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Soliton Generation at Resonance and Density Modification in Laser-Irradiated Plasmas

Hsing-Hen Chen and Chuan-Sheng Liu
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 29 June 1977)

Analytic results are obtained for generation of solitons and their effects on density-profile modifications at critical density for resonance absorption process.

For a laser beam of frequency ω_0 obliquely incident onto a nonuniform plasma slab with polarization in the plane of incidence, there is a component of the electric field along the density gradient driving the density oscillation. Because of Budden tunneling, the residual electric field beyond the cutoff can drive the plasma wave at resonance where $\omega_p(x)=\omega_0$. This transformation of the electromagnetic wave into electrostatic waves which are subsequently absorbed by the plasma particles constitutes an important absorption mechanism of the laser radiation.[1]

In the usual linear theory, a fixed density profile is assumed and the modification of the density profile by the large-amplitude plasma wave generated at resonance is neglected.[2] In reality, even for a relatively weak pump wave, the resonantly driven plasma wave can significantly modify the density profile near the critical density region by the ponderomotive force.[3-6] Because of the localized structure of the field near the resonance, the ponderomotive pressure tends to drive the plasma out of the resonance region, thus depleting the local plasma density, forming a caviton (soliton). The successive generation of solitons and their subsequent downward motion along the density gradient results in a quasistationary density step with steep density gradient.[3,4,6] The formation of this sharp density step can stabilize many parametric processes,[3,4] affecting the absorption and scattering of laser light

In this Letter, we present an analytic theory of these nonlinear processes of soliton generations and profile modifications at the resonance. Threshold conditions for N-soliton formation are obtained. A closely related problem is the saturation of the linearly transformed wave. In the linear theory, the thermal convection of the plasma wave and its subsequent Landau damping provides the saturation after a time $t_c \approx (L/\lambda_D)^{2/3}\omega_{pe}^{-1}$, where L is the initial (unmodified) density scale length and $\lambda_D=(T/4\pi ne^2)^{1/2}$ is the Debye length. Because the soliton generation involves ion motion which takes place after several ion plasma periods $[\omega_{pi}^{-1} = (M/4\pi ne^2)^{1/2}]$, it occurs after the convective saturation if $(L/\lambda_D)^{2/3} < (M/m)^{1/2}$ $(\omega_{pi}t_c < 1)$ and the saturated wave can thus be used as the initial condition for determining how may solitons can be generated as well as the final scale length of the modified density profile. On the other hand, for $(L/\lambda_D)^{2/3} > (M/m)^{1/2}$, solitons are generated during the growth phase and contribute dominantly to the saturation process by detuning the resonance. Excluded from the present discussion is the wave breaking as a nonlinear saturation process which occurs in a timescale $t_B \approx [(L/\lambda_D)4(\pi nT)^{1/2}/E_d]^{1/2}\omega_{pe}^{-1}$, where ω_{pe} is the electron plasma frequency, $E_d \approx E_0\varphi(s)/(2\pi k_0 L)^{1/2}$, with E_0 and k_0, respectively, the amplitude and wave number of the laser light; $\varphi(s)$ will be defined after Eq. (1). Thus the present analysis is valid for relatively long initial scale length and not-so-strong pump wave so that $t_B \gg$ the smaller of t_c and ω_{pi}^{-1}, which is, for $t_c\omega_{pi} < 1$,

$$(|E_0|^2/16\pi nT)\lambda_0/\lambda_D < (L/\lambda_D)^{1/3},$$

where λ_0 is the laser wavelength, easily satisfied for an expanding plasma with unmodified scale length of ~ 100 μm. Extensive numerical simulations have been performed in the opposite limit, complementing the present analytic work.[3,4]

Consider an electromagentic wave $E_{in} = E_0 \times \exp(ik_{0y}y + ik_{0x}x - i\omega_0 t) + \text{c.c.}$, obliquely incident onto a plasma slab with increasing density $n = n_0(1 + x/L)$. The nonlinear equation for the slowly varying amplitude $E(x,t)$ of the driven Langmuir wave $\mathcal{E}(x,t) = E(x,t)\exp(-i\omega_p t + ik_{0y}y) + \text{c.c.}$ in a nonuniform plasma near the critical density $(x = 0)$ is written in dimensionless form as

$$i\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} + 2\left(|E|^2 - \alpha x + \frac{i\Gamma}{2}\right)E = E_d\theta(t). \qquad (1)$$

Here $E_d = E_0\varphi(s)/(2\pi k_0 L)^{1/2}$; $\varphi(s)$ is the usual resonance function with $s = (k_0 L)^{1/3}\sin\beta$ and $\beta = \tan^{-1}k_{0y}/k_{0x}$; $\alpha = 1/2L$; $t = \tau - \omega_p y/3k_y v_e^2$ is the shifted time variable; Γ the collisional damping rate; and $\theta(t)$ is the Heaviside step function. The dimensionless variables are expressed in units of ω_{pe}^{-1} for time, λ_D for spatial length, and $4(\pi nT)^{1/2}$ for E.

