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Evidence for the Lorentz-Lorenz Effect in Low-Energy Pion-Nucleus Scattering

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We point out the relatively clean geometrical aspect of the Lorentz-Lorenz effect and use it to extract the strength of the effect from an analysis of the elastic scattering of 50-MeV π^+ on ^{16}O. We find that for a finite-range π-N interaction there is strong evidence for the effect with a value $0.2 \leq \xi \leq 0.6$.

The existence of the Lorentz-Lorenz (L-L) effect in pion-nucleus scattering was first pointed out by the Ericsons.[1] They estimated the influence of this effect upon the optical-model potential by using a zero-range pion-nucleon interaction and an infinitely repulsive nucleon-nucleon correlation with a range limiting to zero. Eisenberg, Hüfner, and Moniz[2] showed that (at zero energy) this result could be derived from the usual expansion of the optical potential in terms of the short-range correlations among nucleons. They also found that the size of the effect was considerably reduced when the relative size of the correlation range is small compared to the pion-nucleon interaction range. This factor is expressed as a parameter ξ which they found to be ~ 0.2. The parameter ξ is normalized to be unity for the original Ericson-Ericson calculation. Garcilazo[3] has calculated in a similar manner (to second order) at finite energy and finds that the L-L effect is an excellent approximation to the second-order optical potential. Recently Baym and Brown[4] have suggested that exchange of ρ mesons might lead to an enhancement of ξ.

Attempts to determine ξ from the data include an analysis by Krell and Ericson[5] of pionic atoms (which tends to indicate a value of about unity) and comparisons of an optical model with pion elastic scattering at 50 MeV by Digiacomo *et al.*[6] (which is inconclusive).

Here we present a different technique for extracting the L-L effect from experimental data than that employed in Ref. 6 and illustrate it by application to the best available data, elastic scattering of 50-MeV π^+ from ^{16}O.[7] We note that the L-L effect changes not only the strength but also the effective geometry of the optical potential. The difficulty in determining ξ comes from our inability to disentangle the L-L effect from other corrections. The largest of these corrections, the angle transform and energy shift,[8,9] affect only the strength and not the geometry. The uncertainty in most of the strengths as a result of using different methods of making these transformations is very great. For this reason we treat the strengths as free parameters and ask

whether the geometrical aspects of the L-L effect can be seen.

There is one additional correction which might strongly affect the geometry: the "true" two-nucleon absorption of the pion. The size of this effect is not known. However, we may hope to utilize the absorptive parts of the strength parameters obtained by this analysis to provide an indication of the relative importance of true absorption.

We make use of χ^2 fitting procedures and absolute values of χ^2 so that it is very important that we understand the errors of the data very well. Since it is almost impossible to give a truly accurate estimate of experimental errors, aside from counting statistics, it is fortunate that there exists a technique for determining the average error utilizing the data itself. We make a least-squares fit to the data using the exact phase-shift expression, i.e., an expansion of the amplitude in Legendre polynomials up to order $l \sim 2KR$, where K is the wave number and R is the nuclear radius. The precise procedure for doing this will appear elsewhere.[10] While many interesting results may be obtained from these fits, the only number of interest to us here is the χ^2 per degree of freedom, which is 1.158 for the $^{16}O(\pi^+,\pi^+)$ data at 49.7 MeV.[7] This means that the errors are correctly assigned and that there are not likely to be hidden biases in the data. It was for this reason that this set of data was chosen for this analysis.

We must choose an optical model containing enough physics and flexibility to be capable of representing well the true physical process of pion elastic scattering, yet simple enough to permit "fitting" of the data in a reasonable length of time. The pion-nucleon t-matrix used to calculate the first-order optical potential[11] was chosen to have the form

$$t(E(k),\vec{q},\vec{q}') \propto [b_0(E)+b_1(E)\vec{q}\cdot\vec{q}'](k^2+\alpha^2)^2/[(\alpha^2+q^2)(\alpha^2+q'^2)]. \quad (1)$$

It is the effective values of b_0 and b_1 which are difficult to compute for pion-nucleus scattering and which are therefore treated as free parameters. The value of the pion-nucleon–interaction range parameter α may also be affected slightly by the presence of the other nucleons. The values obtained by Londergan, McVoy, and Moniz[12] (and independently by Reiner[13]) are ~300 MeV/c for the (3, 3) channel and ~700 MeV/c for the other p waves. The s waves have yet different values. Only a single value of α is used here and, since it must represent an average value, it is expected to lie within the range of 300–700 MeV/c [probably closer to 300 MeV/c since the (3, 3) is the strongest channel].

