Muon and Electron Number Nonconservation in a $V-A$ Gauge Model

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We analyze muon and electron lepton-number nonconservation in a pure $V-A$ gauge model. The rates for $\mu\rightarrow e\gamma$, $\mu\rightarrow ee\bar{e}$, and $K_L\rightarrow\mu\bar{e}$ are computed for this model. We find that for a reasonable range of neutral heavy-lepton mass these rates are in accord with, but not extremely small compared to, present experimental bounds. We comment on the nonorthogonality of ν_e and ν_μ, and interesting features of the L^- decays.

Some time ago we discussed a six-quark model[1] with only left-handed currents. This is a minimal extension of the "standard" four-quark Weinberg-Salam $SU(2)\otimes U(1)$ gauge model[2] which allows CP nonconservation to be incorporated. The alternatives are right-handed currents[3] or proliferation of Higgs bosons.[4] Such a model leads to approximate superweak (or microweak[5]) predictions for CP nonconservation. The model also includes a pair of leptons (L^0,L^-), both massive, coupled to the W's through a left-handed current, in order to cancel anomalies. The L^- can be tentatively identified with the heavy lepton of mass ≈ 2 GeV reported at SPEAR[6] and corroborated at DORIS.[7] This model gives the same predictions for atomic-physics parity nonconservation as the Weinberg-Salam model.

The general form of the leptonic current is

$$J_\mu=\bar{l}_c\gamma_\mu(1-\gamma_5)Ul_n, \tag{1}$$

where $l_c=(e^-,\mu^-,L^-)$ and $l_n=(\nu_1,\nu_2,L_0)$. U is a general unitary matrix. The massless neutrino produced in association with the electron, ν_e, is given by

$$(|U_{11}|^2+|U_{12}|^2)^{1/2}\nu_e=U_{11}\nu_1+U_{12}\nu_2;$$

mutatis mutandis for the muon neutrino ν_μ:

$$(|U_{21}|^2+|U_{22}|^2)^{1/2}\nu_\mu=U_{21}\nu_1+U_{22}\nu_2.$$

The known limits on hadron-lepton universality, μe universality, and nonorthogonality between ν_e and ν_μ,

$$\langle\nu_e|\nu_\mu\rangle=\frac{-U_{13}{}^*U_{23}}{(|U_{11}|^2+|U_{12}|^2)^{1/2}(|U_{21}|^2+|U_{22}|^2)^{1/2}}\approx -U_{13}{}^*U_{23}, \tag{2}$$

imply that[8]

$$|U_{13}{}^*U_{23}|<0.055.$$

If $|U_{13}{}^*U_{23}|$ is nonzero, there are several interesting consequences.

$\mu\rightarrow e\gamma$ decay.—The diagram which contributes in leading order to the decay $\mu\rightarrow e\gamma$ is shown in Fig. 1. We have calculated this amplitude to be[9]

$$M(\mu\rightarrow e\gamma)=ie(G_F/\sqrt{2})(m_\mu/32\pi^2)\epsilon U_{23}{}^*U_{13}\bar{e}\sigma_{\alpha\beta}(1+\gamma_5)\mu\epsilon^\alpha q^\beta, \tag{3}$$

where $\epsilon=(m_{L^0})^2/m_W{}^2$. The branching ratio of $\mu\rightarrow e\gamma$ to $\mu\rightarrow e\bar{\nu}_\mu$ is then

$$B(\mu\rightarrow e\gamma)=(3\alpha/32\pi)\epsilon^2|U_{23}{}^*U_{13}|^2. \tag{4}$$

If we take $|U_{23}{}^*U_{13}|^2$ to be 0.3×10^{-2} (see below) and $m_W\simeq 60$ GeV, we find $m_{L^0}\simeq 12$–30 GeV for $B=10^{-9}$.

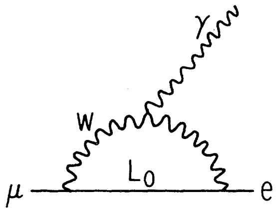

FIG. 1. One-loop diagram contribution in $\mu \to e\gamma$ via L_0.

Such a value for B can be tested very soon by an experiment in progress at the Swiss Institute for Nuclear Research.[10] The angular distribution of the decay of the polarized muon is given by $1+\cos\theta$, where θ is the angle between the direction of the electron momentum and the direction of the muon polarization. This is due to the left-handedness of our weak currents.

$\mu \to ee\bar{e}$.—In the $SU(2)\otimes U(1)$ gauge theory, there are three classes of diagrams contributing to this process: the photon exchange, the Z exchange, and the W^+W^- exchange; the calculation involved is very similar to that previously performed for the process $s \to d\mu\bar{\mu}$.[11] The final result is

$$M(\mu \to ee\bar{e}) = i\frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}\epsilon \ln\epsilon\, U_{13}{}^*U_{23}\,\bar{e}\gamma_\beta\left(\frac{1-\gamma_5}{2}\right)\mu\bar{e}\gamma^\beta e \quad (5)$$

to leading order in $\ln\epsilon$. From this we calculate a branching ratio

$$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to e\bar{\nu}_e\nu_\mu)} = \frac{3\alpha^2}{16\pi^2}\epsilon^2 \ln^2\epsilon\, |U_{13}{}^*U_{23}|^2. \quad (6)$$

