Maximum Metallic Resistance in Thin Wires

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It is argued that electronic states should be localized in any wire whose impurity resistance is greater than about 10 kΩ. At sufficiently low temperatures this will lead to a T^{-2} increase in resistance because one-dimensional phonons or excited electrons are needed to cause transitions between localized states. An estimate is made of the temperature needed to observe this effect.

Mott and Twose[1] showed that electrons moving in a one-dimensional static potential would be in localized states if there were any disorder in the potential. This has been shown with greater rigor by later workers (see the review by Ishii[2]) and there is no doubt that it implies that the electrons can only transport electric current if energy is available to enable them to hop from one localized state to another; as a result the electrical conductivity goes exponentially to zero as the temperature goes to zero. It is also known that for the case of weak disorder the length of the localized state is identical to the quantity which would normally be calculated for the mean free path for backward scattering.[3] The proofs of this result have all been restricted to electrons moving in a strictly one-dimensional potential (or to equivalent tight-binding models), but it has seemed likely that the result that all states are localized is more general. An argument to show that any system of a given cross section will eventually behave one-dimensionally, with localized states only, if it becomes sufficiently long was given by Yuval.[4] This argument is similar to the one used by Edwards and Thouless[5] and subsequently used by Licciardello and Thouless[6] to discuss the minimum metallic conductivity in two-dimensional systems. In this Letter the argument is developed further for one-dimensional systems. It is shown that once the zero-temperature resistance of a wire exceeds a critical value, which is likely to be 10 or 20 kΩ, the resistance should increase exponentially with length instead of linearly. At higher temperatures the localization of electron states will not be apparent because the phonons will cause electrons to hop from one state to another, but as the temperature is lowered the conductance should drop in a characteristic fashion—initially as a power of the temperature. Estimates are made of conditions under which this effect might be experimentally observable.

Consider a metallic wire of length L and cross-sectional area A. In this system suppose that the electrons move in a potential which is macroscopically uniform but which has sufficient irregularity to give the conductivity of the bulk metal a finite value σ at zero temperature. The density of electron states is dn/dE per unit volume per unit energy. The individual energy levels are sensitive to the boundary conditions applied at the ends of the wire, and can be shifted by an amount of order $\hbar/t$, where t is the time it takes for an electron to diffuse to the end of the wire. This time is equal to L^2/D, where D is the diffusion constant for an electron, and this can be related to the conductivity by the Einstein relation for a degenerate electron gas $\sigma = \frac{1}{2}e^2 D\, dn/dE$. Combining these two we get

$$\frac{\hbar}{t} = \frac{\hbar}{e^2}\,\frac{2\sigma}{L^2}\,\frac{dE}{dn} = \frac{\hbar}{e^2}\,\frac{2}{r}\,\frac{dE}{dN}, \tag{1}$$

where r is the resistance of the wire and dE/dN is the average spacing between energy levels.

This relation between resistance, density of states, and the strength of coupling to the boundary can be obtained in another way. If the wire is connected between two metallic reservoirs with a high density of degenerate electrons held at different potentials, current will flow by means of the $eV\,dN/dE$ levels in the wire lying between the Fermi levels at the two ends, and each level on the average carries a current of order e/t.

Now consider what happens when two similar (but not identical) wires of length L are joined end to end. There is a matrix element of order $\hbar/t$ connecting an energy level in one wire to any level in the other wire. When $\hbar/t$ is greater than the spacing between levels, the energy levels of the combined system will be some complicated combination of energy levels of the two individual subsystems. Under these conditions it is not implausible that $\hbar/t$ for the combined system should be a quarter of its value for each subsystem; but it follows from Eq. (1) if the resistance of the two

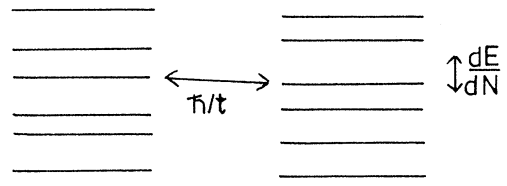

FIG. 1. Energy levels on two adjacent similar blocks of material. The spacing between energy levels is dE/dN and the coupling between levels in the two blocks is $\hbar/t$ where t is given by Eq. (1).

wires in series is the sum of their individual resistances. If, on the other hand, $\hbar/t$ is much less than the spacing between levels the energy levels of the combined system are only slightly perturbed from the levels of the indiviual subsystems. Figure 1 shows how the levels in the two subsystems, which will be uncorrelated between one subsystem and the other, may be arranged. A typical spacing between a level on one subsystem and the nearest level with the same spin direction on the other subsystem is dE/dN. If this is greater than $\hbar/t$ we can expect the levels in a long system made up of many such subsystems arranged in a line to be obtainable by perturbation theory from a level on a single subsystem, and so to be exponentially localized with a range no greater than the length of a single subsystem. From Eq. (1) the condition necessary for this argument to go through is

$$r \gg 2\hbar/e^2. \tag{2}$$

Since $\hbar/e^2$ has a value of about 4 kΩ we conclude that above a resistance of 10 kΩ or so a wire at zero temperature will have a resistance that increases exponentially instead of linearly with its length.