The "strengths" $b_0(E)$ and $b_1(E)$ may be calculated from free-pion–nucleon phase shifts. The uncertainties in these phase shifts lead to considerable uncertainties in the values of b_0 and b_1. It was also pointed out above that a number of corrections must be applied because of the presence of the nuclear medium. The largest of these are the angle transform and energy variation due to the motion of the nucleons. These primarily alter b_0; b_1 is relatively insensitive to these effects.

The two-nucleon absorption is expected to affect primarily the imaginary parts of the strengths. Since the size of this effect is largely unknown, we must regard $\mathrm{Im}b_0$ and $\mathrm{Im}b_1$ as very uncertain. This leaves $\mathrm{Re}b_1$ as the best-understood strength.

To include the L-L effect the density is replaced by

$$b_1\rho' = b_1\rho/(1+b\rho), \quad b = \xi\hat{b}. \quad (2)$$

For the case of a Woods-Saxon density,

$$\rho = \frac{N(c,a)}{1+\exp[(r-c)/a]},$$

we have

$$b_1\rho' = \frac{b_1 N(c,a)}{1+bN(c,a)} \frac{1}{1+\exp[(r-c')/a]}, \quad (3)$$

$$c' = c + a\{\ln[1+bN(c,a)]\}, \quad (4)$$

so that the transformation alters only the half-density radius and the normalization. This remarkably simple result shows that a nonlinear form in the density can be replaced with a linear form containing an altered value of c. We assume that we are at sufficiently low energy that a scattering-volume approximation is reasonable for $\hat{b}$; hence it (and c') is real.

Modern values of the rms body radius of ^{16}O cluster very closely around 2.58 fm (generally within 10 mfm) so this value is fixed. Then the ratio a/c is adjusted, so that the charge form factor (including a Gaussian proton form factor of radius 0.81 fm) is matched to the form factor obtained from the charge density of Sick and McCarthy.[14] The comparison of these two functions

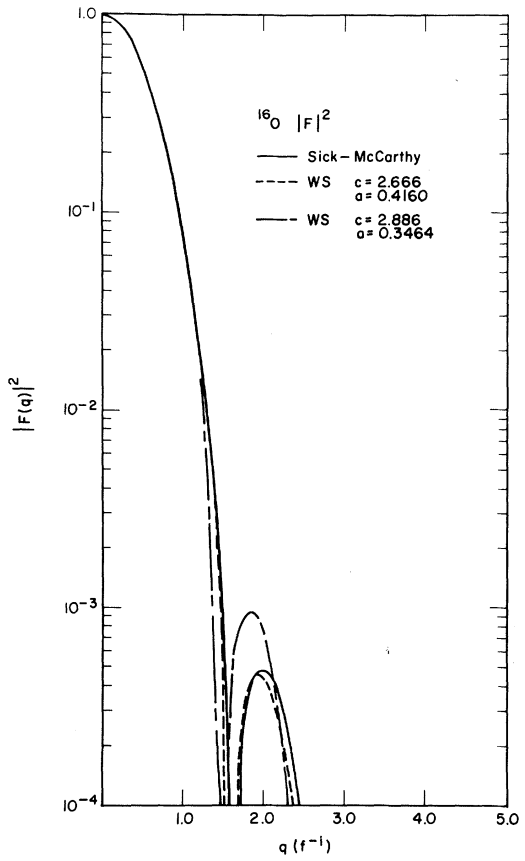

FIG. 1. Comparison of form factors (obtained from the convolution of a proton charge density with two different body densities) with the form factor from Ref. 14. The dashed curve corresponds to the density actually used in the analysis while the unevenly dashed curve corresponds to the geometry required to fit the data with $\alpha \sim 300$ MeV/c and no Lorentz-Lorenz effect.

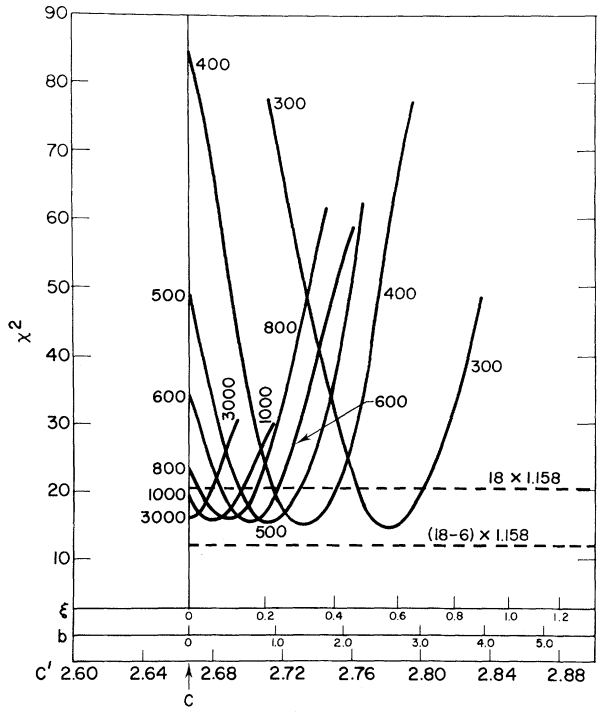

FIG. 2. The loci of minimum χ^2. The proper χ^2 value should lie between the two dashed lines.

