$2p\pi$ MO proved to be in agreement with the measured K-shell–vacancy production cross sections.[3] However, as