Coulomb Displacement Energies and Neutron-Proton Density Distributions

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We call attention to the fact that an accumulation of recent data concerning r_n, the rms radius of neutron density distributions, in neutron-rich nuclei indicate that values of $\Delta r = r_n - r_p$ are considerably smaller than those obtained from simple shell-model prescriptions or predicted by common Hartree-Fock calculations. We show that values of Δr consistent with these data can explain the long-standing Coulomb energy problem (Nolen-Schiffer anomaly).

In recent years there has been an accumulation of experimental evidence concerning values of r_n and r_{ex}, the root-mean-square radii of neutron and excess-neutron distributions, respectively, in nuclei. Whereas good agreement is obtained between experiment and theory for r_p, the rms radius of proton distributions, discrepancies have been observed between experiment and theory regarding neutron radii. In this Letter we show that very recent experimental results fit into a rather consistent picture and that the problem with neutron radii is linked with the Coulomb energy anomaly.

The experimental results regarding neutron radii can be summarized as follows: (1) Experimental values of $\Delta r = r_n - r_p$ in neutron-rich nuclei are found to be significantly smaller than predictions. Table I summarizes results of recent experiments[1-8] concerning Δr in ^{48}Ca and ^{208}Pb. These results show that $\Delta r \sim 0.1$ fm for these nuclei compared to predictions of Hartree-Fock (HF) calculations[9,10] of $\Delta r \sim 0.2$ fm [the simple shell-model (SM) value is $\Delta r \sim 0.3$ fm]. Also, the very recent analysis[11] of the scattering of 1-GeV protons by $^{58-64}$Ni leads to $\Delta r \sim 0.0$ fm for these nuclei. It is emphasized that, although these results involve the strong interaction and some model is required for the analysis, the large degree of consistency between the widely different methods support the conclusion that Δr is small. Also, in several experiments, ratios of cross sections were measured which serve to reduce the model dependence of the results. (2) Very recent analysis[12] of large-angle electron scattering on ^{87}Sr (odd neutron) and ^{93}Nb (odd proton) implies that the rms radius of the $1g_{1/2}$ neutron orbit is significantly smaller (by ~ 0.2 fm) than that of the corresponding proton orbit, in disagreement with HF predictions. This result, obtained using an electromagnetic probe, is in a qualitative agreement with the previous results, yielding a neutron radius smaller than HF predictions. (3) For several cases,[13] such as the isotopes of Cr, Sn, and Fe, the HF predictions for

TABLE I. Summary of recent experimental values for $\Delta r = r_n - r_p$ in ^{48}Ca and ^{208}Pb.

Nucleus	$\Delta r = r_n - r_p$ (fm)	Method	Ref.
^{48}Ca	0.08 ± 0.05	$\pi^{\pm}$ total cross section (90–240 MeV)	1
	0.12 ± 0.05	p elastic scattering (1 GeV)	2
	0.03 ± 0.08	α elastic scattering (79 MeV)	3
	0.17 ± 0.05	α scattering (1.37 GeV)	4
^{208}Pb	0.0 ± 0.1	π^- reaction cross section (20–60 GeV)	5
	-0.05 ± 0.1	$\pi^{\pm}$ reaction cross section (1–2 GeV)	6
	-0.05 ± 0.05	p elastic scattering (1 GeV)	2
	0.0 ± 0.1	α scattering (104 MeV)	7
	0.0 ± 0.1	bremsstrahlung-weighted cross section	8

the isotope shifts of r_p tend to be smaller than the experimental values (by ~30%). (4) Values of r_{ex} were deduced recently from analyses of (i) electron scattering on isotones, isotopes, and other neighboring nuclei,[12,14-16] and (ii) sub-Coulomb nucleon transfer reactions.[17-19] The values deduced for r_{ex} are significantly larger than the corresponding values of r_p. For example, $r_{ex} \sim 4.0$ and 6.1 fm for ^{48}Ca and ^{208}Pb as compared to $r_p \sim 3.5$ and 5.5 fm, respectively. We note that the experimental values of r_{ex} seem to be somewhat smaller than the corresponding HF predictions (see also Ref. 12).

In this Letter we suggest that the above discrepancies between experiment and HF predictions are intimately related to another discrepancy between experiment and HF predictions, namely the long-standing problem in Coulomb displacement energies, ΔE_c. The problem is observed when the simple SM picture is adopted and the expression,

$$\Delta E_c^{\text{dir}} = [e/(N-Z)] \int V_c(\vec{r}) \rho_{ex}(\vec{r})\, d^3r \qquad (1)$$

is used for the direct term of ΔE_c. $V_c(\vec{r})$ is the Coulomb potential due to the Z protons of the core and

