PHYSICAL REVIEW LETTERS

VOLUME 39 | 14 NOVEMBER 1977 | NUMBER 20

Simple Quantum-Chromodynamics Prediction of Jet Structure in e^+e^- Annihilation

Howard Georgi and Marie Machacek
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 26 September 1977)

We propose a simple alternative to the sphericity as a measure of jet structure in e^+e^- annihilation. Our variable has the property that it can be reliably calculated in perturbation theory in quantum chromodynamics (QCD) for large Q^2. It is not sensitive to the details of quark and gluon decay into color-singlet hadrons. We discuss the nonperturbative effects which are important at moderate Q^2.

The experimental discovery of jet structure in the hadronic final states produced in e^+e^- annihilation[1] sets the stage for an important test of QCD (quantum chromodynamics). As the center-of-mass energy Q increases, the effective $\langle p_\perp\rangle$ (the momentum transverse to the jet axis) must grow because of gluon bremsstrahlung. Eventually, three jet structures should become identifiable.

Meanwhile, theoretical analyses of jet structure in QCD have evolved. In an early analysis of sphericity,

$$S=\tfrac{3}{2}\min\{\textstyle\sum p_\perp^2/\sum p^2\}, \qquad (1)$$

Ellis, Gaillard, and Ross[2] found it necessary to supplement perturbation theory with quark and gluon decay functions which describe how the colored partons fragment into color-singlet hadrons. The theoretical status of decay functions in QCD is uncertain,[3] but at any rate these functions cannot be reliably computed in perturbation theory. They must be extracted from experimental data.

In a recent analysis, Sterman and Weinberg[4] identify jet structure completely within the context of perturbation theory. They define a "jet differential cross section" as the probability distribution that all but a small fraction ϵ of the energy Q is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$. They argue that this quantity, computed in perturbation theory in QCD, is free of infrared divergences as the quark masses go to zero, which suggests that the perturbation theory calculation is reliable.[5]

In this Letter, we seek to combine the nice features of these two analyses. We define a single variable which, like sphericity, measures the deviation from perfect two-jet structure. But like the Sterman-Weinberg jet cross section, our variable should be free of infrared logarithmic divergences in perturbation theory, and thus independent of the quark and gluon decay functions.

Our variable which we call *spherocity* (S') is defined as follows:

$$S'=(4/\pi)^2\{\textstyle\sum|p_\perp|/\sum|p|\}^2, \qquad (2)$$

where the jet axis is chosen to minimize $\sum|p_\perp|$ (typically it will be the direction of the largest particle momentum).[6] In the analysis below, we will assume that the sum runs over all particles. In experimental reality, it will usually be convenient to sum only over charged particles. Hopefully, this distinction will be unimportant.

Before computing the spherocity, we discuss the logical connection between the absence of infrared logarithms in perturbation theory and nondependence on parton decay functions. For an experimentally measurable quantity y to be independent of the details of parton decay functions, it must depend only on the properties of each jet as a whole. For example, consider two events which are identical except that in event 1 one of the jets consists of a single fast hadron with momentum $\vec{p}$ (plus soft particles), while in event 2 the corresponding jet consists of two fast particles, each with momentum $\frac{1}{2}\vec{p}$ (again neglecting soft particles and also small transverse momenta). To be independent of decay functions, y must be the same for events 1 and 2; so it can depend only on the total jet momentum $\vec{p}$. From this point of view, the advantage of spherocity over sphericity is clear. If we neglect the transverse-momentum spread within each jet, then spherocity depends only on the total momentum of each jet, while sphericity depends on how the jet momentum is parceled out among the hadrons.

