published).
[3]R. Blankenbecler and S. J. Brodsky, Phys. Rev. D 10, 2973 (1974).
[4]J. Gunion, Phys. Rev. D 10, 242 (1974).
[5]G. Farrar, Nucl. Phys. B77, 429 (1974).
[6]See, for example, F. E. Close, F. Halzen, and D. M. Scott, Phys. Lett. 68B, 447 (1977).
[7]G. Altarelli and G. Parisi, to be published.
[8]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 25, 316 (1970).
[9]J. Kuti and V. Weisskopf, Phys. Rev. D 4, 3418 (1971); F. E. Close, Nucl. Phys. B80, 269 (1974).
[10]R. Carlitz and J. Kaur, Phys. Rev. Lett. 38, 673, 1102(E) (1977).
[11]F. E. Close, Phys. Lett. 43B, 422 (1973); R. Carlitz, Phys. Lett. 58B, 345 (1975); G. Farrar and D. R. Jackson, Phys. Rev. Lett. 35, 1416 (1975).
[12]However, for a calculation in the M.I.T. bag in which the sea is generated by mechanism similar to ours, see J. F. Donoghue and E. Golowich, Phys. Rev. D 15, 3421 (1977). (i) In the bag, the $q\bar{q}$ sea is always at least partially polarized, regardless of bag excitation (in agreement with the parton analysis). (ii) However, the bag calculation holds for a *rest-frame hadron*, so direct comparison with parton calculations is not possible.
[13]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 24, 181 (1970); G. B. West, Phys. Rev. Lett. 24, 1206 (1970).

Hadron Production in Nuclear Collisions—a New Parton-Model Approach

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We consider a quark-parton model in which wee partons of the projectile interact with wee partons of essentially independent nucleons in the nuclear target. The ratio of multiplicities in the central region is $\langle n\rangle_{HA}/\langle n\rangle_{HN} = \frac{1}{2}\bar{\nu} + \bar{\nu}/(\bar{\nu}+1)$, where $\bar{\nu}$ is the mean number of inelastic collisions of the projectile H, in agreement with experiment. We also predict nucleus-nucleus multiplicities, the multiplicity distribution, and the absence of shadowing in large-q^2 and large-p_T reactions.

Although the quark-parton model has been very successful in predicting the short-distance behavior of hadronic interactions, the underlying mechanisms involved in the production of hadrons in ordinary high-energy collisions have never been specified. In the case of particle production on nuclear targets, this fundamental uncertainty of the parton approach becomes amplified, and this has led to an extraordinary range of divergent predictions for even the most basic experimental parameters.[1] In this Letter we present a new approach to this problem based on a straightforward application of parton-model concepts. The resulting picture for nuclear collisions is very simple and in good agreement with experiment. It is based upon (1) the assumption that each inelastically excited nucleon in the nuclear target produces hadrons independently of the others, and (2) a specific hadronic collision model based on wee-parton interactions[2] analogous to the Drell-Yan[3] pair-production process.

We begin with a simple parton-model description of hadron-hadron interactions. Each hadron has a Fock-space decomposition in terms of multiparton states. An interaction occurs via a collision of a parton in the beam (B) with a parton in the target (A). The cross section takes the typical Drell-Yan form[3,4]

$$\sigma_{BA} = \sum_{\substack{a\in A\\ b\in B}} \int_0^1 dx_a \int_0^1 dx_b\, G_{a/A}(x_a) G_{b/B}(x_b) \hat{\sigma}_{ab}(\hat{s}_{ab}), \tag{1}$$

where $x_b = (k_b^{\,0}+k_b^{\,z})/(p_B^{\,0}+p_B^{\,z})$ and $x_a = (k_a^{\,0}-k_a^{\,z})/(p_A^{\,0}-p_A^{\,z})$ are the light-cone fractions ($p_B^{\,z}>0$; $p_A^{\,z}<0$) of the beam and target momenta, respectively, and $\hat{s}_{ab} = x_a x_b s + m_a^2 m_b^2/x_a x_b s$ is the square of collision energy for the subprocess. (For simplicity we do not display the transverse-momentum dependence.) We presume that $\hat{\sigma}_{ab}$ falls rapidly with increasing $\hat{s}_{ab}$, as would be typical of quark-parton exchange[2,5] or q-$\bar{q}$ annihilation processes,[6] and that each distribution $G(x)$ has the Feynman[2] wee-parton distribution $xG(x) \to C \neq 0$ at $x \to 0$. In this model $\sigma_{BA}(s) \propto \ln s$, and the location in rapidity of the parton-parton collision, $\hat{y}$, is distributed uniformly throughout the central region, where neither x_a nor x_b is forced into the finite-x, power-law–damped regions of $G(x)$. In inelastic collisions, the partons in the beam materialize as hadrons for $\hat{y} \lesssim y < Y_B$, and those in the target materialize throughout the interval $Y_A < y \lesssim \hat{y}$. Note that real hadron production from the beam partons cannot extend much below $\hat{y}$ since this forces propagators off shell where interactions are suppressed.

