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Evidence of a Large Superfluid Vortex in ^{4}He

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Optical interferometry was used to measure surface profiles of He II. Depressions observed following rapid container spinup agreed with theoretical profiles of a superfluid vortex with circulation $\approx 400h/m$.

The energetically preferred state of rotating He II is a uniform distribution of vortex lines, each with a circulation Γ predicted to be $\kappa = h/m$, where h is Planck's constant and m is the ^{4}He atomic mass.[1] Photographs of vortex lines by Williams and Packard[2] show a nearest-neighbor spacing roughly consistent with that Γ. Quantization of Γ, however, does not rule out the energetically less favorable creation of a vortex having $\Gamma = N\kappa$ with integer $N > 1$. Feynman[3] noted that if a vortex with large N were created, the liquid surface above it would be noticeably depressed. This Letter describes observation of depressions in He II pools with a thickness of several microns. Depressions were accurately described by a model with superfluid flow in the bulk liquid being that of a vortex with $N \approx 400$. At the center of each depression, the substrate was covered by a thin He II film and details of flow there could not be determined.

When calculating a vortex's surface profile, it is important to include the influences of the surface tension σ and the wetting of the container. For a quasisteady profile, there is a pressure balance at each point on the surface given by Laplace's formula: $P = P_v + \sigma(R_1^{-1} + R_2^{-1})$, where P and P_v are the pressures on the liquid and vapor sides of the interface and R_1 and R_2 are principal radii of curvature. For purely azimuthal flow about the center of a cylinder of radius b, R_1 and R_2 lead to a differential equation[4] for the surface height $Z(r)$ at radius r. When linearized for $Z' \ll 1$, it becomes[5]

$$Z'' + Z'r^{-1} - a^{-2}Z = -\sigma^{-1}f(r), \tag{1}$$

$$f(r) = \int^r (\rho_s v_{s\theta}^2 + \rho_n v_{n\theta}^2) r^{-1}\, dr, \tag{2}$$

where differentiation with respect to r is indicated by a prime; $a = [\sigma/g(\rho - \rho_v)]^{1/2} \approx 0.5$ mm is the capillary constant; g is the acceleration of gravity; ρ, ρ_s, ρ_n, and ρ_v are the densities of the liquid, superfluid, normal fluid, and vapor, respectively; and $f(r)$ is the centrifugal contribution to P by the azimuthal superfluid and normal-fluid velocities, $v_{s\theta}(r)$ and $v_{n\theta}(r)$. The boundary conditions on (1) are specified by Z' at two radii and the constant of integration in (2) has therefore been omitted. The van der Waals attraction of the liquid to the cylinder walls causes $Z'(b)\approx\infty$. Rayleigh[6] has shown that this can be accounted for in the region $r<b-a$ by taking $Z'(b)=4\tan(\pi/8)\exp(2^{1/2}-2)\approx 0.92$ as a boundary condition on (1). His method applies to rotating He II if[5] $[v_{s\theta}(b)^2+v_{n\theta}(b)^2]/bg \ll 0.92$. The other boundary condition is $Z'(r_0)=0$, where r_0 is the radius of the region at the center of the depression where the cylinder bottom is covered by the thin film. The van der Waals attraction to the bottom is omitted in (1); taking $Z'(r_0)=0$ approximates its influence on the $r>r_0$ profile.

The solution of (1) and (2) for $r_0<r<b$ with these inhomogeneous boundary conditions is[5]

$$Z(r)=-\sigma^{-1}\int_{r_0}^{b} f(\tilde r)G(r,\tilde r)\tilde r\, d\tilde r - bZ'(b)G(r,b), \qquad (3)$$

where $G(r,\tilde r)=A[I_0(r_</a)+BK_0(r_</a)][K_0(r_>/a)+CI_0(r_>/a)]$, $r_<$ and $r_>$ are the smaller and larger of r and $\tilde r$; I_0, K_0, I_1, and K_1 are modified Bessel functions; $B=I_1(r_0/a)/K_1(r_0/a)$; $C=K_1(b/a)/I_1(b/a)$; and $A=(BC-1)^{-1}$. The second term in (3) gives the influence of the wetting of the walls. For a superfluid vortex in a stationary cylinder, $v_{s\theta}=N\kappa/2\pi r$ and $v_{n\theta}=0$, and the integral can be evaluated with use of polynomial approximations. For an $N=1$ vortex in bulk He II, the profile can be estimated by taking $r_0=1$ Å, the usual core radius; if $b\gg a$, the integral in (3) gives a central depression depth of 50 Å at 1.6 K.