is given in Fig. 1.

A series of χ^2 minimization procedures is performed treating the real and imaginary parts of b_0 and b_1 as free parameters, for a selection of values of α and c'. In Fig. 2 we show the curves for fixed α as a function of c'. We may immediately translate from c' to b using Eq. (4) and, employing a scattering-volume estimate for $\hat{b}$ of 4.4 fm^3; a corresponding value of ξ is obtained. All three scales are displayed in the figure.

Since the system is highly nonlinear, the number of degrees of freedom is not clearly defined. However, it should lie between 18 (the number of data points) and 18 – 6 (the total number of parameters being varied). These numbers would be correct if χ^2 per degree of freedom were 1 in the amplitude analysis. They are corrected by multiplying by the actual χ^2 per degree of freedom from that analysis. These two values are shown as horizontal dotted lines and the proper value of minimum χ^2 should lie between them. The continuous ambiguity of acceptable solutions is immediately visible. The solution for small values of α is slightly better than for large values. We note that a fit can be obtained for very large values of α with zero L-L effect, but this is contrary to all present beliefs, since with zero-range pion-nucleon interactions we would have $\xi = 1$ (or greater). For very small values of α we should have no L-L effect and this is also contrary to our results. The only acceptable solution is to have intermediate values of α, around 400 MeV/c, as anticipated.

Table I shows the strength parameters extracted. Judging from the most reliable parameter, Reb_1, we would also conclude that the range of α is 400–500 MeV/c for acceptable fits.

For the results described here, inner Coulomb effects were calculated with a uniform charge density inside the half-density radius. Modifying this density to account for the finite size of the proton and the pion leads to different values of b_0 and Imb_1, but the χ^2 vs c' curves were unchanged.

The present technique of separating the Lorentz-Lorenz effect by geometrical considerations

TABLE I. Values of the strength parameter corresponding to the minima of the χ^2 curves in Fig. 2. The "free" and "angle-transformed" quantities are also shown for comparison.

α (MeV/c)	ξ	Reb_1	Imb_1	Reb_0	Imb_0
300	0.58	8.71	1.98	−4.06	−0.01
400	0.32	7.83	1.65	−3.76	−0.01
500	0.21	7.43	1.48	−3.62	−0.02
600	0.15	7.19	1.38	−3.53	−0.06
800	0.10	6.87	1.30	−3.43	−0.12
1000	0.05	6.61	1.19	−3.34	−0.14
3000	0.00	5.67	0.96	−3.09	−0.19
	Free	6.85	1.02	−1.01	0.79
	Angle transformed	7.56	0.91	−2.55	0.50

clearly contains sufficient sensitivity to observe even fairly small values of ξ. From the present analysis it seems clear that a value of ξ as large as 1.0 is definitely ruled out. While a maximum value of 0.7–0.8 is permitted, the preferred range is 0.2–0.6.

We have removed the Lorentz-Lorenz effect from the strength parameters and find that the best-known one (Reb_1) agrees with predictions while the others fall within reasonable ranges. These strengths can be of use as intermediate quantities to be directly predicted from more fundamental theories.

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Nucleon Knockout by Kaons

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The reaction (K^+,K^+p) is considered as a probe of the deep hole states in nuclei. Distorted-wave impulse-approximation calculations are presented for the knockout of a $1s_{1/2}$ proton from ^{40}Ca. We conclude that the experiment should be valuable.

The knockout of nucleons from nuclei by protons[1] and electrons[2] has proved to be a very useful tool in demonstrating the shell-model structure of both light nuclei and the surface of medium-weight nuclei. Binding energies, hole-state widths, form factors, and spectroscopic strengths have all been extracted with varying degrees of success using these reactions.

Recent proposals[3] have suggested knockout by K^+ mesons as a probe of nuclear structure. This is particularly relevant at the present time in conjunction with the possibility of upgrading current kaon beams.[4] In this Letter we present absolute distorted-wave calculations for knockout