Nuclear-Structure Effects in Pion Double Charge Exchange

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We show that nuclear structure can have a strong effect on the calculated double-charge-exchange cross section. The measured ratio of cross sections at forward angles on an ^{18}O target relative to an ^{16}O target is $R = 2.3 \pm 1$. Our calculated value is $R = 16$ with the most simple nuclear model, but a well known, more sophisticated model leads to $R = 3$.

Recently measurements of zero-degree cross sections have been reported for the (π^+, π^-) double-charge-exchange (DCE) reaction with incident pion energies of about 140 MeV for both an ^{18}O target[1] and an ^{16}O target.[2] The ratio of the observed cross sections to the respective ground states [^{18}Ne$(T=1)$ and ^{16}Ne$(T=2)$] is

$$R = d\sigma(18)/d\sigma(16) = 2.3\,^{+1.2}_{-0.8}.$$

This result is contrary to the expectation that the transition between the $T=1$ isobaric analog states would be very much stronger than the transition from $T=0$ to a $T=2$ state. The purpose of this Letter is to show that a possible explanation of the observed result can be found in the nuclear structure of the states involved.

Calculations for the DCE reaction ^{18}O$(\pi^+, \pi^-)^{18}$Ne have been carried out by using the optical-model,[3] fixed-scatterer approximation,[4] and the Glauber theory.[5] However, no comparative effort has been devoted to study the reaction ^{16}O$(\pi^+, \pi^-)^{16}$Ne. To proceed, we will make the simplest assumption, namely, that the dominant DCE mechanism is due to two successive πN charge-exchange scatterings. Our approach, which is similar to that of Refs. 3–5, does not take into account all of the possible DCE mechanisms which may be important in determining the absolute value of the DCE cross section. However, it should be sufficient for our present purpose, which is to demonstrate the effect of nuclear structure on the ratio R. Within the impulse approximation of multiple-scattering theory,[6] the DCE transition amplitude is

$$T_{fi}(\vec{k}_-, \vec{k}_+, E) = \langle \Phi_f{}^A(Z+2)\varphi^{(-)}(\vec{k}_-) | T^{\mathrm{DCE}}(E) | \Phi_i{}^A(Z)\varphi^{(+)}(\vec{k}_+)\rangle, \tag{1}$$

where

$$T^{\mathrm{DCE}} = \sum_{\lambda} \sum_{i \neq j} t_i(\omega)(\vec{\tau}\cdot\vec{I})\, \frac{|\Phi_\lambda{}^A(Z+1)\rangle\langle\Phi_\lambda{}^A(Z+1)|}{E - E_\lambda - K_\pi - U_\lambda(E)}\, t_j(\omega)(\vec{\tau}\cdot\vec{I}). \tag{2}$$

Here $t(\omega)$ is the πN scattering matrix for the elementary single-charge-exchange process, τ and I are isospin operators for the nucleon and pion, respectively, and the optical potential operator $U_\lambda(E)$ describes the pion propagation within the eigenstate of the intermediate nucleus. The πN collision energy ω is determined from the incident pion energy E within the frozen-nucleon approximation.[7] The pion kinetic energy is K_π and the nuclear excitation energy is E_λ.

A useful approximation to evaluate Eq. (1) is based on the premise that the only intermediate states important in the sum over λ are those having strong similarity to the initial and final nuclear states. In particular, the transition between the ^{18}O and ^{18}Ne ground states has the isobaric analog state (0^+, $T=1$) in ^{18}F as a possible intermediate state. If this is taken to be the only intermediate state, the calculation can be done equivalently by using a coupled-channel optical model[3] which includes an isospin-dependent interaction $(\vec{\tau}\cdot\vec{I})V$. However, for the ^{16}O-to-^{16}Ne transition, no similar simply related intermediate state exists, which is why one would expect a much weaker transition in this case. In view of the experimental result, it seems that other intermediate states must be included in order to use this approach.

