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Overlap of Bounce Resonances and the Motion of Ions in a Trapped-Ion Mode

Gary R. Smith
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
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Motion near a separatrix (the boundary between closed and open orbits) is studied both theoretically and numerically for parameters appropriate to the dissipative trapped-ion mode. I observe disappearance of an invariant in a stochastic layer surrounding the separatrix, as a result of overlapping bounce resonances. The fraction of ions lying within the stochastic layer is large even for a mode of relatively small amplitude.

Dissipative trapped-particle instabilities[1] are thought to cause anomalous transport in tokamaks. The transport rates are determined by the amplitude of the fluctuating electric field of the nonlinearly saturated instability. It is therefore important to consider nonlinear processes which might lead to saturation at relatively low levels.

In this Letter I report studies of a nonlinear process not previously considered, as far as I know, for the trapped-ion mode. Some stabilizing effect of this process have already been studied by us, but estimates of saturation levels are reserved for publication elsewhere, because details of trapped-particle instability theory are required.

The equations of motion which we study describe other physical situations of current interest. The equations were first used[2] to study the one-dimensional motion of a particle in two electrostatic waves with different amplitudes and phase velocities. This configuration is of interest[3] in the theory of radio-frequency heating. The equations were used[4] for a tokamak to estimate the fraction of a magnetic island (due to a tearing mode) in which field lines would be braided. In the general theory of stochasticity,[5] study of motion near a separatrix is recognized as being of fundamental importance.

We study the guiding-center motion of an ion in a magnetic field whose amplitude varies sinusoidally, the usual model[6] for a tokamak of large aspect ratio and circular cross section. The inclusion of guiding-center drifts off a field line would not change the physics of the process being studied, so we ignore those drifts. The calculations are thus in the spirit of local theory, in which radial excursions are assumed to be sufficiently small. We ignore collisions in order to isolate the effects, including pitch-angle scattering, due to the electric field of the mode.

A trapped-ion mode causes perturbations in the motion of the ion. We assume that a single mode is present, with toroidal and poloidal mode numbers l and m, respectively. As the ion moves along a field line, it feels the potential

$$\Phi(\Theta, t) = -\Phi_0 \cos[(m - lq)\Theta - \omega t + \eta], \tag{1}$$

where Θ is the poloidal angle, q the safety factor on the magnetic surface on which the ion moves, ω the mode frequency, and η a phase angle related to the ion's initial toroidal angle.

The equations of motion are derivable from the Hamiltonian

$$\begin{aligned} H(\Theta, p, t) &= H_0(\Theta, p) + e\Phi(\Theta, t), \\ H_0 &= (p/qR_0)^2/2M - \mu\Delta B \cos\Theta, \end{aligned} \tag{2}$$

where p is the momentum conjugate to Θ, R_0 the major radius of the tokamak, e and M the ion charge and mass, ΔB the modulation amplitude of the magnetic field, and μ the magnetic moment. Since μ is conserved during the motion, it plays the role of a parameter here.

A more convenient form of Hamiltonian (2) for analytic work uses action and angle variables. We express the unperturbed Hamiltonian H_0 in terms of the longitudinal action

$$J(H_0) = (2\pi)^{-1} \oint p \, d\Theta,$$

which, except for constant factors, is $\oint v_\parallel \, ds$, in standard notation. The explicit expressions[2,7] for J and the canonically conjugate angle variable φ involve elliptic integrals and κ, defined by $2\kappa^2 \equiv (1 + H_0/\mu\Delta B)$. The variables J and φ are defined in such a way that an ion which changes from trapped to circulating does not change its J or φ

discontinuously. We denote by ω_b the frequency $\partial H_0/\partial J$ of the unperturbed motion. For a trapped ion ($\kappa<1$), ω_b is the frequency of bouncing between magnetic mirrors. For a circulating ion ($\kappa>1$), it is half the frequency of transiting from one minimum of the magnetic field to the next. The bounce frequency of a deeply trapped ion ($\kappa=0$) is $\omega_b(0)=(\mu\Delta B/M)^{1/2}/qR_0$.

In terms of the action-angle variables, Hamiltonian (2) appears as

$$H(\varphi, J, t)=H_0(J)+e\Phi(\varphi, J, t),$$
$$\Phi=-\Phi_0\textstyle\sum_n v_n(J)\cos(n\varphi-\omega t+\eta). \tag{3}$$

The nth bounce resonance occurs when the nth term is slowly varying in time. From H_0 we have $\dot{\varphi}=\omega_b(J)$, and thus the condition for bounce (or transit) resonance is

$$\omega_b(J)=\omega/n. \tag{4}$$

Since $\omega_b(J)\to 0$ as $\kappa\to 1$ for both trapped and circulating ions, there is a double infinity of values of J near the separatrix ($\kappa=1$) which satisfy (4). The separation between resonances n and $n+1$ is

$$\delta_n\equiv\omega_b(J_n)-\omega_b(J_{n+1})=\omega/n(n+1),$$

where J_n is the value of J satisfying (4).

