

sults of $A_{\max} \approx 0.4-0.6$.^{11,12} In conclusion, reflectance measurements and plasma-simulation calculations exhibit similarities unexpected only a short time ago.

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Note added.—Since the original submission of this paper a related work has appeared.¹³

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Universal Jump in the Superfluid Density of Two-Dimensional Superfluids

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We observe that recent theories of phase transitions in the two-dimensional XY model predict a universal jump in the superfluid density of ⁴He films as T_c is approached from below. Specifically, we find that $\lim_{T \rightarrow T_c^-} \rho_s(T)/T = 3.52 \times 10^{-9} \text{ g/cm}^2 \cdot \text{K}$. Analogous results should hold for two-dimensional planar magnets and liquid crystals.

A number of theories have been advanced dealing with the critical properties of the classical two-dimensional XY model.¹⁻⁶ Although the lack of long-range order in such systems has been proven rigorously,⁷ evidence from high-temperature-series expansions⁸ suggests the possibility of a transition at finite temperature into a low-temperature phase without long-range order. Indeed, the theories of Refs. 1-6 all predict a low-temperature region which can be characterized as a "phase" of critical points with continuously variable critical exponents. Within this phase, order-parameter correlation functions are expected to fall off at large distances as power laws with temperature-dependent exponents.

There is considerable disagreement, however, on the predictions near the critical temperature T_c , above which correlations are expected to decay exponentially. Because *experimental* realizations of two-dimensional XY behavior may actually be available in ⁴He films,⁹ in planar magnets,¹⁰ and in liquid crystals,¹¹ it seems desirable to have a simple criterion for determining which of the theories, if any, are correct.

In this Letter, we point out that the theories advanced in Refs. 2b and 5 predict a *universal* jump in the superfluid density of ⁴He films. The size of the jump is related to the critical exponent $\eta(T_c)$, which governs the power-law decay of correlations at the critical temperature. This result

can be transcribed into analogous predictions for two-dimensional planar magnets and for liquid-crystal films. To the extent we understand Refs. 1, 3, and 4, these theories would lead to quite different predictions. Because many of the predictions of Wiegmann⁶ agree with the results of Kosterlitz^{2b} and José *et al.*,⁵ we expect that the same universal jump in $\rho_s(T)$ would result from

his theory.

A crucial feature of the theories of Kosterlitz and Thouless² and of José *et al.*⁵ is the mapping of a two-dimensional lattice of planar spins onto a neutral Coulomb gas. A Coulomb-gas problem is also suggested by the application of statistical mechanics to a superfluid ⁴He film. We assume that the relevant probability distribution for the superfluid velocity v_s is

$$P \propto \exp \left\{ - \int d^2r \left[\frac{1}{2} (\rho_0/k_B T) v_s^2 - m^2 \ln y_0 |\Delta \times \vec{v}_s|^2 / (2\pi\hbar)^2 \right] \right\}, \quad (1)$$

where, in addition to the usual kinetic-energy term, we have included a term proportional to the total squared vorticity in the fluid, or "enstrophy." As pointed out by Kraichnan,¹² conservation of "enstrophy" is a feature peculiar to two-dimensional hydrodynamics, and leads to the modified statistical-mechanical description implied by (1). The parameter $\ln y_0$ is the thermodynamically conjugate variable to the enstrophy, and was introduced in a rather *ad hoc* fashion for planar magnets in Ref. 5. We shall assume that y_0 is less than unity.

