Beam-Density Effect on the Stopping of Fast Charged Particles in Matter

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Greatly enhanced stopping powers result from the interaction of sufficiently dense charged particle beams with matter. Stopping power is proportional to a well-defined number of beam particles that interact coherently. The effect is not due to turbulence, but relies on a phase mixing of the polarization wakes produced by the beam particles. The beam-density effect can drive a self-dissipative instability.

The stopping of fast charged particles by ambient matter has been, and continues to be, a problem of central concern with applications in widely varying areas. Presently, there is much interest in anomalous stopping power for intense electron or ion beams, the application being thermonuclear fusion.[1-5] The acceleration of ion bunches by coherent or collective effects arising from the interaction of electron beams and matter is a subject of extensive investigation.[6,7] The pumping of short-wavelength lasers using intense electron beams may be possible provided energy coupling from the beam to an ambient plasma can be made sufficiently strong.[8]

It has long been recognized that the passage of fast charged particles through matter exhibits a medium "density effect"[9-11] in which collective polarization of the medium produces a partial cancellation of the fields of the incident particle, thereby reducing the energy loss rate by the particle. This effect is most pronounced at highly relativistic speeds for which the fields of the particle extend preferentially in the lateral direction and thereby influence matter far from the path of the particle. In this Letter, a new density effect, due to collective behavior by a dense beam of charged particles, is shown to give rise to strikingly anomalous behavior with respect to the coupling of beam energy to the ambient material. In particular, for beam densities sufficiently high, an enhancement of the stopping power occurs. The stopping power can be orders of magnitude higher than the value for independent particles. This effect is not due to turbulence, but instead relies upon a phase mixing of the polarization wakes produced in the ambient media by the beam particles. The beam-density effect is shown to give rise to a dissipative instability.

Bohr first suggested[12] that the stopping of a fast charged particle in matter can be related directly to the polarization wake set up by the particle. Utilization of this concept[13-16] has provided a description of stopping power (energy loss per unit path length). In particular, the stopping power for a single charged particle of charge Ze and speed v may be expressed[17] as

$$S_0 = -(ZE/a)^2 K_0(\rho/a), \tag{1}$$

where $a \equiv v/\omega_p$ and K_0 is the zeroth-order modified Bessel function of the second kind. The ambient polarization has been characterized by a simple plasma dielectric response of $1-\omega_p^2/\omega^2$ where $\omega_p^2 = 4\pi n e^2/m$ is the relevant electron plasma frequency. For case of interest here, $v > e^2/\hbar$. Then $\rho = \hbar/\gamma m v$ for ions and $\rho = \hbar(2/\gamma - 1)^{1/2}/mc$ for electrons.[18] Planck's constant is $2\pi\hbar$, m is the electron mass, c is the speed of light, and $\gamma = (1-\beta^2)^{-1/2}$ with $\beta = v/c$. Thus $\rho/a \ll 1$ and[19] $K_0(\rho/a) \simeq \ln(a/\rho)$ yielding the familiar Coulomb logarithm.

More recently, both experimental and theoretical examination of the penetration of fast ion clusters through solids[20-30] has shown interference effects attributable to a superposition of wake fields. This behavior was shown to give rise to a range of new phenomena: a change in the stopping power of a cluster from that to be expected from considerations based on independent particle behavior, an alignment of diatomic clusters along the direction of motion with a concomitant reduction in relative angular scattering experienced by the components of the cluster, and evidence for bound electron states in the wake of fast ions in solids. The particular effect of present interest is the alteration of stopping power due to wake interference.

In particular, it has been shown[21,22] that the influence of a neighboring member of a charged particle cluster influences the stopping of the particle of interest through an interference term $g(r/a)$ where r is the separation of the two particles and $a = v/\omega_p$ (as given above). The interference term, which represents an average over all cluster orientations, is given to sufficient accuracy for present purposes by[21,22]

$$g(r/a) \simeq \tfrac{1}{2} \quad \text{for } r \leq a,$$
$$g(r/a) \simeq 0 \quad \text{for } r > a. \tag{2}$$

In this description, the stopping power for an electron in the presence of a nearby electron of the same velocity is

$$S = S_0[1 + g(r/a)], \tag{3}$$

with S_0 given by Eq. (1) with $Z = 1$.

For the case of a dense electron beam in passage through matter, the wake forces may be superposed by considering the influence of the system as a whole and the influence of the immediate local neighborhood.[31] The former can be estimated based on a "smoothed out" distribution of beam particles while the latter will include relatively rapid fluctuations. The former influence is the dominant one and is estimated here by neglecting correlation effects.[32] Then the number of beam particles surrounding a particle of interest at a distance between r and $r+dr$ from the latter is $4\pi r^2 n_b dr$ where the beam particle density is n_b. Thereby, the total influence of the beam is specified by

$$S_b = S_0[1 + \int_0^\infty g(r/a) 4\pi r^2 n_b dr],$$
$$\simeq S_0(1 + 2\pi n_b a^3/3), \tag{4}$$

where use was made of Eq. (2). Here, S_b is the stopping power of a single electron that is part of a beam of density n_b.