A similar situation prevails with respect to infrared divergences in perturbation theory. In QCD with massless quarks, consider two events which are identical except that a final-state quark which in event 1 has momentum $\vec{p}$ is replaced in event 2 by a quark and a gluon, each with momentum $\frac{1}{2}\vec{p}$. The differential cross section for event 2 is divergent because the quark and gluon are collinear (the total momentum is still on the massless-quark mass shell). In the total cross section for e^+e^- annihilation, the infrared divergences associated with event 2 are canceled by divergences in the virtual corrections to event 1 (in which virtual quark and gluon momenta are collinear). If the quantity y has the same value for events 1 and 2, the cancellation of divergences which occurs for the total cross section should also occur in the calculation of $\langle y\rangle$. Again the criterion is that y must depend only on the total momentum of any jet of particles with parallel (not antiparallel) momenta. Again, spherocity satisfies this criterion while sphericity does not.

The above argument is a modest refinement of the comment by Sterman and Weinberg that a calculable quantity must not distinguish between states which are physically indistinguishable, such as a massless quark versus a quark plus gluon with the same total four-momentum.

We will show below that if the quark masses can be neglected, the average spherocity to lowest nontrivial order in perturbation theory is

$$S_P' = (4/\pi)^2 k[\alpha_s(Q^2)/\pi], \qquad (3)$$

where

$$k = \tfrac{4}{3}(64 \ln\tfrac{3}{2} - \tfrac{229}{9}) \simeq 0.674. \qquad (4)$$

At very large Q^2, we expect Eqs. (3) and (4) to be an accurate prediction of the experimental spherocity. At moderate Q^2, such as probed at SPEAR, we do not expect Eqs. (3) and (4) to be accurate. There must be important effects due to quark confinement which are undetectable in perturbation theory.

We can easily estimate the size of the nonperturbative effects by assuming that the cross section is dominated by two-jet events with the transverse-momentum distribution observed in jets in hadron-hadron scattering, with $\langle p_\perp\rangle \simeq 300$ MeV. For such events we expect a spherocity

$$S_{NP}' \simeq (4/\pi)^2[(300\text{ MeV})/Q]^2\langle n\rangle^2, \qquad (5)$$

where n is the multiplicity. At moderate Q^2, we believe the spherocity will be well approximated by the sum

$$S' = S_P' + S_{NP}'. \qquad (6)$$

As Q^2 increases, both S_P' and S_{NP}' decrease, but S_{NP}' decreases much faster than S_P'. At sufficiently large Q^2, the nonperturbative effects will be negligible compared to the perturbative prediction. In particular, this is true in the range of PEP and PETRA energies[2]; for example, at $Q = 18$ GeV, the nonperturbative and perturbative effects contribute equally to S'. Figure 1 summarizes our predictions.[7]

The derivation of Eqs. (3) and (4) is simplified by the fact that the spherocity vanishes in zeroth order, for q-$\bar{q}$ production. To first order in α_s, we need only compute the average spherocity for the three-particle (q-$\bar{q}$-gluon) final state. If the

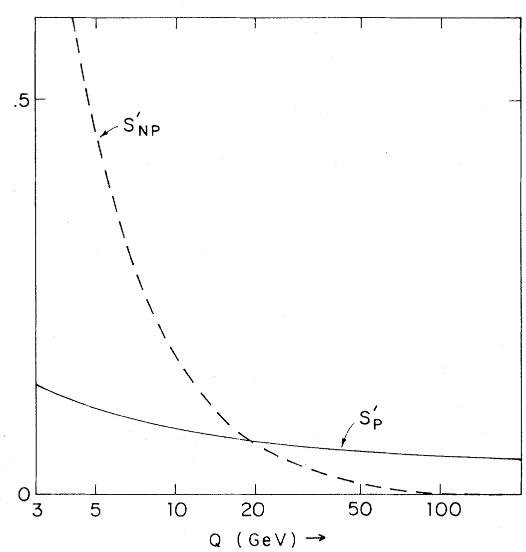

FIG. 1. Average value of spherocity vs Q. The solid line is the perturbative prediction, S_P'. The dotted line is an estimate of the nonperturbative effect, S_{NP}'.