We now decompose the superfluid velocity field into a potential flow term and a part due to vortices:

$$\vec{v}_s(\vec{r}) = (\hbar/m) \nabla \varphi(\vec{r}) + (2\pi\hbar/m) (\hat{z} \times \nabla) \int d^2r' n(\vec{r}') G(\vec{r}, \vec{r}'), \quad (2)$$

where $\varphi(\vec{r})$ is a smoothly varying function, $n(\vec{r})$ is an integer-valued vorticity field, and m is the mass of a helium atom. The unit vector \hat{z} is perpendicular to the plane of the film, ∇ represents a three-dimensional gradient, and the Green's function $G(\vec{r}, \vec{r}')$ satisfies

$$[(\partial^2/\partial x^2) + (\partial^2/\partial y^2)] G(\vec{r}, \vec{r}') = \delta^{(2)}(\vec{r} - \vec{r}'). \quad (3)$$

Although we have used a convenient continuum notation, we shall, in fact, impose an ultraviolet cutoff, and require in particular that the logarithmic interaction between two vortices cease at separations smaller than some core radius a . On substituting (2) into (1), we find a probability distribution of the form $P \propto e^{\bar{H}}$, with

$$\bar{H} = -\frac{1}{2} K \int d^2r (\nabla \varphi)^2 + \pi K \iint_{|\vec{r}-\vec{r}'|>a} \frac{d^2r}{a^2} \frac{d^2r'}{a^2} n(\vec{r}) n(\vec{r}') \ln \frac{|\vec{r}-\vec{r}'|}{a} + \ln y \int \frac{d^2r}{a^2} n^2(\vec{r}), \quad (4)$$

where $K = \rho_0 \hbar^2 / m^2 k_B T$ and $y = y_0 e^{-CK}$. In deriving (4), we have approximated $G(\vec{r}, \vec{r}')$ by $(1/2\pi) \ln(|\vec{r}-\vec{r}'|/a) + C$, where C is a positive constant related to the core energy whose precise value is unimportant. The kinetic-energy term is finite, provided that we allow only those vorticity complexions with $\int d^2r n(\vec{r}) = 0$. Equation (4) is, of course, precisely the spin-wave Coulomb-gas Hamiltonian found for magnetic systems in Refs. 2 and 5. To calculate correlation functions with (4), one must average over smoothly varying velocity fields determined by $\varphi(\vec{r})$, and over distinct vorticity complexions determined by $n(\vec{r})$. At low temperatures (or small y_0), we expect that $n(\vec{r})$ will be zero nearly everywhere, and ± 1 at isolated points. The vortex-interaction part of (4) is just that proposed years ago by Onsager.¹³ We refer readers to Ref. 12 for a discussion of the relation between continuous and discrete vorticity in classical hydrodynamics.

We shall demonstrate the universality of $\rho_s(T)/T$ by calculating this quantity directly from (4). This will be done by using a renormalization transformation to relate a difficult calculation in the presence of vortices to a more tractable one in a vortex-free environment. The superfluid density can be expressed in terms of a correlation function^{13,14}:

$$K_R^{-1} \equiv m^2 k_B T / \hbar^2 \rho_s(T) = (m^2 / \hbar^2) \int d^2r \langle \vec{v}_s(\vec{r}) \cdot \vec{v}_s(\vec{0}) \rangle. \quad (5)$$

By substituting the velocity decomposition (2) into (5), we can easily derive a formal expression for K_R^{-1} as a power series in y :

$$K_R^{-1} = K^{-1} + 4\pi^3 y^2 \int_a^\infty (dr/a) (r/a)^{3-2\pi K} + O(y^4), \quad (6)$$

where we have made use of the relation⁵ $\langle n(\vec{0}) n(\vec{r}) \rangle = -2y^2 r^{-2\pi K} + O(y^4)$. To this order, only vortex pairs

with quantum numbers ± 1 need be taken into account. An expression similar to (6) was obtained in Ref. 5,¹⁵ where a very different (and much more complicated) correlation function was considered. In Ref. 5, it was shown that, by integrating over length scales between a_0 and $a_0 e^l$, the series (6) can be re-written,

$$K_R^{-1} = K^{-1}(l) + 4\pi^3 y^2(l) \int_a^\infty (dr/a)(r/a)^{3-2\pi K(l)} + O(y^4(l)), \quad (7)$$