$$\rho_{ex}(\vec{r}) = \rho_n(\vec{r}) - \rho_{nc}(\vec{r}), \qquad (2)$$

where $\rho_n(\vec{r})$ is the density distribution of all neutrons and $\rho_{nc}(\vec{r})$ is that of the Z neutrons of the core. Nolen and Schiffer[20] have used (1), including the exchange and the electromagnetic spin-orbit terms, to determine r_{ex}, the rms radius of ρ_{ex}. They found that in order to reproduce experimental values of ΔE_c, the values of r_{ex} had to be rather close to the corresponding values of r_p, in disagreement with theory (and with later experiments[17-19]). For example, in order to reproduce the value of $\Delta E_c = 7.28$ MeV for the ^{41}Sc-^{41}Ca ground states, r_{ex} for the $1f_{7/2}$ orbit must be as small as r_p, ~3.5 fm. The problem persists in more sophisticated calculations.[10,21-23] Calculated values of ΔE_c are smaller than experimental ones by ~7% over a wide range of nuclei (see Ref. 23 for details). It has been shown[22] that two-body charge-symmetry-breaking (CSB) interactions, which are compatible with the contributions of charge-asymmetric meson-exchange processes, can account for the discrepancy[24] of ~100 keV in the ^3He-^3H case but only for less than half of the discrepancy for heavier nuclei. Corrections to wave functions, such as those due to two-body short-range correlation and isospin mixing in the core,[25] also produce values which are too small to resolve the anomaly.[10,26] We add, however, that the uncertainties in these estimations are quite large[23] because of the lack of knowledge of the nuclear response and of the effective interaction.

In order to demonstrate that the origin of the Coulomb-energy anomaly is directly related to the difficulties with HF calculations regarding values of Δr, we recall that Talmi and Shlomo have shown[14] that if one *assumes* that for any nucleus

$$r_n = r_p \quad (\Delta r = 0), \qquad (3)$$

then for any reasonable value of r_{ex}, such that $r_{ex} > r_p$, there exists an additional term in the Coulomb displacement energy due to the compression of the Z "core" protons in the analog state *relative* to the parent state (for a constant mass radius). The energy associated with this core compression can be approximated, using uniform charge distributions, by

$$\Delta E_{cc} = (\tfrac{3}{5})^{3/2} e^2 [Z(Z-1)/r_p](\Delta r_{cc}/r_p), \qquad (4)$$

with $\Delta r_{cc} = r_p - r_p'$, where r_p' are the rms radii of the density distributions of the Z "core" protons of the parent and the analog states, respectively. Note that the term multiplying $\Delta r_{cc}/r_p$ in (4) is of the order of 70 MeV for $Z = 20$. Consider for example the case of ^{41}Sc-^{41}Ca. Assuming that the valence $1f_{7/2}$ proton in ^{41}Sc has $r_{ex} \sim 4.2$ fm (the SM value), we observe that the twenty protons in the core of ^{41}Sc are compressed (relative to ^{41}Ca) by $\Delta r_{cc} \sim 0.04$ fm. This corresponds to $\Delta E_{cc} \sim 0.8$ MeV which is large enough to explain the discrepancy of 0.6 MeV. Although (1) and (4) separately depend on the value of r_{ex}, their sum was found to depend *only* on $r_n - r_p$, and *not* on r_{ex}.

It may well be that the above assumption of $r_n = r_p$ does not hold exactly for all nuclei. However, the recent data (Table I) show that values of Δr are indeed significantly smaller than theoretical predictions. Detailed calculations within the energy-density formalism show[27] that a rather unique relationship exists between values of ΔE_c and Δr which leads to agreement with experimental results for ΔE_c if the values of Δr are sufficiently small (i.e., $\Delta r \approx 0.05$ fm for ^{48}Ca and ^{208}Pb). The magnitude of the relative core-compression effect depends on the values of Δr and r_{ex}; it increases with decreasing Δr and with increasing r_{ex}. Considering, for example, the ^{40}Ca and ^{48}Ca nuclei and using the fact that their proton rms radii are equal to within 0.1 fm, it is found that the twenty neutrons in the core of

^{48}Ca are compressed relative to those of ^{40}Ca by more than 0.1 fm, if one assumes that $\Delta r < 0.1$ fm and $r_{ex} > 4.0$ fm for ^{48}Ca. By symmetry, there is a proton core compression in the analog state which is roughly $(N-Z)$ times smaller, and detailed calculations show[27] that it is sufficient to resolve the Coulomb energy anomaly. An important result[27] is that ΔE_c is independent of r_{ex}. In a more formal way, ΔE_c is given[28] (except for the Thomas-Ehrmann term) by

$$\Delta E_c = E_A - E_\pi = N_1^{-1}\{\langle\pi|[T_+,[H,T_-]]|\pi\rangle + \langle\pi|[H,T_-]T_+|\pi\rangle\}, \quad (5)$$

where $N_1 = \langle\pi|T_+T_-|\pi\rangle$. For a parent state $|\pi\rangle$ of a pure isospin, $N_1 = 2T = N-Z$ and the second term in (5) vanishes. If $|\pi\rangle$ is approximated by a Slater determinant the first term in (5) can be written as a sum of direct plus exchange Coulomb terms, where the direct term is given by

$$\Delta E_c^{\,\mathrm{dir}} = [e/(N-Z)]\int V_c(\vec{r})\Delta\rho(\vec{r})\,d^3r, \quad (6)$$

with

$$\Delta\rho(\vec{r}) = \rho_n(\vec{r}) - \rho_p(\vec{r}). \quad (7)$$

For sufficiently small values of Δr the rms radius of $\Delta\rho$ becomes smaller than r_{ex}. Consequently, the direct term (6) becomes larger than (1). This suggests that the Coulomb energy anomaly can be attributed to the approximation $\Delta\rho(\vec{r}) = \rho_{ex}(\vec{r})$ [or $\rho_{nc}(\vec{r}) = \rho_p(\vec{r})$], adopted in previous calculations.