The surface profile of ^{4}He contained in a 2.15-cm-diam rotatable cylinder was measured with Fizeau optical interferometry.[5,7] The sealed copper cylinder was surrounded by a He II bath with a temperature T regulated by a manostat. The free surface was illuminated through a window in the top of the cylinder by an expanded laser beam with a wavelength $\lambda=0.6328$ μm and an intensity of 0.3 mW/cm^2. Light reflected from the free surface interfered with light reflected from the cylinder bottom and produced fringes where the liquid thickness ζ had distinct values. Adjacent fringes normally indicated a change in ζ of $\lambda/2n=0.3072$ μm, where n is the refractive index of He II. To reduce thermal gradients and light absorption, the bucket bottom consisted of a single piece of crystalline quartz. It was antireflection coated on the upper side since its reflection coefficient had to be similar to the free He II surface if there were to be adequate fringe contrast. The local height of the free surface Z is the sum of the liquid thickness ζ and the substrate height ψ. Optical measurement techniques made it possible to align the incident optical beam, the axis of rotation, the gravitational acceleration, and the normal to the center of the substrate, to be all parallel to one another to within 10^{-4} rad. The profile $\psi(r)$ of the coated substrate was determined with Twyman-Green interferometry[5] to have a small constant curvature[8] $\psi''\approx-0.029$ m^{-1}. The rotation period of the cylinder was measured by independent photoelectric and electromechanical methods with an accuracy of 0.5% and the angular velocity ω could be traced with a chart recorder.

Before describing the metastable vortexlike depressions, we describe stable equilibrium profiles[9] which were measured by gradually increasing ω from 0 with the rate of change $\dot\omega<0.1$ sec^{-2}. After ω was held constant, the movement of fringes ceased within 30 sec. For an equilibrium distribution of $N=1$ vortices,[10] $\langle v_{s\theta}\rangle=v_{n\theta}=\omega r$ and $\langle f\rangle\approx\frac{1}{2}\omega^2r^2$, where angular brackets denote an average over a region containing several vortices. When the entire substrate is covered by bulk He II, taking $r_0=0$ in (3) gives $\langle Z\rangle\approx\omega^2r^2/2g$ for $r<5$ mm. In addition, there should be a depression above each $N=1$ vortex. Fizeau interferometry with $T=1.6$–1.8 K failed to detect them in keeping with their predicted[4] 50 Å depth. Profiles were measured with $0\le\omega\le 2.7$ sec^{-1} and minimum ζ of 4–12 μm. They were found to deviate from $\omega^2r^2/2g$ by ϵr^2, where $\epsilon=(-1.7\pm0.4)\times10^{-2}$ m^{-2}. The sign of ϵ, its lack of ω dependence, and the observation that $|\epsilon|<0.003$ m^{-2} at $T=2.16$ K are consistent with the cause of the deviation being radial internal convection with $v_{nr}>0$, and $\langle v_{sr}\rangle<0$. We estimate that 10^{-2} mW/cm^2 of background thermal radiation was absorbed by the substrate and this was the cause of the convection.

The equilibrium liquid thickness at the center of the cylinder $\zeta_0(\omega)$ can be calculated to within ± 2 μm for any ω by first measuring the lowest ω for which $\zeta_0(\omega)\lesssim\lambda/2n$ ($\lambda/2n=0.3072$ μm is the thickness change associated with each fringe). If ω is slowly increased, fringes originate at the center and expand as circles outward, until $\zeta_0(\omega)$ has decreased below approximately 0.3 μm. When the liquid volume is assumed to be constant and is calculated with the observed parabolic profiles, we find

$$\zeta_0(\omega) = (\omega_0^2 - \omega^2)b^2/4g \qquad (4)$$

for $\omega < \omega_0$, where ω_0 can be identified as the lowest ω for the liquid to be sufficiently thin that increasing ω no longer causes fringes to originate at $r = 0$.