The assumption we use to evaluate Eq. (1) is that the propagator of Eq. (2) is approximately independent of the intermediate state, λ. Therefore we replace E_λ by an average nuclear excitation energy, $\bar{E}$, and replace U_λ by an average pion potential $\bar{U}$. Then we are able to eliminate the intermediate nuclear states by closure, so that the nuclear model is required only to furnish wave functions for the initial and final states. We rewrite Eqs. (1) and (2) for the special case of angular momentum zero in both initial and final states. Using the notation of second quantization, we have

$$T_{fi}(\vec{k}_-, \vec{k}_+, E) = \sum_{J j_1 j_2 j_3 j_4} G^0([j_2 j_4]^J [j_1 j_3]^J) F^0(\vec{k}_-, \vec{k}_+, [j_2 j_4]^J [j_1 j_3]^J), \tag{3}$$

where the nuclear structure of initial and final states is contained in matrix element G^0,

$$G^0 \equiv \langle \Phi^{0\,T_f}(Z+2) | \{ [a^{j_2\dagger} \times a^{j_4\dagger}]^J \times [h^{j_1\dagger} \times h^{j_3\dagger}]^J \}^0 | \Phi^{0\,T_i}(Z) \rangle. \tag{4}$$

The square brackets represent coupling to a resultant J of proton creation operators or neutron-hole creation operators. The curly bracket represents coupling to total angular momentum of zero. The two-body matrix element F^0 involves integration over the momentum of the intermediate pion and also the coordinates of the nuclear single-particle wave functions,

$$F^0 \equiv \sum_M (JJM\text{-}M|00) \int d^3k_0 (E - \bar{E} - \bar{U} - E_0 + i\epsilon)^{-1} \langle [\varphi^{j_2}(\vec{r}_a)\varphi^{j_4}(\vec{r}_b)]_M{}^J | t_a(\vec{k}_-, \vec{k}_0)$$
$$\times \exp[i(\vec{k}_- - \vec{k}_0)\cdot\vec{r}_a] t_b(\vec{k}_0, \vec{k}_+) \exp[i(\vec{k}_0 - \vec{k}_+)\cdot\vec{r}_b] | [\varphi^{j_1}(\vec{r}_a)\varphi^{j_3}(\vec{r}_b)]_M{}^J \rangle. \tag{5}$$

The incoming and outgoing pion waves are included in the πN operators t_a and t_b. The main feature of Eq. (3) is the separation into two factors, G, which contains the nuclear structure dependence, and F, which contains the dynamics of the pion-nucleon interaction. In the nuclear models we shall consider only the $1d_{5/2}$, $2s_{1/2}$, and $1p_{1/2}$ single-particle orbitals, henceforth denoted as d, s, and p. Furthermore, after evaluating Eq. (4) with our model wave functions for $A=16$ and $A=18$, we find that those matrix elements G wherein the protons (and neutron holes) are coupled to $J=0$ account for more than 90% of the available strength. Therefore we include only $J=0$ in Eqs. (3) and (5).

The calculation of the dynamical factor, F, depends not only on the pion wave functions but also on the choice of average value for $\bar{E}$ and $\bar{U}$. This problem is under investigation, but in the present paper we concentrate on estimating the ratio R of cross sections between the $A=18$ and $A=16$ systems at forward angles. Therefore we believe that distortion effects on the pion waves will not be very different for the two isotopes, and we have evaluated F within the plane-wave approximation, setting $\bar{U}$ equal to zero and including only the p-wave component of the πN interaction. The input t matrices are calculated[7] from a simple phenomenological πN model using relativistic particle quantum mechanics. The radial integrals are evaluated numerically using single-particle oscillator functions. The resulting numerical values of F for $J=0$ are given as an array in Table I, where the row index is the orbital of the paired protons and the column index is that of the paired neutron holes. The main feature is that the nondiagonal elements, which involve a change of orbital, are considerably weaker than the diagonal terms.

