scattered light was observed at these densities and the enhancement (Fig. 3) lasted for longer than the CO_2-laser pulse (Fig. 1). The threshold for the heat-flux instability is $\lambda_e/L = 0.6$ for $T_e = 5T_i$[3], which is close to our observed maximum, $\lambda_e/L \sim 0.5$. However the distribution functions[1] on which this theory is based are unphysical since they become negative on one side in the region of velocity space where the net heat flux occurs ($1.5v_{\rm th} < v < 3v_{\rm th}$) when $\lambda_e/L > 0.02$.

In conclusion we have observed a (2–5)% flux limit to heat flow when $T_e \sim 5T_i$, which can be explained by the observed low-frequency turbulence.

This work was supported by the Science Research Council and the Northern Ireland Department of Education.

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Kinetic Description of Ponderomotive Effects in a Plasma

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The kinetic treatment of the ponderomotive force concept is found from nonresonant quasilinear theory for waves with spatially dependent amplitude. In general, the ponderomotive effect appears as a velocity-space diffusion term, not just as a force. For an unmagnetized plasma, the qualilinear equations are solved directly, and the correct density modification exhibited explicitly. Examples are considered for both a homogeneous and an inhomogeneous magnetic field, and aspects of rf end plugging are discussed.

The effects of electromagnetic waves on a plasma are relevant to problems in both laser fusion and magnetic confinement. In the former case, self-focusing density modifications, parametric instabilities, and magnetic field generation are of interest, and, in the latter, wave heating, end plugging of open systems, and impurity control. For many of these problems, the collisionless regime is appropriate, and both single-particle and fluid treatments have shown that the ponderomotive force plays a key role in nonresonant nonlinear phenomena. In this Letter, the inadequacy of using only a ponderomotive force in a kinetic treatment is demonstrated, and the porper collisionless kinetic theory of nonresonant wave effects in a weakly inhomogeneous plasma is presented.

When fluctuating wave amplitudes are small and autocorrelation times are short compared with diffusion times, the lowest-order wave modifications of the particle distribution are given by quasilinear theory. In this approximation, it is known that nonresonant particles acquire an apparent temperature due to the nonlinear interaction with the waves.[1] This "fake diffusion" has been used in calculating saturation amplitudes of unstable waves,[1,2] and is reconsidered here in order to understand ponderomotive effects in a weakly inhomogeneous plasma.

In one dimension, consider an electrostatic wave, $E=\sum_\omega E_\omega(x)e^{i\omega t}$, and let $\bar f(x,v,t)$ denote the slowly varying part of the distribution function. If the fluctuating part of the distribution is obtained to first order in E and only low-velocity nonresonant particles are considered, the lowest-order kinetic equation for $\bar f$ is

$$(\partial_t + v\partial_x)\bar f + (1/m)(F_{0x} - \partial_x\psi)\partial_v \bar f - (v/m)\partial_x\psi\partial_v^2\bar f - (2\psi/m)\partial_v[v\partial_x + (F_{0x}/m)\partial_v]\partial_v\bar f = 0, \tag{1}$$

where $\psi(x)=(q^2/m)\sum_\omega|E_\omega|^2/2\omega^2$ is the usual ponderomotive potential. A self-consistent electric field associated with the resulting charge separation has been included by keeping the force $F_{0x}=-\partial_x\varphi$, where φ is determined by Poisson's equation.[2] Various terms in Eq. (1) have been consistently neglected in previous treatments, and it is useful to identify the implications of these approximations in some detail. With the boundary condition that $\bar f$ be Maxwellian where $\psi=0$, the relevant solutions are

$$\bar f_P = (n_0/\sqrt{\pi}\,\bar v)\exp[-(\tfrac12 mv^2+\psi+q\varphi)/(\tfrac12 m\bar v^2)]; \tag{2a}$$

$$\bar f_Q = [n_0 m\bar v/2\sqrt{\pi}\,(\tfrac12 m\bar v^2+2\psi)]\exp[-(\tfrac12 mv^2+q\varphi)/(\tfrac12 m\bar v^2+2\psi)]; \tag{2b}$$

$$\bar f = \frac{n_0}{\pi\bar v\delta^{1/2}}\int_0^\infty \frac{dt\exp[-t\delta^{-1} - v^2/\bar v^2(1+t)]}{t^{1/2}(1+t)}. \tag{2c}$$