where $K(l)$ and $y(l)$ obey differential recursion relations,^{2,5}

$$\begin{aligned} dK^{-1}(l)/dl &= 4\pi^3 y^2(l), \\ dy(l)/dl &= [2 - \pi K(l)]y(l), \end{aligned} \quad (8)$$

subject to the initial conditions $K(l=0)=K$ and $y(l=0)=y$. Comparing Eqs. (6) and (7), we arrive at an important relation between K_R calculated with the bare parameters K and y , and K_R calculated with the partially "dressed" quantities $K(l)$ and $y(l)$, namely,

$$K_R^{-1}(K, y) = K_R^{-1}(K(l), y(l)). \quad (9)$$

A direct determination of $K_R^{-1}(K, y)$ from the series (6) can be difficult, especially for $2\pi K \gtrsim 4$, where the integrals appearing in (6) are poorly convergent. Equation (9), however, relates such a calculation to one with altered parameters $K(l)$ and $y(l)$. For $T \leq T_c$, the differential equations (8) drive $y(l)$ to zero^{2,5}, i.e.,

$$\lim_{l \rightarrow \infty} y(l) = 0.$$

This suggests that (9) can be exploited below T_c by taking the limit $l \rightarrow \infty$,

$$\begin{aligned} K_R^{-1}(K, y) &= \lim_{l \rightarrow \infty} K_R^{-1}(K(l), y(l)) \\ &= \lim_{l \rightarrow \infty} K^{-1}(l) \quad (T \leq T_c), \end{aligned}$$

where we have made use of (7) with $y(l)=0$. The critical temperature is identified^{2b,5} with the largest temperature such that

$$\lim_{l \rightarrow \infty} y(l) = 0.$$

As T goes to T_c from below, it is easily shown from Eqs. (8) that

$$\lim_{T \rightarrow T_c^-} K_R^{-1} = \lim_{T \rightarrow T_c^-} m^2 k_B T / \hbar^2 \rho_s(T) = \frac{1}{2}\pi. \quad (10)$$

Thus, $m^2 k_B T / \hbar^2 \rho_s(T)$ approaches a *universal* constant as $T \rightarrow T_c^-$ —a constant which is independent of the initial values of K and y .

It is instructive to produce plots of $\rho_s(T)$ vs T for the Coulomb-gas model (4) by integrating the recursion relations (8) numerically until $y(l) \approx 0$. We have done this for different values of ρ_0 , which could be varied in an experiment by, say, changing the thickness of the ^4He film. Although

the model produces a series of flat curves without much structure in $\rho_s(T)$ below T_c , there is a characteristic square-root cusp¹⁶ as $T \rightarrow T_c^-$, which depresses $\rho_s(T_c)$ by about 10% of its value at $T=0$. Of course, in a real film, many other processes will tend to depress $\rho_s(T)$ below its $T=0$ value; however, we expect that an additional 10% effect due to the cusp will still be important. The various values of $\rho_s(T_c)$ and T_c all lie on a straight line through the origin with slope $2m^2 k_B / \hbar^2 \pi = 3.52 \times 10^{-9}$ g/cm² °K. We predict that this slope will be a universal constant for ^4He films despite the fact that $\rho_s(T_c)$ and T_c will depend on coverage, the details of the wall potential, etc. Although the amplitude of the square-root cusp is nonuniversal, we can estimate its size from our model calculations. Near T_c , we find $\rho_s(T)/T \approx [\rho_s(T_c)/T_c][1 + 0.50(1 - T/T_c)^{1/2} + O(T_c - T)]$. Presumably, $\rho_s(T)$ is zero above T_c .