In order to demonstrate the effect of nuclear structure, we consider two models within our (dsp) space. In the simplest model we assume a closed p shell for ^{16}O, a pair of d nucleons to represent ^{18}O and ^{18}Ne, and for ^{16}Ne a pair of d protons coupled to a pair of p neutron holes. For this model, the resulting array for G, with indices corresponding to those of Table I, has unity in the (dd) position for $A=18$, the rest being zeros. Similarly for $A=16$ the only nonzero value is unity in the (dp) position. The ratio, R, of cross sections is then simply the ratio of absolute squares of the corresponding elements of F in Table I. This value is $R=16$, so that the simple model gives the expected dominance of the transition between analog states, in contradiction to the experimental observation.

As a second nuclear model we take that of Zuker, Buck, and McGrory (ZBM),[8] which allows multiparticle, multihole excitations within the (dsp) space. In particular we use the interaction[9] of Zuker with which he was able to get a good representation of the properties of levels for nuclei from $A=15$ to $A=18$. The resulting wave functions differ considerably from those of the simple model. For example, the ^{16}O ground state has 66% intensity to be a closed p shell and

TABLE I. Numerical values of $F^0(\vec{k}_-, \vec{k}_+ [j_2{}^2]^0 [j_1{}^2]^0)$ for $E_\pi = 140$ MeV $\theta = 0°$ in a $(1d_{5/2} 2s_{1/2} 1p_{1/2})$ space. Rows are labeled by the proton orbital, columns by the neutron-hole orbital.

	d	s	p
d	$-0.245+0.352i$	$-0.064+0.058i$	$0.054-0.091i$
s	$-0.064+0.058i$	$-0.165+0.315i$	$0.029-0.030i$
p	$0.054-0.091i$	$0.029-0.030i$	$-0.188+0.374i$

TABLE II. Numerical value of $G^0([j_2^2]^0[j_1^2]^0)$ for $A=18$ and $A=16$ resulting from the nuclear model of Ref. 8. Labels are are in Table I.

	$A=18$		
	d	s	p
d	−0.773	−0.368	0.386
s	−0.368	−0.159	0.220
p	0.386	0.220	−0.242
	$A=16$		
	d	s	p
d	−0.281	−0.095	0.689
s	−0.211	−0.060	0.536
p	0.135	0.067	−0.229

TABLE III. Numerical value of $G^0([j_2^2]^0[j_1^2]^0)$ for the excited 0^+ state of ^{18}Ne.

	d	s	p
d	−0.371	0.565	0.004
s	−0.175	0.238	−0.018
p	−0.091	−0.365	0.187

29% intensity for having two-particle, two-hole excitations. We have calculated the nuclear transition elements G of Eq. (4) with this model for all values of J using the Argonne National Laboratory shell-model programs.[10] We find that both for $A=18$ and $A=16$, matrix elements with $J=0$ provide more than 90% of the available strength. The values for $J=0$ are given in Table II. When combined with the F values of Table I according to Eq. (3) they lead to a calculated value of $R=3.0$ for the ratio of cross sections.

Since this value of R is much closer to observation than that of the simple model, it is interesting to see the source of this difference. By comparing Tables I and II one sees that for $A=18$ all the corresponding terms have the same relative sign, so that all the contributions add constructively. The relative signs of the terms in G are exactly those one would obtain with a paring model for the neutrons and for the protons. In fact if one constructs the ground states for $A=18$ from a model of degenerate single-particle levels plus a pairing interaction within the (dsp) space, the elements corresponding to those of the G array in Table II would be

$$0.1(-1)^{l_2+l_1+1}[(2j_2+1)(2j_1+1)]^{1/2}.$$

This array is very similar to the result of the ZBM interaction and demonstrates the importance of the pairing correlation.

For $A=18$, the diagonal terms contribute 56% of the cross section so that the off-diagonal terms are also important. Furthermore, although the magnitudes of the elements of G are different for $A=16$, the relative signs are the same as for $A=18$, so that the influence of pairing is also important for this case. A more dramatic effect can be seen by looking at Table III where the values of G are given for transition to the excited 0^+ state of ^{18}Ne. When this array is combined with the F values of Table I there is clearly destructive interference, and the calculated cross section is only 1% of that for going to the ground state.