For the width of resonance n we ignore all other resonances and calculate from (3) the width of the "secondary" separatrix of resonance n. This separatrix divides the φ-J plane into regions in which the phase $n\varphi-\omega t$ is either bounded or unbounded in time. The width (in frequency) of resonance n is

$$\Delta_n=4\left|e\Phi_0V_n(J_n)\partial\omega_b/\partial J_n\right|^{1/2}.$$

When resonances overlap, an invariant[8] ceases to exist, which allows the motion to be stochastic.[9] A rough guide for the onset of stochastic motion is that the resonance width exceeds the separation between resonances:[5]

$$\tfrac{1}{2}(\Delta_n+\Delta_{n+1})\approx\Delta_n>\delta_n. \tag{5}$$

I choose the typical[10] value $m-lq=\frac{1}{2}$ and look up[11] the explicit formulas for $V_n(J)$. Condition (5) then predicts overlap of all resonances with n greater than a critical n depending on Φ_0. After some manipulation, I find from (5) how close an ion must be to the primary separatrix for its motion to be stochastic:

$$|E-\mu B_M|<(128/\pi^2)e\Phi_0\omega/\omega_b(0). \tag{6}$$

In (6), E is the ions's energy and B_M is the maximum field on the magnetic surface on which the motion occurs. The theory based on (5) actually predicts that the part of the stochastic layer lying outside the separatrix should be roughly *half* the width given by (6) for the inside part. I find, however, that (6) describes our numerical results for *both* parts of the layer. The agreement is best for ions near the separatrix, where approximations used in the theory are valid.

References 3 and 4 gave a method for calculating the width of the stochastic layer for other values of $m-lq$. An expression analogous to (6) given in those works is valid in the limit of large $\omega/\omega_b(0)$. In deriving (6) I have used the opposite limit, which is appropriate to motion in a trapped-ion mode.

For the numerical calculations I choose

$$e\Phi_0=0.05(\Delta B/B_0)T_i,$$
$$m-lq=\tfrac{1}{2},\quad \omega=\tfrac{1}{2}\omega_{bi}, \tag{7}$$

where $\omega_{bi}=(T_i\Delta B/B_0M)^{1/2}/qR_0$, with T_i the ion temperature and B_0 the field at the magnetic axis. I have chosen Φ_0 comparable to the mode amplitudes at which other[12] nonlinear processes become important. We integrate the equations of motion derivable from (2) and convert the resulting $\Theta(t)$ and $p(t)$ to $\varphi(t)$ and $J(t)$ using the explicit expressions mentioned above.

Stochastic effects can be strong for low-energy ions even for the relatively weak mode amplitude (7). The strength of the perturbation in (2) is measured by the parameter $e\Phi_0/\mu\Delta B$, which, for given Φ_0, increases as μ decreases. Note also that $\omega/\omega_b(0)=(T_i/\mu B_0)^{1/2}\omega/\omega_{bi}$ increases as μ decreases.

In Fig. 1 we show the increase in width of the stochastic layer as μ is decreased. Using the trajectory information $\varphi(t)$ and $J(t)$, we plot a point whenever ωt is a multiple of 2π. When the points for a given trajectory lie on a curve, an invariant exists, while if they fill an area, an invariant does not exist. This surface-of-section method was used in earlier work[8] on the overlap of cyclotron resonances, where more discussion appears.

The analytic form of the invariant is not simply J when a trapped-ion mode is present, as can be seen from Fig. 1. Near the large islands, which represent the resonances $\omega=\omega_b$, even the topology of the invariant curves changes, from open curves to closed ones. But the invariant ceases to exist, *only* where multiple resonances overlap. The bounce resonances discussed above cause disappearance of the invariant in the vicinity of

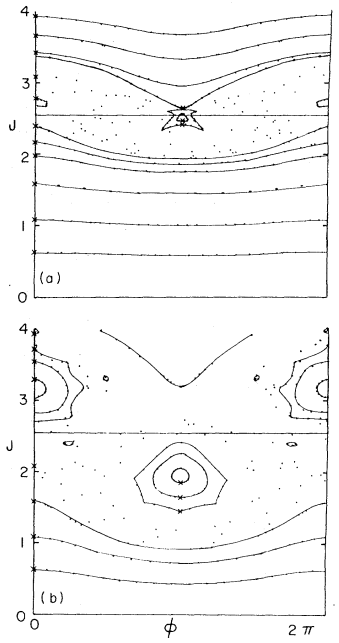

FIG. 1. Trajectories of ions in the presence of a trapped-ion mode with parameters given by (7). ×'s indicate initial conditions for the trajectories, which are represented by plotting a point each wave period. A hand-drawn curve connecting the points shows the existence of an invariant. The line at $J = 8/\pi$ is the separatrix. The parameter η equals 0. (a) $\mu B_0 = T_i$; (b) $\mu B_0 = \frac{1}{2} T_i$.