Turning to nuclear collisions, we shall assume that, aside from small binding corrections and Fermi-motion effects, each nucleon in the nucleus independently develops its own parton distribution. Thus the partons of different nucleons interact with each other only minimally and do not shadow or coalesce with one another. We emphasize that nuclear-binding and Fermi-motion corrections are not related to the "$A^{2/3}$" surface or nuclear-size effects characteristic of shadowing. In a high-energy collision, the various wee partons of the projectile can interact with the wee partons of different nucleons. The rapidity locations of the parton-parton collisions, $\hat{y}_i$, are uncorrelated and uniformly distributed in the central region. Each nucleon in the nucleus A participates in at most one interaction, whereas the mean number of inelastic collisions of the beam hadron H is $\bar{\nu} = A\sigma_{HN}^{\text{inel}}/\sigma_{HA}^{\text{inel}}$. On the average, then, the rapidity separation between parton collisions is $\Delta y \cong Y_c/(\bar{\nu}+1)$, where Y_c is the total length of the central rapidity region. A typical multiparticle distribution for $\bar{\nu} = 3$ collisions is illustrated in Fig. 1. Since the collision rapidities are uncorrelated, each inelastically excited nucleon produces hadronic multiplicity on the average halfway across the central region. As the number of collisions increases, the range of the projectile hadron distribution extends further and further into the central region to the minimum $\hat{y}_i$—on the average over a rapidity length $\bar{\nu}\Delta y$

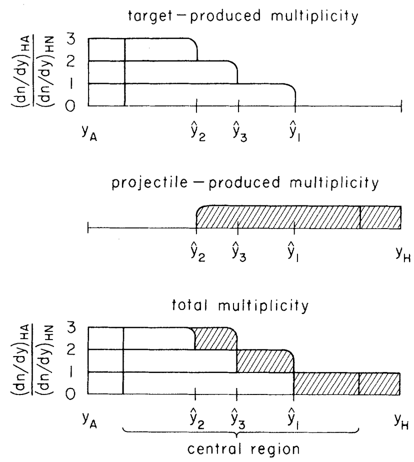

FIG. 1. Idealized multiplicity distribution for an H-A collision with $\bar{\nu} = 3$ inelastic excitations. The y_i are uniformly distributed in rapidity and can be produced in any sequence.

$= [\bar{\nu}/(\bar{\nu}+1)]Y_c$. Thus we obtain, for the ratio of multiplicities in the central region,

$$\langle n\rangle_{HA}/\langle n\rangle_{HN} = \bar{\nu}/2 + \bar{\nu}/(\bar{\nu}+1), \tag{2}$$

where the only dependence on the projectile H is through the definition of $\bar{\nu}$.

The distribution of particles averaged over events produced from the excitation of the nuclear partons is wedge shaped. The ratio of distributions $R_A(y)$ in the central region for hadron-nucleon to hadron-nucleus collisions is simply ($y_A \equiv 0$)

$$\frac{(dn/dy)_{HA}}{(dn/dy)_{HN}} = \bar{\nu}\left(1 - \frac{y}{Y_c}\right) + \left[1 - \left(1 - \frac{y}{Y_c}\right)^{\bar{\nu}}\right]. \tag{3}$$

Although Eqs. (2) and (3) are derived assuming a uniform plateau height in the central region in H-N collisions, corrections to this shape (which can arise, for example, from multiple parton interactions) tend to cancel in the ratio.