This result of course conflicts with common experience, and we must examine what conditions are necessary to observe this effect. At high temperatures the localization of the electron eigenstates is of no practical importance, since phonons will cause transitions between the localized states long before the electrons can travel a distance comparable with the localization length. Under these circumstances one can describe the motion of an electron in terms of a wave packet made up of localized wave functions much larger than the size of the wave packet, which will then diffuse until the emission or absorption of a phonon causes a transition to another wave packet. Then the mobility of an electron in the wire will be essentially the same as the mobility in the bulk metal, since the diffusion of the electron over distances of the order of the distance between phonon-scattering events will not be affected by the existence of a much longer localization length. As the temperature is lowered the scattering by phonons becomes less effective and eventually electrons are able to diffuse a distance of the order of the localization distance between phonon-scattering events. The size of the localized states is given by the length of wire for which the resistance is of order $2\hbar/e^2$. If we take the standard formula for the conductivity of an electron gas

$$\sigma = ne^2\tau/m = (e^2/\hbar)(k_F^2\lambda/3\pi^2), \tag{3}$$

where λ is the mean free path and k_F is the wave number of an electron at the Fermi surface, we find that the size of a localized state is given by

$$L = 2Ak_F^2\lambda/3\pi^2, \tag{4}$$

and the time taken for an electron to diffuse over this distance is

$$t = (2/3\pi^4)A^2k_F^4\tau. \tag{5}$$

Therefore the condition for the localization of the eigenstates to be apparent is approximately

$$\tau_{ph} > (2A^2k_F^4/3\pi^4)\tau. \tag{6}$$

The ratio of τ to τ_{ph} at room temperature is $\Gamma - 1$, where Γ is the resistance ratio, and the low-temperature behavior of τ_{ph} can be estimated on the basis of models of electron-phonon scattering. In a clean bulk material τ_{ph} increases as T^{-3} at low temperatures, but if $q\lambda \ll 1$, where $q = k_BT/\hbar c$ is the wave number of thermal phonons, then it increases as T^{-4}. An estimate of τ_{ph} under these conditions gives

$$1/\tau_{ph} = 44(\Gamma - 1)\alpha q^4\lambda k_F^{-3}\tau^{-1}, \tag{7}$$

where α is the ratio of room temperature to the Debye temperature. For a clean material this formula must be divided by $2q\lambda$, and if Aq^2 is less than unity the phonons are one-dimensional and the formula should be divided by Aq^2. It is also necessary to consider electron-electron scattering, which gives a relaxation rate of the form

$$1/\tau_{ee} = 2m\beta(k_BT)^2/\hbar^3k_F^2; \tag{8}$$

β is a numerical constant which is probably not very far from unity.[7] Clearly a high level of impurity scattering (low resistance ratio) will make the localization easier to observe, and a low resistance ratio is almost inevitable for a very thin wire. If we substitute Eq. (7) in (6) we find that

the temperature at which localization is apparent is inversely proportional to $A^{1/2}$, or the diameter of the wire. For reasonable values of the parameters this makes Aq^2 greater than unity, but not large, so a one-dimensional–phonon behavior sets in at a slightly lower temperature. When electron-electron scattering predominates τ_{ph} in Eq. (6) should be replaced by τ_{ee} as given in Eq. (8), and then the temperature is inversely proportional to A. Below this temperature the electrons are free to diffuse over the distance L, but can then go no further until a phonon or another electron causes a transition to a new wave packet of states. In this temperature range the resistance will increase as T^{-2}, whether the transitions are caused by electrons or one-dimensional phonons. Finally we may be concerned with temperatures so low that $k_B T$ is of the order of the spacing between overlapping localized states, and below such temperatures we should describe the conduction as activated hopping between localized states; in one dimension[8] the resistance for this process increases exponentially with T^{-1}.

If we consider a system with a cross-sectional area of 2.5×10^{-15} m^2, an impurity mean free path of 0.5×10^{-9} m, and $k_F = 1.2\times10^{10}$ m^{-1}, then the length of localized states given by Eq. (4) is about 0.012 mm. With $\Gamma - 1$ equal to 0.01 and the sound velocity c equal to 4000 m/s, the condition (6) is satisfied at about 1.7 K, while if τ_{ph} is replaced by τ_{ee} the condition is satisfied at about 1.2 K. In this example the electron-electron scattering is slightly more important, and so the T^{-2} increase in resistance should occur below about 1.2 K. The energy spacing between overlapping localized states is of order $5\times10^{-5}k_B$ under these conditions, so that the exponentially rising resistance should be observed in the region of 50 μK.

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Electron-Hole Liquid and Biexciton Pocket in AgBr

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A new emission band in AgBr is attributed to electron-hole–liquid recombination. Excitonic molecules appears only at higher excitation and temperature, corresponding to a limited region of the (T, n) plane called biexciton pocket.

The occurrence of a liquid phase of degenerate electron-hole plasma (EHL) is now well established in Ge,[1] Si,[1] and GaP.[2] The properties of the liquid are in good agreement with theory, provided the detailed band parameters of these semiconductors (in particular their indirect–band-transition structure) is properly taken into account. Recently, it has been shown by Keldysh and Silin,[3] and Beni and Rice,[4] that the interaction between free carriers and the lattice in polar materials increases the stability of EHL, and makes it possible to observe it even in direct-gap compounds. Experiments performed in CdS[5,6] and CdSe[6] confirm these predictions. The values of carriers density n and liquid binding energy φ are here also in fair agreement with calculations.

In this Letter, we present experimental evidence for the existence of EHL in AgBr. However, in contrast to the other cases mentioned above, the binding energy of this strongly polar compound with indirect-gap structure is significantly larger than derived from the theory.[4] This makes it possible to observe the liquid with temperatures in excess of 100 K and even suggests a much higher critical temperature of the EHL.

But AgBr also exhibits another novel character-