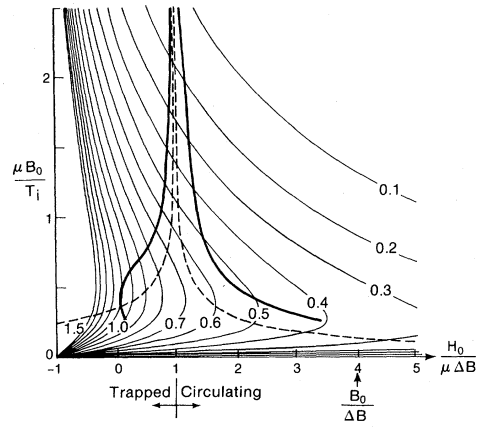

FIG. 2. Extent in velocity space of the stochastic region (between the heavy lines) for a trapped-ion mode given by (7).

the separatrix.

In Fig. 2 I show the region in velocity space in which stochastic effects are important. The horizontal axis is the parameter $H_0/\mu\Delta B$ which measures the depth of trapping in the magnetic well. The heavy lines surrounding the separatrix ($H_0 = \mu\Delta B$) give the observed width of the stochastic layer for a range of μ values. The widths were measured from plots such as Fig. 1. The dashed lines show the location of the ions satisfying $\omega = \omega_b$ and thus the location of the large islands in Fig. 1. The width of these islands has not been indicated on Fig. 2. The light lines are contour levels showing the number of ions in each small area on Fig. 2; a Maxwellian distribution and $B_0/\Delta B = 4$ were assumed. We see that a large fraction of the ions lies within the stochastic region.

The single-mode assumption used in (1) is not crucial for the effects discussed above. If many modes were present, many more resonances (4) would appear, increasing the tendency for the motion to be stochastic. Plots like Fig. 1 show no qualitative changes when we multiply (1) by $g(\Theta) = \frac{1}{2}(1 + \cos\Theta)$ to model the ballooning of the mode on the outside of the torus.

I believe that the overlap of bounce resonances is the physical mechanism behind some of the results observed by Coppi and Taroni,[13] where no explanation was given. Detrapping of particles due to ion sound turbulence[14] also relies on the presence of electric field energy at multiples of ω_b. In contrast to Ref. 14 I have shown here that stochastic effects can occur in the presence of even a single mode.

The trapped-ion mode tends to be stabilized by the stochastic effects discussed here in the following ways. Pitch-angle diffusion of ions is enhanced, leading to more frequent transitions between the trapped and circulating states. Ions are thus able to take energy from the mode (see Ref. 12) more effectively than if collisions alone caused pitch-angle diffusion. Also, the greater pitch-angle diffusivity prevents, for some trapped ions, the $\vec{E}\times\vec{B}$ motion which determines the real part of the mode frequency ω.[1] This effective decrease in the number of trapped ions causes a reduction of ω, which in turn leads to decreased collisional growth by electrons. Finally, the net transfer of energy to ions which is associated with Landau damping of the mode is increased by irreversible changes in the distribution function which are more rapid than found in linear theory.

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Modification of the Low-Density Phases of Adsorbed Helium Monolayer Films Produced by Changes in the Adsorbing Substrate*†

S. B. Crary and O. E. Vilches
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 14 February 1977)

The specific heats of monolayer ^{4}He and ^{3}He films adsorbed on Ar-plated Grafoil show profound differences from films on bare Grafoil. The apparent liquefaction temperature of ^{4}He is doubled, and the interacting Fermi gas behavior of ^{3}He is modified to show either condensation or strong band effects.

Surface phases of monolayer helium films adsorbed on Grafoil[1-6] have been the subject of considerable experimental study since the work of Bretz and Dash.[7] Some of these phases have been successfully described in terms of two-dimensional (2D) models. Heat capacity measurements of two-layer films[8,9] have provided some information on the behavior of helium adsorbed on a solid substrate other than Grafoil. The adsorbent is then helium-plated Grafoil. The "second layer" studies have shown a marked *decrease* in the 2D ^{4}He condensation temperature, almost no change in the thermodynamic behavior of ^{3}He, the absence of ordering transitions for both isotopes, and the appearance of melting at densities comparable to those of the first layer.

In this Letter we report specific-heat measurements of both ^{4}He and ^{3}He monolayers adsorbed on a radically different surface, argon-plated Grafoil. Argon plating increases the depth and the variation of the depth and changes the translational symmetry of the adsorbing potential while still providing a well-characterized surface. Our measurements show that by modifying the substrate it is possible to change the thermodynamic properties of the adsorbed films. We have produced a 1-K *increase* in the apparent condensation temperature of ^{4}He. In addition, we have observed either condensation or the existence of band-type effects in ^{3}He.

Previous experimental work on argon-plated Grafoil has consisted of measurements of adsorption isotherms at $T \geq 4.2$ K.[10,4] The 4.2-K results for ^{4}He have been theoretically analyzed by Novaco,[11] who deduced the second virial coefficient. In addition, the density of states of a single adsorbed helium atom on rare-gas–plated graphite has been calculated,[12] as has the effect of the substrate potential on the ground state of adsorbed helium.[13]

Measurements were carried out using a well-characterized Grafoil cell (cell *B* of Ref. 2) plated at nitrogen temperature with 85 cm^3 of argon. The argon monolayer capacity of this cell at 77 K