Equation (1) contains the nonlinear effect of the ponderomotive force important for soliton generation and density-profile modification. Strictly speaking the nonlinear term is present only after several ion plasma periods. Woithout the nonlinear term, Eq. (1) can be solved analytically by Green's function technique after a variable transformation. The solution is,

$$E(x,t) = iE_d\int_0^t \exp[\tfrac{1}{3}(4i\alpha^2)(t'-t)^3 + (2i\alpha x + \Gamma)(t'-t)]dt'. \qquad (2)$$

Initially, i.e., $t \ll (3x/2\alpha)^{1/2}$, the wave convection is not important and we have

$$\begin{aligned} E(x,t) &= iE_d(2i\alpha x + \Gamma)^{-1}\{1 - \exp[-(2i\alpha x + \Gamma)t\} \\ &= E_d(\alpha x - i\Gamma/2)^{-1}\sin[(\alpha x - i\Gamma/2)t]\exp[-(i\alpha x + \Gamma/2)t], \end{aligned} \qquad (3)$$

which shows that the wave amplitude grows linearly in time and the width shrinks as t^{-1} for $|\alpha x - i\Gamma/2| \ll 1$. Asymptotically, for large time, i.e., $t \gg (3x/2\alpha)^{1/2}$, a steady state is attained when the linear growth is balanced by the convection:

$$\begin{aligned} E(x,t) &= iE_d\int_0^\infty ds\,\exp[-\tfrac{4}{3}i\alpha^2 s^3 - (2i\alpha x + \Gamma)s] \\ &= -iE_d\pi(2\alpha)^{-2/3}\mathrm{Gi}((2\alpha)^{-2/3}(2\alpha x - i\Gamma)) + i\mathrm{Ai}((2\alpha)^{-2/3}(2\alpha x - i\Gamma)), \end{aligned} \qquad (4)$$

where Ai, Gi are the Airy functions.[7] Thus near $x = 0$ the amplitude is saturated by convection after a time $t_c \sim (3d/2\alpha)^{1/2} \sim \alpha^{-2/3}$ at a level

$$E_c \cong iE_d\pi(\alpha)^{-2/3}[\mathrm{Gi}(0) + i\mathrm{Ai}(0)], \qquad (5)$$

where $d \sim \alpha^{-1/3}$ is the width of the main peak of the Airy function. At large x, the wave number $k(x) \simeq (2\alpha x)^{1/2}$ in the WKB approximation, and Landau damping becomes important for $k\lambda_D \approx 0(1)$ or $\alpha x \approx 1$, beyond which the wave is heavily damped.

For $\omega_{pi}t_c < 1$, or $(m/M)^{1/2}(L/\lambda_D)^{2/3} \ll 1$, convective saturation takes place much before the ions start to move by ponderomotive pressure. The linear Eq. (1) then treats properly the saturation until several ion plasma periods after which the nonlinear term in Eq. (1) begins to play a role. We may therefore treat the nonlinear equation (1) as an initial-value problem with the convectively saturated amplitude (4) as the initial wave amplitude and ask how does the nonlinearity arising from the ponderomotive force change this initial amplitude after the ions have moved to establish the pressure equilibrium between thermal and ponderomotive forces.

Because the saturated field level is much higher than E_d, we may neglect E_d in Eq. (1) as a first approximation. Without the driving field, Eq. (1) with the nonlinear term is exactly solvable by the inverse scattering method to yield N-soliton solutions.[8] The particular one-soliton solution takes the form of $E = Ae^{i\varphi}$, with

$$A = 2\eta\,\mathrm{sech}2\eta(x + 2\alpha t^2 - 4\xi t - x_0),$$
$$\varphi = 2(\xi - \alpha t)x - 4[\tfrac{1}{3}\alpha^2 t^3 - \alpha\xi t^2 + (\xi^2 - \eta^2)t] + \varphi_0, \quad (6)$$