Thus far we have ignored the effects of the fragmentation regions. Equation (1) predicts that the fast (e.g., valence) partons interact only weakly[8] and thus $R_A(y) = 1$ in the projectile fragmentation region, and $R_A(y) = \bar{\nu}$ in the target fragmentation region. Let $\langle n_{\text{frag}}\rangle_H$ and $\langle n_{\text{frag}}\rangle_N$ be the average number of particles produced in the projectile and nucleon fragmentation regions (i.e., within $\Delta y_{\text{frag}} \sim 2$ units of the incident rapidity). Then, instead of Eq. (2), we obtain

$$\frac{\langle n_{\text{tot}}\rangle_{HA}}{\langle n_{\text{tot}}\rangle_{HN}} = \frac{[\bar{\nu}/2 + \bar{\nu}/(\bar{\nu}+1)]\langle n_{\text{central}}\rangle + \bar{\nu}\langle n_{\text{frag}}\rangle_N + \langle n_{\text{frag}}\rangle_H}{\langle n_{\text{tot}}\rangle_{HN}}$$

$$= \left(\frac{\bar{\nu}}{2} + \frac{\bar{\nu}}{\bar{\nu}+1}\right) - \left(\frac{\bar{\nu}}{2} - \frac{1}{\bar{\nu}+1}\right)\frac{\langle n_{\text{frag}}\rangle_H}{\langle n_{\text{tot}}\rangle_{HN}} + \left(\frac{\bar{\nu}}{2} - \frac{\bar{\nu}}{\bar{\nu}+1}\right)\frac{\langle n_{\text{frag}}\rangle_N}{\langle n_{\text{tot}}\rangle_{HN}}, \tag{4}$$

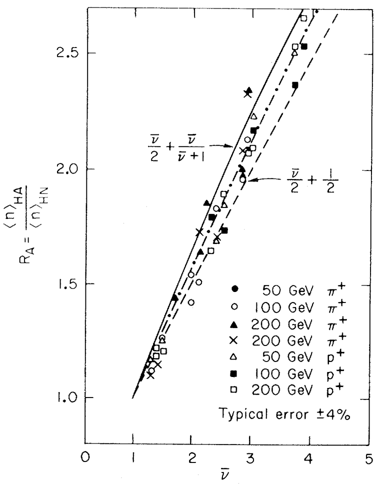

FIG. 2. The variation of $R_A = \langle n\rangle_{HA}/\langle n\rangle_{HN}$ with $\bar\nu$ for pion and proton beams. The data are for charged multiplicities from Ref. 9. The solid curve is the $s\rightarrow\infty$ prediction $R_A = \frac{1}{2}\bar\nu + \bar\nu/(\bar\nu+1)$. The dashed curve is the line $R_A = \frac{1}{2}\bar\nu + \frac{1}{2}$ corresponding to no central region. The prediction of the model, Eq. (4), for $E_{\text{lab}} = 200$ GeV (taking $\langle n_{\text{frag}}\rangle_{HN}/\langle n_{\text{tot}}\rangle = 0.2$) is the dash-dotted curve, $R_A = \bar\nu/2 + \bar\nu/(\bar\nu+1) - 0.2(\bar\nu-1)/(\bar\nu+1)$.

where $\langle n_{\text{tot}}\rangle_{HN} = \langle n_{\text{central}}\rangle + \langle n_{\text{frag}}\rangle_N + \langle n_{\text{frag}}\rangle_H$ is the total produced multiplicity for the H-N collision. In practice, the fragmentation correction terms are small, of order $(\Delta y)_{\text{frag}}/Y_{\text{tot}} \sim O(1/\ln s)$, compared to $\bar\nu/2$.

This result [Eq. (4)] is shown in Fig. 2 and is in good agreement with the data for charged-pion and proton collisions. In addition, the shapes of the observed multiplicity distributions are consistent with the predicted forms of Eq. (3) and Fig. 1, except possibly at $y \sim y_A$ where a small amount of cascading is probably occurring. The slight energy dependence predicted in Eq. (4) is also consistent with the trend of the data.[10]

We have analyzed the total nuclear cross section in this model and have found it to be consistent with the usual Glauber theory.[11] In this picture the incident hadron, which is represented by its Fock-space parton distribution, can interact elastically (diffractively) via elastic parton interactions in the central region and can continue to propagate and interact as a coherent hadron through the nuclear medium.[12] Thus one obtains the usual multiple-scattering Glauber series. Nonetheless, the multiplicity density dN/dy produced from the incident-projectile parton distribution is not increased by the repeated collisions. Because of the Glauber series, the cross section does not factorize: $\sigma_{\pi A}{}^{\text{inel}} \sim \sigma_{pA}{}^{\text{inel}}$ approach the geometric limit.