For $m_{L^0}=10$ GeV, $m_W=60$ GeV, and $|U_{13}{}^*U_{23}| \simeq 0.055$, this branching ratio is equal to 0.3×10^{-10}, safely smaller than the experimental limit 6×10^{-9}. It is interesting to observe that although this result depends sensitively on the parameter of the theory, the ratio

$$\Gamma(\mu \to ee\bar{e})/\Gamma(\mu \to e\gamma) = (2\alpha/\pi)\ln^2\epsilon \quad (7)$$

varies only slowly as one changes m_{L^0}. For the values of m_{L^0} and m_W given, this ratio is equal to 0.06, somewhat larger than the level $\sim\alpha/\pi$ which one might, *a priori*, expect. The reason for this is the $\ln(1/\epsilon)$ term in the Z-exchange contribution to $\mu \to ee\bar{e}$.

Decay of L^-.—If the neutral lepton L^0 associated with the charged lepton L^- of mass 2 GeV is indeed as massive as 10 GeV there are several unusual effects. Decays of L^- such as $e^-\bar{\nu}_e L_0$ are forbidden. We have (summing over neutrino and antineutrino species)

$$\Gamma(L^- \to e\nu\bar{\nu}) \simeq (G_F{}^2/192\pi^3)(m_{L^-})^5(|U_{31}|^2+|U_{32}|^2) = \Gamma(\mu \to e\nu\bar{\nu})(m_{L^-}/m_\mu)^5(|U_{31}|^2+|U_{32}|^2). \quad (8)$$

This rate is suppressed considerably by the smallness of the mixing angles; taking $(|U_{31}|^2+|U_{32}|^2)\simeq10^{-2}$, we find $\tau(L^- \to e\nu\bar{\nu})\sim10^{-10}$ sec.

The decays $L^- \to e^-\gamma$ and $\mu^-\gamma$ are expected.[12] We have

$$\frac{\Gamma(L^- \to e\gamma)}{\Gamma(\mu \to e\gamma)} = \left(\frac{m_{L^-}}{m_\mu}\right)^5\left|\frac{U_{13}U_{33}{}^*}{U_{13}U_{23}{}^*}\right|^2 \simeq \left(\frac{m_{L^-}}{m_\mu}\right)^5|U_{23}|^{-2}. \quad (9)$$

Combining Eqs. (8) and (9), we deduce that

$$\frac{\Gamma(L^- \to e\gamma)}{\Gamma(L^- \to e\nu\bar{\nu})} = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})}\frac{1}{|U_{23}|^2(|U_{31}|^2+|U_{32}|^2)} \approx 10^{-5} \quad (10)$$

if $\Gamma(\mu \to e\gamma)/\Gamma(\mu \to e\nu\bar{\nu})$ is about 10^{-9}, and $|U_{31}|\approx|U_{32}|$.

Neutrino Reactions.—We predict a nonzero coupling of the muon neutrino to e^- and L^-. For sufficiently high incident neutrino energies where the mass differences may be neglected, we get

$$\sigma(\nu_\mu N \to \mu^-X):\sigma(\nu_\mu N \to e^-X):\sigma(\nu_\mu N \to L^-X) = (1-|U_{23}|^2)^2:|U_{23}U_{13}{}^*|^2:|U_{23}U_{33}{}^*|^2. \quad (11)$$

The second reaction gives the upper bound for $|U_{23}U_{13}{}^*|^2$ which we estimate[8] as no bigger than 10^{-2}. The third reaction is very interesting, because the L^- tracks may be observable in bubble-chamber experiments. This model does not give rise to a large high-y anomaly in the reaction $\bar{\nu}_\mu N \to \mu^+X$.

In the version of the model presently discussed, there is no neutrino oscillation. However, it is possible to endow ν_1 and ν_2 with finite, nondegenerate masses in the model; in that case, there will be neutrino oscillations, as discussed in Ref. 8.

Other phenomena.—There are several classical effects[13] discussed in the literature associated with muon number nonconservation, such as $\mu^- N \to e^- N$ and $\mu\bar{e} \to e\bar{\mu}$, but these effects are too small to have a chance for detection in this model. The decays $K_L \to \mu\bar{e}$ (or $e\bar{\mu}$) or $K_L \to \pi e\bar{\mu}$ are also difficult to detect; for the former, we have[11] in the free-quark approximation

$$M(K_L \to \mu\bar{e}) \sim \frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi}\left(\frac{m_c}{38\ \mathrm{GeV}}\right)^2 \sin\theta_c \cos\theta_c\, U_{13}{}^* U_{23} f_K \left[\bar{\mu}\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)e\right] p_\mu, \tag{12}$$

where f_K is the kaon decay constant and p^μ is the kaon four-momentum. This leads to the prediction in this model that

$$\Gamma(K_L \to \mu\bar{e})/\Gamma(K_L \to \mu\bar{\mu}) \simeq |U_{13}U_{23}{}^*|^2 \lesssim 10^{-2}. \tag{13}$$

Experimentally, $B(K_L \to \mu\bar{e}) < 2.0\times 10^{-9}$, a bound five times lower than the one on $B(K_L \to \mu\bar{\mu})$.

Note added.—After the submission of this work, we received preprints by S. Glashow and H. Fritzsch on similar matters.

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