The universal result (10) can be couched in terms of a prediction for the critical exponent $\eta(T)$ as $T \rightarrow T_c^-$. Since it is an immediate consequence of the theories of Refs. 2b and 5 that

$$\eta(T) = \lim_{l \rightarrow \infty} [1/2\pi K(l)],$$

we have $\eta(T) = m^2 k_B T / 2\pi \hbar^2 \rho_s(T)$, which gives $\eta = \frac{1}{4}$ at T_c . (There are actually logarithmic corrections to the power-law decay of correlations at T_c ,^{2b,6} but these need not concern us here.) An identical relation between η and ρ_s appears to us to be a consequence of the theories of Refs. 1, 3, and 4. Since Zittartz³ predicts transition at T_c 's such that $\eta(T_c) = 2$ and 4, and Luther and Scalapino⁴ predict a nonuniversal $\eta(T_c)$ [in particular, $\eta(T_c)$ can assume the value $1/\sqrt{8}$], precision experimental measurements of $\rho_s(T)$ in ^4He films could presumably decide between the various theories. Berezinskii¹ predicts that $\rho_s(T)$ vanishes at T_c , which can also be checked experimentally. Measurements of third-sound propagation in films¹⁷ are not inconsistent¹⁶ with the prediction (10).

Predictions for the spin-wave stiffness in two-dimensional magnetic systems analogous to (10) follow by making the standard analogy between XY magnetism and superfluidity.¹⁸ Two-dimensional nematic crystals can be described by a

free-energy density of the form¹¹

$$F = \frac{1}{2}K_1(T)[\nabla \cdot \vec{n}(\vec{r})]^2 + \frac{1}{2}K_3(T)|\vec{n}(\vec{r}) \times [\nabla \times \vec{n}(\vec{r})]|^2, \quad (11)$$

where $\vec{n}(\vec{r})$ is a two-dimensional director field. The K_2 Franck coefficient corresponding to "twist" distortions is absent for $d=2$. A renormalization-group analysis by Nelson and Pelcovits¹⁹ has shown that deviations from the one-Franck-constant approximation $K_1=K_3$ are irrelevant variables in the low-temperature XY-model phase characterized by power-law decay of correlations. In this limit, Eq. (11) reduces to a continuum version of the two-dimensional XY model.¹¹ Taking over the analysis for superfluids, we find

$$\lim_{T \rightarrow T_c^-} K_i(T)/k_B T = 2/\pi, \quad i=1,3.$$

De Gennes¹¹ has suggested that a nematic film floating on the surface of a fluid may give a good realization of a liquid-crystal system described by (11). "Soap-bubble" films of a small number of smectic liquid-crystal layers may also provide such an experimental realization.²⁰

Although we have demonstrated explicitly only that $\rho_s(T)/T$ should approach a universal constant for the model defined by (4), we believe our conclusions remain unchanged when other irrelevant variables are taken into account. Indeed, if we denote by $\{y_i\}$ a collection of additional perturbations representing, say, vortices with quantum numbers $\pm 2, \pm 3, \dots$, or interactions between vortices and the field $\varphi(\vec{r})$, it seems clear that the renormalization groups constructed in Refs. 2b and 5 would lead to a simple generalization of (9): $K_R^{-1}(K, y, \{y_i\}) = K_R^{-1}(K(l), y(l), \{y_i(l)\})$. In d dimensions this result reads $K_R(K, y, \{y_i\}) = e^{(2-d)l} K_R(K(l), y(l), \{y_i(l)\})$, which is just the Josephson scaling relation²¹ in disguise. It is easy to show using the methods of Refs. 2 and 5 that the perturbations $\{y_i(l)\}$ as well as $y(l)$ all tend to zero as $l \rightarrow \infty$ in the low-temperature phase of the two-dimensional XY model. An analysis in the limit $l \rightarrow \infty$ then leads immediately to the universal result (10).

It is worth noting that the universal (minimum) value of $\rho_s(T)/T$ which is approached as $T \rightarrow T_c^-$ bears a striking resemblance to the predicted²² minimum conductivity in two-dimensional metals.