In Eq. (4), the quantity $2\pi n_b a^3/3$ may be interpreted as the number of electrons surrounding the electron of interest that can participate cooperatively in interacting with the ambient matter. This number is designated as N_c and the stopping of a beam electron may be written as

$$S_b = S_0(1 + N_c). \tag{5}$$

Of particular importance is the fact that we have restricted our attention to a uniform velocity beam, $\Delta\vec{v} = 0$. The interaction in this case is hydrodynamic and is quite strong. For example, since $N_c = 2\pi n_b a^3/3 \simeq 10^{18} n_b \beta^3/3n^{3/2}$, with $n_b \sim 10^{12}$ cm^{-3}, $n \sim 10^{14}$ cm^{-3}, and $\beta \sim 1$, the cooperation number is $N_c \sim 10^9/3$. This effect is in fact that of coherent Cherenkov radiation of plasma oscillations.[33]

A simple perturbation analysis shows that the anomalous stopping behavior can drive the slow space-charge wave. The instability is a self-dissipative one in which a perturbational enhancement of beam density increases the stopping power of the beam with a subsequent slowing of the beam. This slowing again augments the local beam density, thereby providing positive feedback. The growth rate is easily estimated to be

$$\omega_i \simeq \frac{S_0 N_c k}{2m\omega_b \gamma^{3/2}}, \tag{6}$$

for a wave number k and beam plasma frequency $\omega_b = (4\pi n_b e^2/m)^{1/2}$. Then, in the hydrodynamic limit,

$$\frac{\omega_{ih}}{\omega_b} \simeq \frac{akK_0(\rho/a)}{4\pi\gamma^{3/2}} \sim \frac{2ak}{\gamma^{3/2}}, \tag{7}$$

since $K_0(\rho/a) \sim 20$, typically. Maximum growth will occur for k maximum, i.e., $k \sim \omega_p/v_e$, where v_e is the ambient plasma electron thermal speed. Then, using the definitions of a, ω_p and ω_b,

$$\frac{\omega_{ihm}}{\omega_p} \sim \left(\frac{n_b}{n}\right)^{1/2} \frac{2v}{\gamma^{3/2} v_e}. \tag{8}$$

Reasonable values for n_b/n and v/v_e give growth rates in excess of ω_p, thereby indicating strongly growing instabilities.[34] It is apparent, then, that the beam-density effect in the hydrodynamic limit can explain anomalous effects found by Kharchenko *et al.*[35] for example.

The production and focusing of intense relativistic electron beams generally results in beams with appreciable angular scatter, $\Delta\Theta$, about the beam axis as well as finite energy spreads, $\Delta\gamma$. Since the wake interference phenomena is basically a kinetic effect, these departures from $\Delta\vec{v} = 0$ will reduce the strength of the effect. In particular, the angular spread of the beam directly reduces the influence of the wake electric field by $\langle\cos\Theta\rangle$. For beams of interest, however, this factor is still of order unity. More importantly, the spread in drift velocity of the beam can wash out the coherent effects of interest. A compatibility condition[22] for "wake-riding" effects has been given. For a beam of particles, this requirement restricts the energy due to the spread in drift velocities to a value less than the potential energy of a beam particle in the wake-potential well. The wake-potential well depth is[20] $e^2K_0(\rho/a)/2a$. In most cases, the fractional energy spread of the beam is considerably less than $(\gamma\Delta\Theta)^2$. Then, the energy due to spread in drift velocities is attributable to the angular spread $\Delta\Theta$ and is $\sim\gamma m v^2 \times \Delta\Theta^2$. Thereby, the fractional number of cooperative participants is reduced by this ratio, $e^2K_0(\rho/a)/2a\gamma m v^2 \Delta\Theta^2 \equiv f$. The new cooperation number is

$$N_{ck}=N_{ch}f \simeq n_b a^2 r_0 \frac{K_0(\rho/a)}{\gamma\beta^2\Delta\Theta^2} \sim \frac{2n_b}{n\gamma\Delta\Theta^2}, \tag{9}$$

where $r_0=e^2/me^2$ is the classical electron radius. Even in this case, with $\gamma\beta^2\sim 1$ and $\Delta\Theta^2 \leqslant 0.1$, $N_{ck} \sim 20n_b/n$ and thus an increase in stopping power by a factor of 2 occurs for $n_b \sim n/20$.