where $4(\xi - \alpha t) = V_g$ is the group velocity of the soliton at time t with initial velocity 4ξ, and φ_0 is its initial phase. For a given initial amplitude $E(x, t=0)$, the contition for N-soliton formation is[8,9]

$$\pi(N + \tfrac{1}{2}) \geq \int |E| dx \geq \pi(N - \tfrac{1}{2}). \quad (7)$$

Substituting the convectively saturated amplitude given by Eq. (5) into the condition of N-soliton generation, Eq. (7), and remembering that Landau damping limits the wave E_c to $x \leq L$, we find the following approximate condition for N-soliton generation:

$$\int |E_c| dx \simeq E_c L = \pi E_d/(2\alpha)^{5/3} > N\pi. \quad (8)$$

The physical significance of the N-soliton generation at resonance can be interpreted as follows[6]: If the threshold condition for N-soliton generation is exceeded for a given initial (unmodified) density scale length and incident power, then a soliton will emerge from the resonance modifying the density profile near the resonance while slowly moves out of the resonance as it is accelerated down the density gradient of the unperturbed profile. A second soliton will emerge on its shoulder where the local density is the new location of critical density with the density scale length there modified by the first soliton. The third soliton will again appear on the shoulder of the second soliton where the density profile is further steepened by the second soliton and so on. This process continues until the Nth soliton is generated and the density profile modified by it is so steep that no more solitons can be generated. Therefore, there will be N solitons present in the final state, each emerging on the shoulder of the previous one and further steepening the density profile. The final density scale length, L_f, can thus be obtained from Eq. (8) by setting $(2\alpha)^{-1} = L_f$ and requiring no further soliton generation:

$$L_f = \frac{1}{E_d^{3/5}} \sim \frac{(k_0 L_f)^{3/10}}{P^{3/10}} \text{ or } L_f \sim \left(\frac{k_0}{P}\right)^{3/7}. \quad (9)$$

This scaling of the density scale with incident power P is consistent with the result of numerical simulations.[10] The soliton is unstable to the transverse perturbation leading to the rippling of the critical surface. The rippling of the surface will broaden the range of incident angles of laser light and therefore affect the rate of linear resonance absorption.

We next consider the case of $\omega_{pi}t_c > 1$, in which the scale length is so long that the convective saturation time is much longer than the ion plasmas period. In this case the soliton forms before the wave convects out and the saturation of the linear growth is primarily due to the detuning of the resonant frequency by the soliton. Initially the wave amplitude grows according to Eq. (3) as the convection is unimportant. As it grows in amplitude, its width shrinks so that the integral $I = \int_{-\infty}^{\infty} |E| dx \cong \pi E_d/\alpha$ is independent of time. Comparing it with Eq. (7), we obtain a threshold driving field for N-soliton formation:

$$N + \tfrac{1}{2} > E_d/\alpha > N - \tfrac{1}{2}; \quad (10)$$

or, in terms of incident power:

$$\pi(N + \tfrac{1}{2}) > (PL\lambda_0)^{1/2} > \pi(N - \tfrac{1}{2}). \quad (11)$$

Below the one-soliton threshold, the linearly converted wave evolves into a stationary Airy-like pattern. Once the threshold is exceeded, then solitons are produced. This threshold condition appears to explain the numerical calculation[5] and experimental observations[6] very well. On the other hand, in order to see how the soliton grows and saturates, we calculate the perturbations due to the driving field E_d to the first order by varying the soliton parameters η and ξ as functions of time. For one-soliton soultions, we can write $E = Ae^{i\varphi}$ with $A = 2\eta(t)\,\mathrm{sech}2\eta(t)[x - x_0(t)]$ and $\varphi = 2[\xi(t) - \alpha t]x + \varphi_0$. Substituting it into Eq. (1), to first order in E_d, we obtain

$$\dot{x}_0 = 4(\xi - \alpha t) \text{ and } \dot{\varphi}_0 = 4[\eta^2 - (\xi - \alpha t)^2] \quad (12a)$$

and from the two conservation laws,

$$i\partial_t \int (E_x^* E - E_x E^*) dx = 4\alpha \int |E|^2 dx,$$
$$i\partial_t \int E^* E\, dx = E_d \int (E^* - E) dx,$$

we obtain also

$$\dot{\eta} = \frac{\pi E_d}{2}\,\mathrm{sech}\frac{\pi(\xi - \alpha t)}{2\eta}\cos[2(\xi - \alpha t)x_0 + \varphi_0],$$
$$\dot{\xi} = (\xi - \alpha t)\dot{\eta}/\eta, \quad (12b)$$