We conclude that the observed ratio of cross sections at 0° can be understood within the closure approximation to the double-scattering expression. The pairing correlations in the nuclear wave functions play an important role in our calculation of this ratio. If one wants to calculate the absolute magnitude for the cross sections, it is necessary to treat the distortion effects and the propagator properly in the calculation of F in Eq. (5). Nuclear-structure effects can play a significant role in the $A=18$ case alone. It may be possible to test nuclear models since our (ZBM) calculation for $A=18$ gives a cross section twice as big as that of the simple model, and the phenomenological model of Lawson, Serduke, and Fortune[11] gives a value lying between these.

It appears that nuclear structure plays an important role in the DCE process even in lowest order, and is likely to be as important as higher-order reaction processes. Therefore it would be beneficial for the development of the understanding of pion-nucleus reactions to devote some experimental and theoretical effort to nuclei for which there exists detailed and well-tested models.

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Diffraction and Mirror Reflection of Ultracold Neutrons

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Ultracold-neutron diffraction by a ruled grating was observed. The measured peak positions, linewidths, and intensities agree with expectation. Mirror reflection experiments yielded, for glass, a reflection curve sensitive to surface contamination, and for gold coating, a typical interference pattern.

We report on experimental investigations of the diffraction of ultracold neutrons (UCN) from an optically ruled grating and on their reflection from neutron mirrors. Diffraction of thermal neutrons by a ruled grating has been observed first by Kruz and Rauch.[1] The motivation behind the present studies was to gain insight into anomalies observed in the containment of UCN's in "neutron bottles" which have thus far persistently yielded shorter containment times than expected.[2,3] We are especially interested in the idea of an "intrinsic coherence length of the neutron wave train," about which there has been much speculation.[4]

In the apparatus used (which may be called "gravity diffractometer") we utilize the fact that the motion of UCN's is strongly affected by gravity since the neutron gravitational potential of $\approx 10^{-7}$ eV per meter of height is of the same order as the kinetic energies considered. Furthermore, we take advantage of the special features of the flight parabola for beam focusing in two spatial dimensions.

Figure 1 shows the principal arrangement. A continuous beam of neutrons slowed down by the "neutron turbine"[5] at the Forschungs Reactor Munich (with a thermal flux of 10^{13} cm^{-2} s^{-1}) is channeled by neutron guides to a horizontal entrance slit. Two beam stops, arranged symmetrically to the slit, select a horizontal UCN beam with small vertical divergence. The neutrons with initial velocity of approximately 3 m/s fall along parabolic trajectories and hit a first vertical mirror consisting of Ni-coated glass. After further reflections from a horizontal and a second vertical mirror with adjustable vertical and horizontal positions, the neutrons pass the exit slit at the maximum height of their ascending flight parabola. The highest point is chosen because it is the focusing point where the spatial beam width is a minimum, being equal to the entrance slit width of 2 cm, provided that the initial divergence and velocity spread are sufficiently small. After passing the exit slit, the neutrons are allowed to acquire some energy by falling in a slightly convergent, vertical guide tube, in order to be able to penetrate the Al window (0.1 mm) of the BF_3 detector (with depleted ^{10}B content), which would totally reflect neutrons with $v < 3.2$ m/s. Along the full flight path the beam is confined to a lateral width of 10 cm by

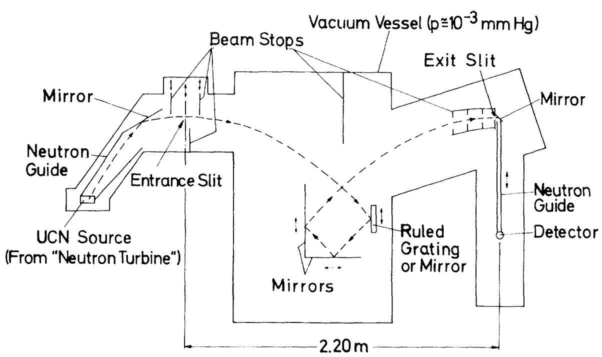

FIG. 1. Scheme of the "gravity diffractometer." A change of the neutron vertical momentum due to diffraction may be sensitively analyzed by measuring the change of the maximum height of the ascending flight parabola.