The model proposed here is consistent with energy and momentum conservation. In the equal-velocity frame, the central particles produced in the projectile direction have a typical total energy of order $\bar\nu m_T$ (with $m_T{}^2 = m^2 + \langle \vec{k}_\perp{}^2\rangle$), which can be compensated by a small loss of energy of the leading particles in the projectile region, a correction of relative order $(\bar\nu - 1)m_T/\sqrt{s}$.

One may also use this picture to predict the multiplicity distributions in nucleus-nucleus collisions.[12] For the central region one obtains

$$\frac{\langle n\rangle_{A_1A_2}}{\langle n\rangle_{NN}} = \bar\nu_{A_1/A_2}\left(\frac{\bar\nu_{A_2/N}}{\bar\nu_{A_2/N}+1}\right) + \bar\nu_{A_2/A_1}\left(\frac{\bar\nu_{A_1/N}}{\bar\nu_{A_1/N}+1}\right), \qquad (5)$$

where $\bar\nu_{A_1/A_2} = A_1\sigma_{NA_2}/\sigma_{A_1A_2}$ is the average number of inelastically excited nucleons in A_1 in the collision with a projectile A_2. Each such excited A_1 nucleon interacts inelastically with $\bar\nu_{A_2/N}$ nucleons in A_2 so that the average rapidity length of excited partons in A_1 is $[\bar\nu_{A_2/N}/(\bar\nu_{A_2/N}+1)]Y_c$. Corresponding statements apply to $\bar\nu_{A_2/A_1}$ and $\bar\nu_{A_1/N}$. The above result predicts, for example, $\langle n\rangle_{\alpha A_2}/\langle n\rangle_{NA_2} \sim 3.8$ for $A_2 > 100$, which is in agreement with cosmic-ray data for α-particle collisions.[13] The predicted rapidity distribution monotonically interpolates between fragmentation regions proportional to $\bar\nu_{A_1/A_2}$ and $\bar\nu_{A_2/A_1}$.

Finally, we wish to point out the connection between our hypothesis of independently interacting and materializing nuclear parton chains and deep inelastic scattering measurements on nuclei. The latter directly probe the parton distributions within nuclei, and, according to our hypothesis, one should obtain

$$\nu W_{2A}(x_{\text{Bj}}) \cong A\,\nu W_2(x_{\text{Bj}}) \qquad (6)$$

for all (including arbitrarily small) $x_{\text{Bj}} = -q^2/2M_N\nu \lesssim 1$ once q^2 is in the Bjorken scaling region.[14] For $x_{\text{Bj}} > 1$, Fermi-motion corrections can be included and computed using quark counting,[15] but otherwise nuclear-binding corrections to (6) are considered negligible. Thus there is neither shadowing nor antishadowing[16] of the partons of one nucleon by the partons of other nucleons. In general, we predict the absence of shadowing —independent of beam energy—for any reaction in the scaling region where the effective collision energy $\sqrt{\hat{s}}$ of the subprocess is large, e.g., for the Drell-Yan process $pA\rightarrow l^+l^-X$ at large $\mathfrak{M}_{l^+l^-}{}^2$, as well as for large-$p_T$ hadrons reactions—ignoring multiple-scattering effects. This also implies that for sufficiently large q^2 (or $\hat{s}$), only one nucleon in the target is "wounded" and the multiplicity will be that appropriate to $\nu = 1$. The absence of shadowing is also apparent in the ratio of distributions $R_A(x) = (dn/dx)_{HA}/(dn/dx)_{HN}$, where x is the Feynman variable $k_{\text{c.m.}}/k_{\text{c.m.}}^{\max}$. At infinite energy $R_A(x)$ reduces in our model to a step function $R_A(x) = \bar{\nu}\theta(-x) + \theta(x)$ since the central region is confined to $x \to 0$. If we identify the nuclear parton distribution shape with the multiparticle distribution for $x<0$, this again corresponds to the absence of shadowing: $(d\sigma/dx)_{HA} = A(d\sigma/dx)_{HN}$.[17]

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[1]For a review, see W. Busza, in *Proceedings of the Seventh International Colloquium on Multiparticle Reactions, Tutzing, Germany, 1976*, edited by J. Benecke *et al.*, (Max-Planck-Institut, Munich, to be published).

[2]R. P. Feynman, *Photon-Hadron Interactions* (Benjamin, Reading, Mass., 1972), and references therein.

[3]S. D. Drell and T.-M. Yan, Phys. Rev. Lett. 25, 316 (1970), and Ann. Phys. (N.Y.) 66, 578 (1971).

[4]A model of this form was considered by P. V. Landshoff and J. Polkinghorne, Nucl. Phys. B32, 541 (1971).

