Quark-Mass Generation by Pseudoparticles[a]

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The pseudoparticle-induced multiquark interaction is shown to produce a spontaneously generated quark mass as calculated by a Hartree-Fock treatment. Hence pseudoparticles lead to a dynamical mechanism for spontaneous breaking of chiral SU(N)$\otimes$SU(N) for N flavors.

Quantum chromodynamics (QCD) is a theory of the strong interactions which starts off with massless quarks and a chiral SU(N)$\otimes$SU(N) symmetry, where N is the number of flavors. In the physical world, however, quarks appear to be massive and the broken chiral symmetry is realized in the Nambu-Goldstone mode. But what is responsible for generating a mass for the quarks and so spontaneously breaking the chiral symmetry has been an open question. Recently Callan, Dashen, and Gross[1] (CDG) have proposed that the answer, at least in two space-time dimensions, is provided by pseudoparticles and their interaction with fermions. I have found that dynamical quark-mass generation via pseudoparticles does occur in four dimensions.

My starting point is the effective Lagrangian found by 't Hooft[2] for a color-SU(2) gauge-theory pseudoparticle[3] interacting with N flavors of massless quarks,

$$\mathfrak{L}_{\rm eff}(z)=(0.64)2^{2+3N}\pi^{2N-2}\left(\frac{8\pi^2}{g^2}\right)^4\int d\rho\,\rho^{-5+3N}\exp\left\{-\frac{8\pi^2}{g^2(\mu_0)}+\ln(\mu_0\rho)[\tfrac{22}{3}-\tfrac{2}{3}N]+2N(0.146)\right\}\langle\prod_{s=1}^{N}(\bar{\psi}_s\omega)(\bar{\omega}\psi_s)\rangle + \text{H.c.}, \quad (1)$$

where ρ is the pseudoparticle size, μ_0 is the renormalization subtraction point, integration over d^4z gives energy-momentum conservation, and

$$\langle\omega_\alpha\bar{\omega}_\beta\rangle=\tfrac{1}{4}\delta_{\alpha\beta}(1+\gamma_5). \quad (2)$$

For the case of two flavors,

$$\langle\prod_{s=1}^{2}(\bar{\psi}_s\omega)(\bar{\omega}\psi_s)\rangle=\tfrac{1}{24}(2\delta_{\alpha_1}{}^{\beta_1}\delta_{\alpha_2}{}^{\beta_2}-\delta_{\alpha_1}{}^{\beta_2}\delta_{\alpha_2}{}^{\beta_1})\epsilon^{st}\bar{\psi}_1{}^{\alpha_1}(1+\gamma_5)\psi_s{}^{\beta_1}\bar{\psi}_2{}^{\alpha_2}(1+\gamma_5)\psi_t{}^{\beta_2}. \quad (3)$$

Because the coefficient of $\ln(\mu_0\rho)$ is that of the Callan-Symanzik β coefficient for $g^2(\mu_0)$, one may to this one-loop order replace g by the running coupling constant $\bar{g}(\rho)$.

Since the $2N$ fermion interaction of this effective Lagrangian has the chiral-symmetry properties of the determinant of an $N\times N$ matrix, it is invariant under chiral SU(N)$\otimes$SU(N). One therefore has a situation similar to that of the Nambu–Jona-Lasinio model[4] and one may ask whether the fermions without a bare mass may nevertheless dynamically develop a mass due to the self-energy from this induced $2N$ interaction. To answer this one looks at the self-consistent or Hartree-Fock equation for the quark mass operator.

For heuristic purposes I shall do the calculation in two different ways: The first is more transparent but is more approximate; the second justifies some of the assumptions and leads to additional results.