It is worth noting that for injection of a beam into a plasma of comparable density, space-charge neutralization within the beam channel can reduce the plasma electron density to a value considerably less than that of the beam, i.e., $n_b/n \gg 1$. Under such conditions, $N_{ck}\gg 1$. Since $n_b \sim 10^{17}$ cm^{-3} is easily obtainable,[8] such conditions can now be realized for comparably dense plasmas. Thereby, anomalous electron-beam heating of magnetically confined plasma[36] can be realized. In particular, for a magnetically self-focused electron beam under space-charge–neutralized conditions, the equation of state for the beam[8] gives $(\Delta\Theta)^2=i_n/\gamma\beta$ where i_n is the net current in units of $mc^3/e = 17$ kA. For such cases,

$$N_{ck}\simeq \frac{2\beta}{i_n}\frac{n_b}{n}=\frac{2}{\pi r_0 \sigma^2 n\delta}, \tag{10}$$

where σ is the beam radius and $\delta = i_n/i$ is the ratio of net current to beam current. Recognizing $\pi r_0\sigma^2 n$ as the Budker parameter[37] for plasma electrons, ν_e, the cooperation number becomes

$$N_{ck}\sim (\delta\nu_e)^{-1}. \tag{11}$$

The passage of a dense relativistic beam through a dense, but finite, plasma again affords a case in which $n_b/n\gg 1$. This occurs on the downstream side of the plasma, i.e., the exit side, for which the decrease in plasma density provides a region where beam and plasma densities are comparable. Again, space-charge neutrilization in this region results in the ambient plasma electron density n falling well below n_b. The "plasma" then consists locally of ions of density n_i and beam electrons of density $n_b \simeq n_i$. One consequence of such an interaction could be "coherent acceleration" of plasma ions in the sense originally proposed by Veksler.[38] In fact, this phenomenon, when viewed from a frame of reference drifting with the electron beam,[39] presents a beam of ions which are within the hydrodynamic limit since these ions have a very small velocity spread in the laboratory frame. Thus, strong cooperative stopping of the "ion beam" in the moving frame would be expected (corresponding to acceleration in the lab frame) with a cooperation number of order $N_{ci}\simeq 2\pi n_i' a'^3/e$. The primes denote values in the moving reference frame and $a'=v/\omega_b'$ where $\omega_b'=(4\pi n_b' e^2/m)^{1/2}$. Thus,

$$N_{ci}\simeq \frac{\gamma^{5/2}\beta^3 n_i}{3n_b^{3/2}}\,10^{18} \sim \frac{\gamma^{5/2}\beta^3}{3n_b^{1/2}}\,10^{18}, \tag{12}$$

the latter expression following since $n_i \sim n_b$ in the region of interest. As acceleration of the ion bunch occurs, the energy spreading experienced by the ions results in a reduction of the accelerating field value since the effect enters the kinetic regime. Although this description may at first appear to conflict with more recent models,[6,7] such may not be the case. In fact, it is readily shown that the same fields calculated in the wake model[20] have far-field values[40] which can be used to obtain the potential wells utilized in these more recent descriptions. This connection between the two models warrants further investigation.

Finally, the self-dissipative instability growth rate in the kinetic limit is found from Eq. (6) utilizing N_{ck} from Eq. (9):

$$\frac{\omega_{ik}}{\omega_b}\approx \frac{r_0 k K_0^2(\rho/a)}{8\pi\gamma^{5/2}\beta^2\Delta\Theta^2}. \tag{13}$$

In this case, the growth rate is significant only for small β, corresponding either to ion beams or low-velocity electron beams.

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High-Density Discharges in the Alcator Tokamak

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Peak plasma densities in excess of 10^{15} cm^{-3} have been obtained in the Alcator tokamak with $60 \leq B_T < 85$ kG. The highest average density so far achieved is $\bar{n}_e = 6\times10^{14}$ cm^{-3}; the corresponding $n_0\tau_E = 2\times10^{13}$ cm^{-3} s. These ultrahigh-density discharges exhibit (i) nearly complete energy equilibration between electrons and ions, (ii) severe attenuation of energetic-neutral-particle fluxes, (iii) a minor role of impurities, and (iv) energy-confinement properties consistent with neoclassical estimates.

The unique combination of high toroidal field, current, and Ohmic power density, together with clean vacuum and wall conditions in the Alcator tokamak has previously enabled discharges with densities ranging from 5×10^{12} cm^{-3} to 5×10^{14} cm^{-3} to be produced. These discharges had particular significance because they made possible a comprehensive study of confinement properties as a function of density, n, which showed that the global energy-confinement time, τ_E, increases roughly in proportion to the density.[1-4] Hence the Lawson parameter, $n_0\tau_E$ (n_0 is the central density), increases in proportion to n^2, and values of $n_0\tau_E$ up to 1×10^{13} cm^{-3} s have been reported.

Recent improvements in the operation of Alcator have permitted further increases in density, and central densities in excess of 10^{15} cm^{-3} have now been produced at toroidal fields between 60