[5]R. Blankenbecler, S. J. Brodsky, and J. F. Gunion, Phys. Rev. D 8, 287 (1973), and 12, 3469 (1975), and references therein.

[6]P. V. Landshoff and J. Polkinghorne, Phys. Rev. D 10, 891 (1974).

[7]The prediction of an approximate wedge shape in the central region was also made in the "two-phase model" of P. M. Fishbane and J. S. Trefil, Phys. Lett. 51B, 139 (1974). Their descriptive approach is specific to the nuclear multiplicity problem, and its relation to a fundamental field-theoretic model and Glauber theory is unclear.

[8]This was first discussed by O. Kancheli, Pis'ma Zh. Eksp. Teor. Fiz. 18, 465 (1973) [JETP Lett. 18, 274 (1973)].

[9]W. Busza *et al.*, in *Proceedings of the Eighteenth International Conference on High Energy Physics, Tbilisi, U. S. S. R. 1976*, edited by N. N. Bogolubov *et al.* (The Joint Institute for Nuclear Research, U.S.S.R., 1977).

[10]Recently, A. Capella and A. Krzywicki, Orsay Report No. LPTHE 7712, 1977 (to be published), proposed an extended multiscattering model in which higher-order Glauber terms correspond to the interaction of n-independent constituent systems within the projectile, each of which shares the incident energy roughly equally, $\hat{s} \sim s/n$. At present energies their model also agrees well with experiment. However, the energy dependence of this model differs from the model discussed here; e.g., the nuclear chains all extend to the projectile fragmentation region and $\langle n\rangle_{HA}/\langle n\rangle_{HN} \to \bar{\nu}$ at infinite energy. The two models can also be distinguished by measurements of the spectrum of leading particles, and predictions for nucleus-nucleus collisions.

[11]The generalized Glauber theory resulting from this model includes inelastic diffractive states which can be treated in the fashion of photon-hadron interactions, as in S. J. Brodsky and J. Pumplin, Phys. Rev. 182, 1794 (1969), and V. Gribov, Zh. Eksp. Teor. Fiz. 57, 1306 (1969) [Sov. Phys. JETP 30, 709 (1970)].

[12]Since the parton scattering is almost forward, the initial Fock-space state is disturbed only minimally. It thus has considerable overlap and remains nearly coherent with the incident-projectile wave function. A complete discussion will be presented in S. J. Brodsky, J. F. Gunion, and J. H. Kühn, to be published.

[13]The experimental ratio is ~4. See M. F. Kaplon and D. M. Ritson, Phys. Rev. 85, 932 (1952), and Phys. Rev. 88, 386 (1952). For other theoretical treatments see A. Bialas, M. Bleszynski, and W. Czyz, Nucl. Phys. B111, 461 (1976), and references therein.

[14]See S. J. Brodsky, F. Close, and J. F. Gunion, Phys. Rev. D 6, 177 (1972). Note that this is contrary to the result predicted by V. I. Zakharov and N. N. Nicolaev [Phys. Lett. 55B, 397 (1975)]. See also J. D. Bjorken, in Proceedings of the SLAC-191 Summer Institute, Stanford, California, 21–31 July 1975, edited by M. Zipf (unpublished).

[15]A. Krzywicki, Phys. Rev. D 14, 152 (1976); S. J. Brodsky and B. Chertok, Phys. Rev. D 14, 3003 (1976); I. A. Schmidt and R. Blankenbecler, SLAC Report No. SLAC-PUB-1881, 1977 (unpublished).

[16]Zakharov and Nikolaev, Ref. 14; N. N. Nikolaev, in Proceedings of the International Seminar on Particle-Nuclear Interactions, International Center for Theoretical Physics, Trieste, Italy, 1976 (unpublished).

[17]Bjorken scaling of the virtual photoabsorption cross sections $\sigma_T(q^2,\nu) \propto (q^2)^{-1}$ at high ν and the scale-invariant behavior for $\sigma(e^+e^- \to \text{hadrons})$ at large s implies, in a generalized vector-dominance model, that the diffractive cross section virtual meson states of $\sigma(\mathfrak{M}^2,\nu)$ decreases at least as fast as $\mathfrak{M}^{-2}$. See Bjorken, Ref. 14; Brodsky and Pumplin, Ref. 11; and Gribov, Ref. 11. Since the high-mass states interact weakly, this again implies the absence of shadowing in deep inelastic lepton scattering, Eq. (6).