Let us consider the quark-mass operator equation for $N=2$ and for pseudoparticle of fixed size ρ. The integral equation for the quark mass operator in lowest-order approximation is represented graphically in Fig. 1, where on the left is shown the mass operator, and the right side the vertex of the pseudoparticle-induced effective four-quark interaction with the internal line being the full massive-quark propagator. The resulting

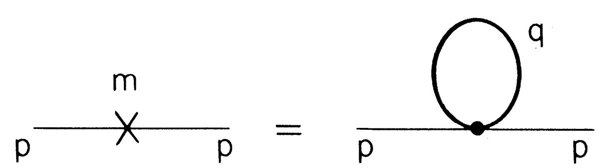

FIG. 1. Equation for the quark mass operator for $N=2$.

equation is

$$m(p)=C\left(\frac{8\pi^2}{\bar{g}^2}\right)^4 e^{-8\pi^2/\bar{g}^2}\rho^2\int\frac{d^4q}{(2\pi)^4}\frac{m(q)F(p^2)F(q^2)}{q^2+m^2(q)},\qquad(4)$$

where $C=2900$ and we have introduced cutoff factors $F(p^2)F(q^2)$ in the definition of the vertex. This is done since $\mathcal{L}_{\rm eff}$ [Eq. (1)] is only valid outside the pseudoparticle (which I have located at the origin). From the structure of (4) it is apparent that $m(p)=mF(p^2)$, hence

$$m=C\left(\frac{8\pi^2}{\bar{g}^2}\right)^4 e^{-8\pi^2/\bar{g}^2}\rho^2\int\frac{d^4q}{(2\pi)^4}\frac{mF^2(q)}{q^2+m^2F^2(q)}.\qquad(5)$$

Disregarding the trivial solution $m=0$, one finds

$$1\approx C\left(\frac{8\pi^2}{\bar{g}^2}\right)^4 e^{-8\pi^2/\bar{g}^2}\rho^2\int^{1/\rho}\frac{d^4q}{(2\pi)^4}\frac{1}{q^2+m^2}.\qquad(6)$$

The result of the Euclidean integration is

$$\left[1-\tilde{C}\left(\frac{\bar{g}^2}{8\pi^2}\right)^4 e^{8\pi^2/\bar{g}^2}\right]=\rho^2m^2\ln\left(\frac{1}{m^2\rho^2}+1\right),\qquad(7)$$

where $\tilde{C}=0.055$. Since the right-hand side of (7) is ≥ 0, there is a spontaneously generated quark mass for

$$0<\tilde{C}(\bar{g}^2/8\pi^2)^4e^{8\pi^2/\bar{g}^2}<1.\qquad(8)$$

The result is, of course, nonperturbative.

The question now is whether for reasonable values of $\bar{g}$ the inequality (8) is satisfied. It turns out that it is for $\frac{1}{13}\leq\bar{g}^2/8\pi^2\leq 1.8$. For example, if we pick the value $\bar{g}^2/8\pi^2=\frac{1}{8}$ ($\bar{g}^2/4\pi\simeq 0.8$) at which CDG predict[5] that hadronic processes occur (i.e., confinement), then $m=4/\rho$. For values of ρ on the order of an inverse pion mass, the quark mass comes out to be several hundred MeV. It is also important that for this value of the coupling the density[5] of pseudoparticles, $0.26(8\pi^2/\bar{g}^2)^4\times\exp(-8\pi^2/\bar{g}^2)=0.36$, is small enough so that the dilute-gas approximation is reasonable. As for the neglect of the $m=0$ solution, one can show[4] that the energy of the vacuum for massive fermions is lower than that for massless ones.

I would like to repeat the calculation now without having to use a cutoff, and also including the integration over all pseudoparticle sizes which I have so far ignored. I begin with the observation that $\mathcal{L}_{\rm eff}$ only approximately reproduces the exact massless-fermion contribution to the vacuum-to-vacuum amplitude computed by 't Hooft. The exact answer has a factor of $(x^2+\rho^2)^{-3}$ which at large distances only is reproduced by massless-fermion propagators in coordinate configuration so that

$$-S_{\rm F}(x)S_{\rm F}(-x)\propto x^{-6},\qquad(9)$$

$$S_{\rm F}(x)=\gamma\cdot x/2\pi^2(x^2)^2.\qquad(10)$$

To reproduce the factor of $(x^2+\rho^2)^{-3}$ exactly, I modify $S_{\rm F}(x)$,

$$\tilde{S}_{\rm F}(x)=\frac{\gamma\cdot x+i\rho}{2\pi^2(x^2+\rho^2)^2},\qquad(11)$$

so that

$$-\tilde{S}_{\rm F}(x)\tilde{S}_{\rm F}(-x)\propto(x^2+\rho^2)^{-3}.\qquad(12)$$

Now $\mathcal{L}_{\rm eff}$ will give the exact answer for the vacuum-to-vacuum amplitude, provided that one uses this fermion propagator $\tilde{S}_{\rm F}(x)$ modified to take into account the presence of the pseudoparticle.