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Unusual Phonon Heat Transport in an Amorphous Superconducting *PbCu* Film

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We report thermal conductivity measurements of a $\text{Pb}_{0.9}\text{Cu}_{0.1}$ film before (amorphous) and after (crystalline) annealing. Below T_c , where electronic effects are negligible, we found that the phonon contribution in the amorphous phase $k_{\text{am}}^{\text{ph}}$ was much larger than that in the crystalline phase. We also found that the temperature dependence of $k_{\text{am}}^{\text{ph}}$ resembles that of amorphous dielectrics, thus providing evidence that this sort of "glassy" behavior does not require the presence of covalent bonds.

Amorphous insulators like glasses, polymers, and α -Se show distinctive thermal properties below 1 K. For these substances contributions to the specific heat proportional to T and T^3 are observed in addition to the Debye term.¹ Furthermore, a T^2 variation of the (phonon) thermal conductivity is found in contrast to the T^3 dependence seen in crystals.¹ This "glassy" behavior, also visible in the k vs T curves of semiconducting α -Ge,² can be successfully interpreted in terms of localized low-energy excitations (LEE) as assumed in the Anderson-Halperin-Varma-Phillips tunneling model.^{3,4} To date, however, this model lacks a microscopic explanation. Thus, since all the mentioned materials are covalently bonded, it is not clear whether the amorphous structure itself or the presence of covalent bonds in a disordered environment is essential for "glassy" behavior.

Amorphous metals, of course, are the natural probe for such an investigation. Very recently, a phonon thermal conductivity $k^{\text{ph}} \sim T^2$ was reported for several bulk metallic glasses.⁵ However, all of these materials, most of which contain metalloids with covalent bonds, are in the normal state (including $\text{Ti}_{0.5}\text{Be}_{0.4}\text{Zr}_{0.1}$ which becomes superconducting only below 0.32 K). In addition, they all contain transition metals which can act as resonance scattering centers of the conduction electrons. Therefore, the influence of the conduction electrons (as heat carriers as well as phonon scatters) is not easy to calculate, and as a result conclusions drawn for k^{ph} are, at most, only of qualitative value. To avoid these difficulties, an amorphous superconducting $\text{Pb}_{0.9}\text{Cu}_{0.1}$ (*PbCu*) film, prepared by quench condensation

from the vapor phase, was chosen for the present investigation: Well below $T_c = 6.5$ K, the influence of the few remaining electronic quasiparticles on k is negligible. Besides, in *PbCu* there are no constituents with covalent bonds.

Another motivation for thermal conductivity measurements on amorphous metal films stems from the fact that in quench-condensed *crystalline* metal films k^{ph} is almost negligible as compared to the electronic contribution k^e .^{6,7} It has been argued⁶ that in this case the phonons are scattered strongly from spatially extended lattice defects, i.e., stacking faults, dislocations, and grain boundaries. However, since these defects should be absent in a solid without long-range order, we hoped to observe a substantial recovery of the ratio k^{ph}/k^e in amorphous (as compared to crystalline) *PbCu*. This should enable us to use k^{ph} as a probe of "glassy" behavior.

The amorphous film was prepared by means of the *Drehrohrofen* technique developed by Hilsch and Martienssen⁸: A mechanically homogenized 90-at.% Pb, 10-at.% Cu mixture was evaporated in small pellets (each producing less than one atomic layer) onto a cooled substrate. As substrate we used a 12.5- μm -thick circular Kapton foil, which sealed an additional Cu tank clamped to the cold part of a He³ cryostat.⁹ During evaporation of the metal film, this Cu tank was filled with He⁴ exchange gas (5 Torr). Thus the substrate temperature could be kept below 15 K, i.e., well below the crystallization temperature (30 K) of *PbCu*. The pressure in the cryostat remained below 10^{-6} Torr during evaporation. Afterwards the He⁴ exchange gas was pumped out and the total thermal conductance K_T of the sub-