From Eq. (1) the ponderomotive force term alone, $(1/m)\partial_x\psi\partial_v\bar f$, has been used as a basis for a kinetic treatment of parametric instabilities of an electromagnetic pump wave.[3] In effect, this is equivalent to keeping only the time-averaged momentum change in a Fokker-Planck derivation of Eq. (1).[4] It gives $\bar f_P$, in (2a), which appears to have the usual exponential density dependence on ψ, $n_P(x)=n_0\exp[-(\psi+\varphi)/(\frac12 m\bar v^2)]$. If the diffusion term, $(v/m)\partial_x\psi\,\partial_v^2\bar f$, which accounts for changes in mean-square momentum,[4] is also considered, the quasilinear solution, f_Q, is obtained, (2b). This solution exhibits "fake heating," $\frac12 m\bar v^2\to\frac12 m\bar v^2+2\psi$, the usual quasilinear effect on nonresonant particles; and a new density dependence, $n_Q(x)=n_0(1+4\psi/m\bar v^2)^{-1/2}\exp(-\varphi/m\bar v^2)$. Use has implicitly been made of the fact that the ambipolar potential, φ, is of the same order as ψ, since (2b) is not a solution to order $\psi\cdot\varphi$. It is now apparent that, for electrostatic waves, an exponential dependence of the density on the ponderomotive pseudopotential appears only in the self-consistent solution to the quasilinear problem. Not only does the solution of the simple ponderomotive force equation, $\bar f_P$, fail to exhibit the expected "fake heating," but it does not give the correct velocity moments of $\bar f$, except for the density to order $\delta=8\psi/m\bar v^2$. The last term in Eq. (1) is higher order in δ, and usually neglected. An exact solution, (2c), has been found for (1) when $\varphi=0$, which is useful in illustrating some limiting features of all these approximations. For $\delta\ll1$, Eq. (2c) yields, in the limit $\delta^{1/2}|v/\bar v|\ll1$,

$$f\simeq\frac{n_0}{\pi^{1/2}\bar v}\left[1+\delta\left(1-\frac{v^2}{\bar v^2}\right)\right]^{-1/2}\exp\left(-\frac{v^2}{\bar v^2}\right), \tag{3a}$$

and, for $\delta^{1/2}|v/\bar v|\gg1$,

$$\bar f\simeq\frac{n_0}{\pi^{1/2}\delta^{1/2}|v|}\exp\left(\frac1\delta-\frac{2}{\delta^{1/2}}\left|\frac{v}{\bar v}\right|\right). \tag{3b}$$

Comparing (3a) with (2b) serves as a reminder that the basic perturbation scheme depends on $\partial_v\bar f$, which, for a Maxwellian, leads to a perturbation parameter, $\delta v^2/\bar v^2$. The limiting form (3b) shows that even weak fields and nonresonant interactions can produce energetic non-Maxwellian tails. At this point, some comments on the entire procedure are appropriate. The last term in Eq. (1) illustrates that an underlying assumption of the quasilinear formulation is that $|\partial_x\psi/\psi|\gg|\partial_x\bar f/\bar f|$. Other higher-order effects which might compete, such as nonlinear Landau damping and resonant wave scattering, have been neglected. However, depending on the wave in question, these typically have a small numerical coefficient. The explicit nonresonant particle assumption used in deriving (1) requires that $|(E_\omega\bar f)^{-1}\partial_x(E_\omega\bar f)|\ll\omega/|v|$, which restricts the range of velocity space under consideration, expecially in the case of (3b). As long as $\delta\ll1$, this difficulty can be avoided by rederiving (1) using particle orbits modified by the self-consistent ambipolar potential, φ. This approach has previously been formulated for localized-electric-field calculations[5]; however, the particle orbits were also modified by the ponderomotive force, which would be inconsistent with the present ordering scheme. In fact, with the assumption of short correlation times, φ can be neglected for resonant particles also.