A vortexlike metastable surface depression was created by stepping ω through a cycle; $\omega(t)$ was held constant at ω_i during the interval $t_i \le t \le t_i^* < t_{i+1}$. For times $t < 0$, both the cylinder and the fringe pattern were motionless. At $t = 0$, power was supplied to a motor and the cylinder accelerated with $\dot{\omega} \approx 7$ sec^{-2} until $t = t_1 < 2$ sec. During the $i = 1$ interval, the bulk liquid moved to within 2 mm of the cylinder wall since we chose $\omega_1 \gg \omega_0$ of (4). At t_1^*, the power was reduced so that $\omega_2 < \omega_0$. The energetically preferred state would be for the profile to be parabolic with a central thickness of $\zeta_0(\omega_2)$. Fourteen cycles were observed with $T = 1.65-1.80$ K, $\omega_1 = 12 \pm 3$ sec^{-1}, $t_1^* - t_1 = 15$ sec, $t_2 - t_1^* < 2$ sec, and liquid volumes and ω_2 characterized by 7 μm $< \zeta_0(0) < 55$ μm and $0 < \zeta_0(\omega_2) < 18$ μm. For each cycle, the profile was *not* parabolic by $t_2 + 120$ sec and for some cycles not by $t_2 + 500$ sec. A few seconds after slowing to ω_2, the inward flow of He II was impeded around a depression in the surface and $Z'' < 0$ over much of the bulk liquid. The center of each depression had $\zeta < 0.3$ μm. It turned with the substrate and was not precisely centered in the cylinder. Diffusion of $\nabla \times \vec{v}_n$ to the substrate by the normal viscosity η should cause $v_{n\theta}$ to approach $\omega_2 r$ with a relaxation time[5,10] $\rho_n \zeta^2/\eta \approx 3 \times 10^{-3}$ sec for $\zeta = 10$ μm. Since the superfluid is assumed to flow around the depressions, however, the superfluid and normal fluids did not have the same axis of rotation so that (3) is not directly applicable. A large-N vortex should have $Z'' < 0$ as can be seen from the $\sigma = 0$ limit of (1): $Z'' \approx (\rho_n \omega^2 - 3\rho_s N^2 \kappa^2 / 4\pi^2 r^4)/\rho g$.

In two cycles, cylinder rotation was stopped at t_3 when the depression was still present. Figure 1 is the fringe pattern of a depression photographed at $t_3 + 47$ sec in a cycle with $t_3 = t_2 + 570$ sec, $\omega_2 + 1.41$ sec^{-1}, and an equilibrium liquid thickness at rest $\zeta_0(0)$ from (4) of 7 μm. Diffusion of $\nabla \times \vec{v}_n$ to the substrate should cause $v_{n\theta} \approx 0$ for $t - t_3 > 10^{-1}$ sec. With $v_{n\theta} = 0$, $v_{s\theta} = N\kappa/2\pi r$, $N = 365$, $\rho_s/\rho = 0.78$, and $r_0 = 0.2$ mm, (2) and (3) described the profile as shown in Fig. 2. The profile was measured along a cylinder diameter with $r = 0$ taken to be the depression center which was 8.7 mm from the wall. By taking $b = 8.7$ mm in (3), the depression's noncentral location is accounted for.[5] The agreement was noticeably degraded if N, the only free parameter, was increased or reduced by more than 10 and profiles did not directly infer that Γ was quantized. The discrepancy for $r > 4$ mm was also present in the $t < 0$ profile and is likewise attributed to convection. The dashed curve illustrates the influence of σ. One other depression was compared with (3) and there was equivalent agreement when $N = 390 \pm 10$. That cycle had a similar rotation history and $\zeta_0(0)$ but was photographed at $t_3 + 60$ sec with $T = 168$ K on a different day.

Other evidence supports the superfluid vortex model and is summarized as follows: (i) Depressions were not formed for small ρ_s/ρ. Ten cycles were observed with rotational histories and

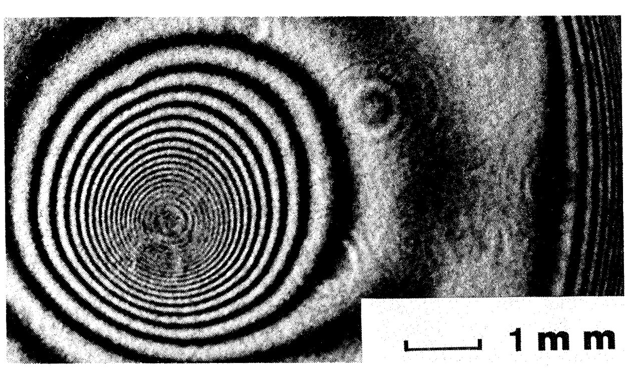

FIG. 1. Fringe pattern of a depression observed in a 7-μm-thick pool with $T = 1.68$ K.