In a microscopic picture, the analog state $|A\rangle$ is obtained from the parent state $|\pi\rangle$ by replacing a valence neutron by a proton without perturbing the core. Obviously the relative core compression is a rearrangement type of effect. Using a certain single-particle basis for both states the wave function $|\pi\rangle$ will include relatively more excited protons than neutrons (and vice versa for $|A\rangle$). When sufficiently high orbits are included, the core-compression effect results from reasonable amounts of configuration mixing.

In conclusion, although the Coulomb energy anomaly for the ^{3}He-^{3}H case can be explained by CSB in the nuclear forces, this effect can explain less than half of the anomaly in heavier nuclei. The recent accumulation of experimental results regarding values of $\Delta r = r_n - r_p$ in neutron-rich nuclei suggests that rhe remaining part of the Coulomb energy anomaly results from the approximations adopted for the wave functions. It will be interesting to see if the Coulomb energy anomaly disappears from results of HF calculations which are made to reproduce the small experimental values of Δr. This will require the introduction of a residual attractive effective $(V_{np}-V_{nn})$ interaction.

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Evidence for the Excitation of Giant Resonances in Heavy-Ion Inelastic Scattering

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A study of the inelastic scattering of ^{12}C from ^{27}Al at 82 MeV, a bombarding energy at which deep-inelastic collisions occur, suggests the population of giant resonances in the target. This result supports the suggestion of Broglia, Dasso, and Winther regarding the mechanism of deep-inelastic collisions.

It has been proposed[1] that the excitation of giant resonances in the target and projectile may play an important role in the mechanism of deep-inelastic heavy-ion collisions. The suggested means by which kinetic energy of relative motion is converted into excitation energy involves the multiple excitation of those degrees of freedom which are strongly coupled to the target and projectile ground states (i.e., the giant resonances). These degrees of freedom may therefore be considered as "doorways" leading to deep inelastic events and thus provide a link between quasielastic and deep-inelastic collisions.

The probability that either fragment will emerge in a single giant resonance depends on the system being considered. For heavy systems,[2] such as Kr + Pb, the large energy loss observed implies a dominance of multiple excitation of the giant resonances which suggests that it is extremely unlikely that either fragment will actually emerge in a single, and therefore identifiable, giant resonance. We do not therefore expect the direct observation of giant resonances in deep-inelastic scattering of heavy systems. For light systems, however, the situation is somewhat simpler. Shorter collision times and higher excitation energies for the giant resonances combine to make it more likely that the fragments separate with one actually still in a giant resonance. These conclusions are supported by the experimental results for systems like $^{16}O + ^{27}Al$, $^{12}C + ^{27}Al$, etc., where the energy loss observed in deep-inelastic collisions leads to excitation energies more characteristic of single excitation of the giant resonances in the target and projectile than of multiple excitation.[3,4] The question then is whether or not the giant resonances are, in fact, selectively populated over the continuum background as implied in the model of Ref. 1.

With the above points in mind, we have studied the inelastic scattering of ^{12}C from ^{27}Al at a bombarding energy of 82 MeV. At this bombarding energy, deep-inelastic collisions form a large fraction of the total reaction cross section[3] ($\sigma_{\text{react}} \approx 1700$ mb, $\sigma_{\text{fusion}} = 1000$ mb), and earlier results[3] show that inelastic scattering of beam particles forms the major fraction of the reaction products. This Letter reports the possible observation of two giant resonances in ^{27}Al thus providing evidence in favor of the suggestion of Ref. 1. It is also, to the authors' knowledge, the first time heavy ions have been used to excite giant resonances in any nucleus.

A 400-μg/cm^2 99.7%-pure Al target was bombarded with a beam of 82-MeV ^{12}C ions from the Yale MP tandem Van de Graaff accelerator. The reaction products were detected in a ΔE-E telescope consisting of two silicon surface-barrier detectors (25 and 300 μm). This detector system was easily able to separate the different isotopes of C, and the overall energy resolution was 300 keV. Carbon build-up on the target was reduced to a negligible amount by placing a Cu plate cooled to liquid-nitrogen temperature in the close vicinity of the target. The absence of carbon on the Al was further checked by frequent runs with a carbon target to provide comparison spectra. Data were obtained in 1° steps from 15° to 25° and 5° steps thereafter to 40°.

Spectra for the reaction $^{27}Al(^{12}C, ^{12}C')$ obtained at angles of 15°, 23°, and 30° are shown in Fig. 1. The raw data have been converted to a Q-value scale thus enabling a direct comparison of the three spectra. Transitions to the low-lying qua-