For the case of transverse electromagnetic waves, $\vec E=\sum_\omega\hat y E_\omega(x)e^{i\omega t}$, suppressing the self-consistent field, φ, the quasilinear equation for low-velocity nonresonant particles is found to be

$$(\partial_t + v_x\partial_x)\bar{f} - (1/m)\partial_x\psi\partial_{v_x}\bar{f} = (q/m)^2\sum_\omega(|E_\omega|/\omega^2)v_x\partial_x|E_\omega|\partial_{v_y}{}^2\bar{f}. \quad (4)$$

Although superficially similar to the electrostatic example, the ponderomotive force term here is new. It is not one part of a quasilinear diffusion operator, but arises because the Lorentz force, $\vec{v}\times\vec{B}_\omega$, has components in the direction of $\vec{E}_\omega$ and $\nabla(\vec{v}\cdot\vec{E}_\omega)$. The former simply modifies the electrostatic result, and the latter generates the ponderomotive force term in (4). Once again, the stationary solution to (4) is obtained with a Maxwellian boundary condition,

$$\bar{f} = (n_0/\pi\bar{v}_x\bar{v}_z)\exp(-v_x^2/\bar{v}_x^2 - v_z^2/\bar{v}_z^2)\int_{-\infty}^{\infty}dt\,(1/2\pi)\exp[itv_y - \tfrac{1}{4}t^2\bar{v}_y^2 - (\tfrac{1}{2} + 1/t^2\bar{v}_x^2)\ln(1 + 2t^2\psi/m)]. \quad (5)$$

For $\delta \ll 1$, this becomes

$$\bar{f} = \frac{n_0}{\pi^{3/2}\bar{v}_x\bar{v}_z}\left[\frac{\frac{1}{2}m}{\frac{1}{2}m\bar{v}_y^2 + 2\psi}\right]^{1/2}\exp\left(-\frac{\frac{1}{2}mv_x^2 + \psi}{\frac{1}{2}m\bar{v}_x^2} - \frac{\frac{1}{2}mv_y^2}{\frac{1}{2}m\bar{v}_y^2 + 2\psi} - \frac{v_z^2}{\bar{v}_z^2}\right), \quad (6)$$

which is similar to the electrostatic result, (2b), except that it has the usual exponential modification of the density in the direction of the ponderomotive force. As before, fake heating occurs in the direction of the oscillating electric field, and the (suppressed) dependence on φ should not be forgotten.

The quasilinear equation for an arbitrary electromagnetic wave in a homogeneous external magnetic field, $\vec{B}_0$, has been obtained, and an example relevant to rf plugging is presented here. Assume an electromagnetic field,

$$\vec{E} = \sum_{k,\omega}\vec{E}_{k,\omega}(s)e^{i(\vec{k}\cdot\vec{x} - \omega t)},$$

with $\vec{k}$, $\vec{E}_\omega$, and $\vec{B}_0$ mutually perpendicular, and a weak spatial variation in the parallel direction $s = \vec{e}_\parallel\cdot\vec{x}$. The nonresonant, quasilinear equation is found to be

$$(\partial_t + v_\parallel\partial_s)\bar{f} - \frac{1}{m}\partial_s\psi\partial_{v_\parallel}\bar{f} = \left(\frac{q}{m}\right)^2\frac{1}{v_\perp}\partial_{v_\perp}\left[v_\perp\sum_{k,\omega>0}|E_{k,\omega}|v_\parallel\partial_s\left(|E_{k,\omega}|\sum_n\frac{(J_n')^2}{(\omega - n\Omega)^2}\partial_{v_\perp}\bar{f}\right)\right], \quad (7)$$

where the ponderomotive potential is

$$\psi = (q^2/m)\sum_{k,\omega}(|E_{k,\omega}|^2/2v_\perp)\partial_{v_\perp}[v_\perp^2\sum_n(J_n')^2/\omega(\omega - n\Omega)], \quad (8)$$

the Bessel-function argument is $k_\perp v_\perp/\Omega$, and Ω denotes the cyclotron frequency. In view of the previous examples, the qualitative features of (7) are evident, i.e., an effective force and nonresonant thermal broadening. In the cold-particle limit, $k_\perp v_\perp/\Omega \ll 1$, the usual single-particle potential is found[6]:

$$\psi = (q^2/m)\sum_{k,\omega}|E_{k,\omega}|^2/2(\omega^2 - \Omega^2). \quad (9)$$

In the opposite limit, $k_\perp v_\perp/\Omega \gg 1$, the potential is found to be reduced by approximately $(k_\perp v_\perp/\Omega)^{-1}$, suggesting that nonresonant rf plugging is likely to to be most effective in the long-wavelength, cold-particle limit.