momenta of the massless quark (antiquark) is $\vec{p}_1$ ($\vec{p}_2$) and the photon energy (in the center-of-mass frame) is Q, define the dimensionless variables

$$\chi_i \equiv 2|\vec{p}_i|/Q. \tag{7}$$

They satisfy

$$1 > \chi_i > 0, \quad \chi_1 + \chi_2 > 1. \tag{8}$$

The differential cross section is[8]

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{d\chi_1 d\chi_2} = \frac{2}{3}\frac{\alpha_s}{\pi}\frac{\chi_1^2 + \chi_2^2}{(1-\chi_1)(1-\chi_2)}, \tag{9}$$

where σ_0 is the total zeroth-order cross section for q-$\bar{q}$ production. The spherocity is

$$S_P'(\chi_1, \chi_2) = 4(4/\pi)^2(1-\chi_1)(1-\chi_2)(\chi_1+\chi_2-1)\min\{\chi_1^{-2}, \chi_2^{-2}, (2-\chi_1-\chi_2)^{-2}\} \tag{10}$$

(the jet axis is taken in the direction of the largest momentum). The result, Eqs. (3) and (4), is obtained by integrating

$$\frac{1}{\sigma_0}\int \frac{d^2\sigma}{d\chi_1 d\chi_2} S_P'(\chi_1, \chi_2)\, d\chi_1 d\chi_2. \tag{11}$$

Many interesting questions about jets remain unanswered by the above analysis. For example, is any memory of the charge of the initial quark preserved by the hadrons in the final jet? We have been unable to incorporate any information on the charge of the hadrons in our jet analysis without spoiling the cancellation of infrared logarithmic divergences. This suggests that the answers to such questions lie buried in the details of quark and gluon decay functions.

Farhi[9] has independently come to many of the same conclusions. We are grateful to him and to S. Weinberg and H. D. Politzer for valuable discussions. One of us (H.G.) wishes to thank the staff of the Aspen Center for Physics for their hospitality in Aspen and to acknowledge useful conversations at the Center with T. Appelquist, A. De Rújula (who invented the name *spherocity*), and G. Ross.

This research was supported in part by the National Science Foundation under Grant No. PHY 75-20427, the Alfred P. Sloan Foundation, and the U. S. Energy Research and Development Administration under Contract No. EY-76-C-02-1112.*000.

[1]G. Hanson *et al.*, Phys. Rev. Lett. 35, 1609 (1975).

[2]T. Ellis, M. K. Gaillard, and G. G. Ross, Nucl. Phys. B111, 253 (1976).

[3]A. Mueller, Phys. Rev. D 9. 963 (1974); C. G. Callan, Jr., and M. L. Goldberger, Phys. Rev. D 11, 1542, 1553 (1975).

[4]G. Sterman and S. Weinberg, Harvard University Report No. HUTP-77/A044, 1977 (to be published).

[5]See also, H. D. Politzer, Harvard University Report No. HUTP-77/A045, 1977 (to be published).

[6]The factor $(4/\pi)^2$ gives an $S'=1$ normalization for a totally spherical event.

[7]We have used $\alpha_s/\pi = \frac{12}{25}[\ln(Q^2/\Lambda^2)]^{-1}$ with $\Lambda = 460$ MeV (chosen to connect smoothly to the three-quark form with $\Lambda = 500$ MeV at $Q^2 = 9$ GeV^2). We have taken the mean multiplicity $\langle n\rangle = 3\ln Q + 4$, which is roughly twice the mean charged multiplicity for $2.4 < Q < 5$ GeV [see J.-E. Augustin *et al.*, Phys. Rev. Lett. 34, 764 (1975)].

[8]T. A. DeGrand, Y. J. Ng, and S.-H. H. Tye, SLAC Report No. SLAC-PUB-1950, 1977 (to be published).

[9]E. Farhi, Harvard University Report No. HUTP-77/A059, 1977 (to be published).