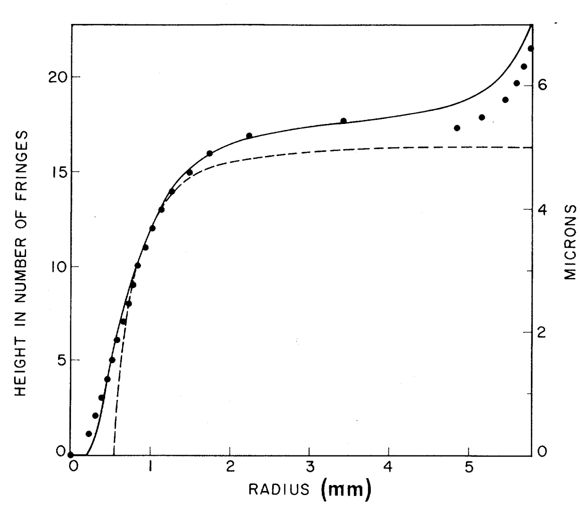

FIG. 2. Surface height above depression center from Fig. 1 (points). Solid curve is vortex fit with $N = 365$. Dashed curve is $\sigma = 0$ limit of Eq. (1) with $N = 365$.

$\zeta_0(0)$ similar but with $T = 2.16$ K ($\rho_s/\rho \approx 0.08$). A parabolic profile was usually formed by $t_2 + 90$ sec and always by $t_2 + 120$ sec. The depressions reported here differ from Andronikashvili's vortex[11] which has been observed in He I and He II with T near T_λ. (ii) Slowing the rate of spinup to ω_1 suppressed the formation of depressions with $T = 1.65-1.80$ K. Eleven cycles were observed with $\zeta_0(0)$, ω_1, and ω_2 similar but with $t_1 > 20$ sec so that $\dot{\omega} < 1$ sec^{-2} for $t < t_1$. A parabolic profile was formed by $t_2 + 120$ sec for all but two cycles and for some cycles by $t_2 + 7$ sec. (iii) Reflected-light intensity measurements indicated that the substrate when covered by a liquid film with $\zeta < 0.3$ μm was noticeably brighter than when completely bare. All parts of the substrate were covered at least by a film of liquid for the entirety of the $T = 1.65-1.80$ K cycles. Since the central region of the depressions was covered by a film, the existence of the depressions is due neither to complete localized evaporation of liquid, nor to poor wetting of the substrates.[5]

Creation of a large vortex may be due to many $N = 1$ vortices becoming pinned to the substrate in the region which eventually became the depression's center. With $t_1 < 2$ sec, ζ had not changed significantly by t_1 from its $t = 0$ value of $\geqslant 7$ μm. The corresponding superfluid critical *relative* velocity of approximately[12] 6 cm/sec was exceeded except near the center of the cylinder. For the cases (ii) with $t_1 > 20$ sec, however, ζ decreased gradually during spinup as liquid moved toward the walls. By t_1, $\zeta \ll 0.3$ μm over much of the substrate and the corresponding critical velocity of approximately 25 cm/sec was exceeded nowhere. The importance of linear critical velocities to rotation and vortex pinning in thin films was recently demonstrated.[13]

The observed decay of the depressions may be related to the dynmaics of the vapor which is omitted in (2) or to the stepwise decrease of N as singly quantized vortices become unpinned from the center and move outward. Thermally induced internal convection, if present, should have enhanced their outward motion.

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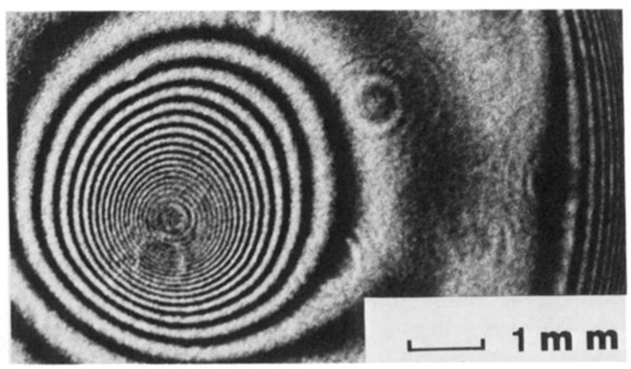

FIG. 1. Fringe pattern of a depression observed in a 7-μm-thick pool with $T = 1.68$ K.