For a plasma confined in an inhomogeneous magnetic field, the kinetic equation can be obtained by the usual techniques, which include averaging over the $\vec{v}_\perp$ phase angle.[7] However, in general, the analysis is extremely complicated, and a single illustrative example is presented here. Consider left-circularly polarized waves, with weak variation in the parallel direction, and $(k_\perp v_\perp/\Omega) \ll 1$. Assuming all of the usual drifts are small, to first order in all spatial gradients the nonresonant quasilinear equation is

$$(\partial_t + v_\parallel\partial_s)\bar{f} + v_\parallel F_\parallel\partial_\epsilon\bar{f} = \left(\frac{q}{m}\right)^2[D + v_\perp^{-1}(\mu)]\sum_\omega|E_\omega|v_\parallel\partial_s\left[\frac{E_\parallel}{(\omega - \Omega)^2}\right]D\bar{f},$$

where

$$\bar{f} = \bar{f}(\epsilon, \mu, s, t), \quad \epsilon = \tfrac{1}{2}mv^2, \quad \mu = mv_\perp^2/2B_0,$$

$$v_\parallel = \pm[2(\epsilon - \mu B_0)/m]^{1/2},$$

and

$$F_\parallel = -(q^2/m)\sum_{\omega>0}[B_0/2\omega(\omega - \Omega)]\partial_s(B_0^{-1}|E_\omega|^2). \quad (10)$$

The operator D arises from $\partial_{v_\perp}|_{v_\parallel}$ on a function of ϵ and μ and is given by

$$D = (2B_0m\mu)^{1/2}(\partial_\epsilon + B_0^{-1}\partial_\mu).$$

The effect of the waves on magnetic confinement is seen to be given by the parallel ponderomotive force, which is no longer the gradient of a scalar and, importantly now, depends on the sign of $B_0\partial_s(B_0^{-1}|E_\omega|^2) = \nabla\cdot(\hat{e}_\parallel|E_\omega|^2)$. The apparent perpendicular thermal broadening can also be estimated as $\Delta\mu \simeq (q^2/mB_0)\sum_\omega|E_\omega|^2/2(\omega-\Omega)^2$. Both of these results are needed to self-consistently evaluate wave plugging in open systems.

In conclusion, the spatial effects of low-amplitude waves on a plasma have been described by a consistent kinetic perturbation theory. A true ponderomotive-force term arises in second-order kinetic theory, but only for electromagnetic waves, for which the time average of products of terms arising from fluctuating electric fields with terms arising from the associated fluctuating magnetic fields produces an effective force in the direction of $\nabla|E_\omega|^2$. The purely electrostatic part of the fluctuations generates a time-average effect best described by a velocity-space diffusion operator, which for the low-velocity nonresonant part of the distribution produces a local, apparent temperature increase in the direction of $\vec{E}$. Some care is required in comparing velocity moments of $\bar{f}$ with macroscopic fluid quantities. Both velocity moments of $\bar{f}$ and the time average of products of fluctuating quantities must be considered; for example, in one dimension, the time-averaged fluid velocity is given by

$$\bar{u} = (\bar{n})^{-1}\int dv\, v\bar{f}[1-(2q^2/m^2)\sum_\omega \omega^{-4}|\partial_x E_\omega|^2].$$

Finally, the treatment here has been for the nonresonant part of the distribution function with particle velocities less than the phase velocity of the waves; similar methods may also be applied in the opposite limit.

This work was supported by the U. S. Energy Research and Development Administration under Contracts No. AT-(04-3)-1018 and No. EY-76-C-02-2277*000.

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Simulations of Nonlinearly Stabilized Beam-Plasma Interactions

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Computer simulations of the kinetic warm-beam instability with finite-mass ions are reported. It is shown that strong turbulence, including Langmuir collapse, stabilized the beam-plasma instability before quasilinear plateau formation. This process decreases significantly the beam-plasma coupling and increases the propagation distance in accordance with laboratory and space observations.

The interaction of a warm electron beam with a plasma in the kinetic regime has been the classic example of the application of quasilinear theory.[1] According to this theory the beam plasma instability is stabilized in a time $t \approx (n_p/n_b)\omega_e^{-1}$ (n_p and n_b are the plasma and beam density) with the beam forming a quasilinear plateau, while releasing one-third of its energy to plasma waves and one-third to sloshing energy of the ambient plasma. However, laboratory beam-plasma interaction experiments indicated much longer energy-coupling time scales than that predicted above.[2] This coupled with the observation of beams propagating over extremely large distances in space[3] without any significant plateau formation has led to an extensive search for a nonlinear mechanism that can stabilize the beam-plasma instability on a time scale faster than that required for plateau formation. It was not till recently[4] that a strong-turbulence theory with