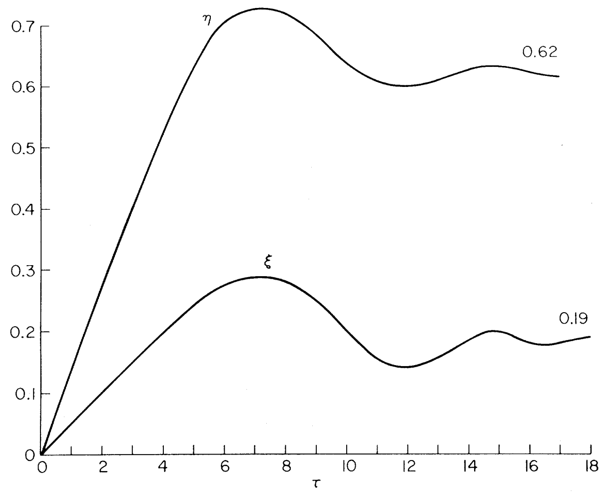

FIG. 1. Growth and saturation of solitons with $E_d = \alpha = 1$.

subject to the initial conditions, $\eta(0) = \xi(0) = x_0(0) = \varphi_0(0) = 0$. From Eq. (12), we observe two stages of soliton growth. In the first stage, when the cosine factor is essentially unity, the soliton grows linearly in time with

$$\eta = \tfrac{1}{2}(\pi E_d t)\operatorname{sech}\theta, \quad \xi \sim \alpha t/2,$$
$$\varphi_0 = [\pi^2 E_d^2 \operatorname{sech}^2\theta - \alpha^2]t^3/3, \tag{13}$$
$$x_0 \simeq -\alpha t^2, \ \theta = \pi(\xi - \alpha t)/\eta \simeq \pi\alpha/E_d,$$

where θ is a small constant in this stage; we can set it to be zero. The subsequent saturation and damped relaxation oscillation stage commences when the phase shift due to nonlinearity detunes the soliton from the driving field as the argument of the cosine factor goes to $\pi/2$. The growth of the soliton then stops with $\dot{\eta}$ going to zero and the phase locking ($\theta = 0$) is broken with θ increasing rapidly. The hyperbolic secant function then limits the oscillation to an exponentially small amplitude around the saturation level as shown in Fig. 1. We can now estimate the saturation parameters of the soliton. The saturation time is found by setting $2(\xi - \alpha t)x_0 + \varphi_0 \simeq t^3[3\pi^2 E_d^2 + 2\alpha^2]/3 \simeq \pi/2$, or

$$t_s \simeq [(3/2\pi)E_d^{-2}]^{1/3}. \tag{14}$$

The saturated η and ξ are then

$$\eta_s = (3\pi^2/16)^{1/3}E_d^{1/3}, \quad \xi_s = \tfrac{1}{2}\alpha[(3/2\pi)E_d^{-2}]^{1/3}, \tag{15}$$

The solitons thus generated will then propagate down the density gradient and be damped by interacting with hot electrons. Since the energy carried away by the solitons is eventually absorbed by the plasma, we can calculate the absorption coefficient of the laser light due to direct soliton generation for this case of $\omega_{pi}t_c > 1$:

$$a = \frac{\text{absorbed power}}{\text{incident power}} = \frac{4\dot{\eta}}{|E_0|^2 c} = \frac{N\int |E_s|^2 dx}{t_s |E_0|^2 c} = \frac{|\varphi(s)|^2}{\pi} \tag{16}$$

with maximum value about 30%, where we have used the relation between the driving field and incident field, also the threshold condition $E_d \sim N\pi\alpha$, and $kc = \omega_p$. We note that the absorption coefficient is indpendent of the temperature, the laser power, wavelength, density scale length, etc.

In summary, we have treated analytically the process of soliton generation and its subsequent effects on density modification for resonance absorption in regions where wave breaking is unimportant. We obtained exact solutions of time evolutions for both the linear and nonlinear wave equations that govern the linearly converted plasma wave. We found the amplitude of the wave initally grows linearly in time and then saturates either by convection or direct soliton formation. In the latter case, the soliton saturates when the nonlinear frequency detunes it from the driving field, while in the former case, the wave growth is balanced by its convection out of the resonance. Even in the case of convective saturation, solitons emerge eventually from the saturated wave amplitude if the N-soliton threshold condition is satisfied. These solitons modify the density gradient length at critical density and yield the scaling $L_f \sim 1/p^{0.3}$ consistent with the simulation results.[10] The interaction of the solitons with electrons and the hot-electron acceleration will be discussed in a forthcoming paper.