The Euclidean Fourier transform of (11) is

$$\tilde{S}_{\rm F}(p)=\rho\left[\frac{i\not{p}}{\sqrt{p^2}}K_1(\rho\sqrt{p^2})+iK_0(\rho\sqrt{p^2})\right],\qquad(13)$$

where the $K_{1,0}$ are modified Bessel functions of integral order which have the following integral representation:

$$K_n(z)=\tfrac{1}{2}(\tfrac{1}{2}z)^n\int_0^\infty e^{-t-z^2/4t}t^{-(n+1)}dt,\qquad(14)$$

and, asymptotically,

$$K_n(z)\underset{z\to\infty}{\longrightarrow}\left(\frac{\pi}{2z}\right)^{1/2}e^{-z}.\qquad(15)$$

For the massive-fermion Euclidean propagator in the presence of an pseudoparticle of size ρ, one has

$$\tilde{S}_m(p)=\rho\left[\frac{i\not{p}+m}{(p^2+m^2)^{1/2}}K_1(\rho(p^2+m^2)^{1/2})+iK_0(\rho(p^2+m^2)^{1/2})\right].\qquad(16)$$

$\tilde{S}_m(p)$ reduces to the massless propagator $\tilde{S}_{\rm F}(p)$ (13) as $m\to 0$. Furthermore, (16) becomes the usual massive-fermion propagator $(i\not{p}+m)/(p^2+m^2)$ at large distances, as can be seen by expanding $K_{1,0}$ in (16) using the series expansions,

$$K_0(z)=-\ln z-\gamma+\ln 2+\dots,\qquad K_1(z)=z^{-1}+\dots,\qquad(17)$$

and keeping the lowest-order term, z^{-1}.

The self-consistent integral equation for the quark mass (Fig. 1), with use of the modified propagator $\tilde{S}_m(p)$, is

$$m = C\left(\frac{8\pi^2}{\bar{g}^2}\right)^4 e^{-8\pi^2/\bar{g}^2}\rho^2 \int \frac{d^4q}{(2\pi)^4} \frac{\rho m K_1(\rho(q^2+m^2)^{1/2})}{(q^2+m^2)^{1/2}}, \tag{18}$$

where for the moment I still keep ρ fixed. In (18) I no longer have cutoff factors as in (4) since $S_m(p)$ explicitly takes the pseudoparticle size into account, and indeed the integral is finite due to the exponential damping provided by $K_1(\rho(q^2+m^2)^{1/2})$. One may also notice that I have taken m to be a constant in (18) independent of momentum. This point is discussed below.

Again disregarding the trivial solution to (18), i.e., $m=0$, the result of the integration is

$$1 = C\left(\frac{8\pi^2}{\bar{g}^2}\right)^4 e^{-8\pi^2/\bar{g}^2} \frac{1}{4\pi^2}\rho m K_1(\rho m). \tag{19}$$

If one again uses $\bar{g}^2/8\pi^2 = \frac{1}{8}$, one finds a result, $m = 6/\rho$, similar to that obtained from (7).

But now one can go further and perform the ρ integration. The integral [to one-loop order in the expansion of $\bar{g}(\rho)$ as in (1)] is

$$\mu_0{}^6 m \int_0^\infty d\rho\, \rho^6 K_1(\rho m). \tag{20}$$

One notes that the integrand goes to zero at both ends of the scale and that it has a fairly sharp maximum at $\rho m = 5.5$ [where, as was seen, $\bar{g}^2/8\pi^2 \approx \frac{1}{8}$ to satisfy (19)]. Hence it was a reasonable approximation to take a single pseudoparticle of fixed size. More importantly, the ir divergence which seemed to be present in $\mathcal{L}_{\rm eff}$ as $\rho \to \infty$ does not appear here because of the exponential damping of $K_1(\rho m)$. (As $\rho \to 0$ the integral converges because of asymptotic freedom.) What appears to be happening is that essentially one size (or at least a small range of sizes) of pseudoparticle is responsible for interacting with fermions to generate a fermion mass.

If one actually performs the integration of (20), then m is expressed in terms of $g(\mu_0)$ and one must pick a renormalization subtraction point μ_0 since the result is

$$1 = C\frac{1}{4\pi^2}\left(\frac{8\pi^2}{g^2(\mu_0)}\right)^4 \exp\left(-\frac{8\pi^2}{g^2(\mu_0)}\right) 2^5\Gamma(4)\Gamma(3)\frac{\mu_0{}^6}{m^6}. \tag{21}$$

If one picks $g^2(\mu_0)/8\pi^2 = \frac{1}{8}$ with $\mu_0 = 100$ MeV, which is consistent with the choice above for $\bar{g}^2(\rho)/8\pi^2$, one finds that again $m \approx 550$ MeV. This also confirms our observation that essentially one pseudoparticle size is involved.

Turning one's attention to the question of the momentum dependence of m, one can argue that if it is generally true that only one pseudoparticle size contributes, then the approach followed in the first calculation (4) is valid. In particular, the factorization of the cutoff factors into $F(p)F(q)$ is allowed. However, I have shown that one pseudoparticle size contributes by performing the second calculation (18) only for constant m. While I believe that this approach is a reasonable approximation and that the result is correct, to establish it rigorously involves a complicated, highly nonlinear integral equation for $m(p)$ which has so far proved intractable.

I have also calculated the quark mass for the cases $N=3,4$. I find masses slightly smaller than that for $N=2$, and they occur for smaller coupling and much lower density. This is in contrast to the two-dimensional work of CDG where there is a phase transition at $N=2$ so that $m=0$ for $N>2$ and m is exceedingly small for $N=2$. I find no evidence for such a phase transition in four dimensions, though there may well be one for high enough N.

All these calculations have been done for a color-SU(2) gauge theory. Of course, QCD has SU(3) for the color gauge group. I expect that the qualitative features of this work will also be found for color-SU(3), but the actual values of the predicted quark mass will probably be somewhat different. These are the values, of course, that should be compared with whatever one can deduce from experimental evidence. This extension to color-SU(3) is currently under investigation and the results will be the subject of a future publication.

The conclusions of this present work are that pseudoparticle interactions with massless quarks have extensive consequences. Not only do they solve the $U_A(1)$ problem[2,6] so that the apparent chiral $U(N)\otimes U(N)$ symmetry of the QCD Lagrangian is really only $SU(N)\otimes SU(N)$, more significantly, this quark-pseudoparticle interaction also produces a spontaneously generated mass for the quark, thus dynamically breaking the chiral $SU(N)\otimes SU(N)$ symmetry. This, as usual, produces a multiplet of massless Nambu-Goldstone pseudoscalar mesons in the adjoint representation of SU(N) which arise from iterated bubble graphs. For the mesons to acquire (different) masses it requires mechanisms for conventional breaking of SU(N) symmetry [hence explicitly breaking SU(N)$\otimes$ SU(N)] as treated, for example, in the Gell-Mann–Oakes–Renner model.[7]

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Search for Narrow Resonant States in e^+e^- Collisions near 6 GeV

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Following reports of anomalous dielectron prediction in the mass region near 6 GeV at 400 GeV, we searched for an enhancement in the reaction $e^+e^- \to$ hadrons and $e^+e^- \to e^+e^-$ at SPEAR in the center-of-mass energy range 5.67–6.43 GeV. The leptonic and hadronic cross sections show no statistically significant peaks. In this mass range, 95% confidence level upper limits for the decay width into electron pairs are less than 200 eV for a narrow resonance which decays predominantly either into hadrons or into electron pairs.

Following reports of an enhancement near 6 GeV in the invariant-mass spectrum of e^+e^- pairs produced in high-energy hadron-hadron collisions,[1,2] we measured the e^+e^- total cross section in 4-MeV intervals in the center-of-mass energy range 5.67 to 6.43 GeV, using the SPEAR electron-positron colliding-beams machine. The detector triggered on both charged and neutral particles. An integrated luminosity of about 10 nb^{-1} was obtained in each energy interval, corresponding to about 75 observed hadronic events. The sensitivity to narrow resonances was about