This work was supported by the U. S. Energy Research and Development Administration.

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Critical Transport Properties of Fluids

P. Calmettes
Service de Physique du Solide et de Resonance Magnêtique, Division de la Physique, Centre d'Etudes Nucléaires de Saclay, 91190 Gif-sur-Yvette, France
(Received 7 December 1976; revised manuscript received 12 July 1977)

An analysis of earlier data on the shear viscosity of pure and binary fluids leads to the expected universal description of the critical transport properties.

It is customary to assume that any physical property F $(F>0)$ of a system near a critical point can be written as the sum of a critical part $F^{\rm crit}$ due to the critical behavior of the system and a regular part $F^{\rm reg}$ present in the absence of criticality.[1] Thus $F^{\rm reg}$ is a positive analytic function of both the temperature T and the other parameter N. This assumption has been used to explain dynamic properties of critical systems such as the thermal conductivity[1] and diffusivity[2] of pure fluids and the shear viscosity of both pure and binary fluids.[3,4] However, this partition does not always give a convincing description of these properties.[4,5] In particular the critical viscosity $\eta^{\rm crit}$ cannot scale as do the other critical properties.[4] The purpose of this Letter is to show how this apparent difficulty can be solved in agreement with the current theoretical expectations.

First, renormalization-group theory predicts that any anomalous kinetic coefficient $K(T,N)$ can be approximated in the form[6,7]

$$K=K_0t^{-y_K}[f_0(x)+\sum_{i=1}K_it^{\omega_i}f_i(x)]+K_B, \qquad (1)$$

where the exponents y_K and ω_i are universal. The leading amplitude K_0, the amplitudes K_i of the nonanalytic corrections, and K_B the background part of K are analytic functions of T and N. $f_i(x)$ (with $i=0,1\ldots$) are functions of the scaling parameter $x=t^{\beta}/|n|$, where $t=(T-T_c)/T_c$, $n=(N-N_c)N_c$, and β is the exponent of the coexistence curve $|n|\simeq B(-t)^{\beta}$. The functions $f_i(x)$ have universal asymptotic behaviors. In particular

$$\lim_{x\to\infty}f_i(x)\equiv 1 \text{ and } \lim_{x\to 0}f_0(x)\propto x^{y_K/\beta}.$$

Secondly, in the particular case for the thermal or mass conductivity Λ of pure or binary fluids and for the shear viscosity η of both systems a recent renormalization-group treatment of the dynamics of critical fluids predicted that[7]

$$y_\Lambda+y_\eta=\gamma-\nu \text{ with } y_\eta\simeq 0.04, \qquad (2)$$

$$\Lambda_0\eta_0=Rk_BT_c\chi_0\xi_0^{-1} \text{ with } R\simeq(5\pi)^{-1}, \qquad (3)$$

where k_B is the Boltzmann's constant, $\chi=\chi_0t^{-\gamma}$ with $\gamma\simeq 1.24$ is the static susceptibility of the order parameter and $\xi=\xi_0t^{-\gamma}$ with $\nu\simeq 0.63$ is the correlation length.

Finally it is worth noticing that Ref. 7 can be regarded as a more accurate statement of the mode-coupling theory[8] in which the approximations $\gamma=2\nu$ and $y_\eta\equiv 0$ were made. Thus $\eta_0\equiv\eta^{\rm reg}$ in this theory. Since more precisely $y_\eta\ll y_\Lambda$ it could be expected that $\eta_0\simeq\eta^{\rm reg}\gg\eta_B$ in a sizable range of temperatures near T_c,[9] but $\lambda_B\simeq\lambda^{\rm reg}$. On the other hand, the nonanalytic corrections are also implicit in the mode-coupling theory where they are generated by the regular coefficients.[7]

A careful analysis of the shear viscosity data for critical fluids will provide a crucial test of the above predictions. To this end I have selected the numerical data reported in literature for ethane and xenon,[10] and for the mixtures of isobutyric acid-water,[11,12] 2,6 lutidine-water,[13] and 3-methylpentane-nitroethane.[4,14] I have estimated, following Oxtoby,[15] the temperature ranges R in which the viscosity data are not affected by shear by more than 1% and I have neglected the data which are not in R. R is given in Table I.

I have paid particular attention to the very care-