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[1]T. G. Trippe *et al.*, Rev. Mod. Phys., Suppl. 48, S1 (1976).

[2]M. H. Shaevitz *et al.*, Phys. Rev. Lett. 36, 5 (1976).

[3]M. H. Shaevitz, Ph.D. thesis, Ohio State University, 1975 (unpublished).

[4]C. Zanfino, M.Sc. thesis, McGill University, 1976 (unpublished).

[5]A $\gamma\gamma$ mass plot which includes all three pairings (not shown) has a similar π^0 peak superimposed on a background; the ratio of peak to background at the π^0 peak is 4:1.

[6]Similar plots of square of the missing mass for events outside the η' peak do not show a clean nucleon peak, but rather a monotonically rising spectrum. This is consistent with our belief that most of the "$\omega\gamma$" events which are not η''s are really $\omega\pi^0$ events with an undetected γ.

[7]A recent experiment by the Amsterdam-CERN-Nijmegen-Oxford Collaboration has unambiguously established the spin and parity of the η' as 0^-: S. O. Holmgren *et al.*, CERN Report No. CERN/EP/PHYS 76-37, 1976 (to be published).

[8]B. T. Feld, *Models of Elementary Particles* (Ginn-Blaisdell, Lexington, Mass., 1969), pp. 92 ff., 341 ff.

[9]K. W. Edwards *et al.*, "Comparison of the Differential Cross Section for $\pi^- p \to \eta' n$ and $\pi^- p \to \eta n$ at 8.4 GeV/c", in Proceedings of the Eighteenth International Conference on High Energy Physics, Tbilisi, U. S. S. R., 1976 (unpublished); K. W. Edwards *et al.*, to be published.

[10]N. Isgur, Phys. Rev. Lett. 36, 1262 (1976).

[11]A. Kanazawa and T. Sukuma, Prog. Theor. Phys. 39, 696 (1968).

[12]D. H. Boal *et al.*, Phys. Rev. Lett. 36, 714 (1976).

[13]P. J. O'Donnell, Phys. Rev. Lett. 36, 177 (1976).

[14]S. Ono, Nuovo Cimento Lett. 14, 569 (1975).

[15]For a review see, e.g., P. J. O'Donnell, "Theory and Phenomenology of Radiative Meson Decays" (Can. J. Phys., to be published).

Estimate of the Pseudoscalar Decay Constant

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We calculate the weak decay constant of paracharmonium (η_c) using a linear potential model. The result is somewhat larger than, but of the order of, F_π and depends on the universal Regge slope α'. A number of amusing features are noted.

The weak decay constants of the pseudoscalar mesons are extremely interesting dynamical quantities which also play a fundamental role in the discussion of the *strong* interactions of these particles at low energies. Unfortunately, it has been very difficult to compute them reliably from basic considerations. For example, to get F_π one would have to solve the highly relativistic bound-state problem for the pion in terms of its constituents.

However, there is one pseudoscalar for which we do have a hope of getting a reasonably reliable estimate of the decay constant. This is the η_c which, on the basis of the success of the charmonium picture[1] for ψ/J, would be an s-wave nonrelativistic bound state of a heavy charmed quark and its antiparticle. η_c is of course distinguished by the fact that it is the only pseudoscalar which does not contain (to lowest order) any light quark. We will carry out this calculation here and find that $F(\eta_c)$ is in fact of the same order as F_π and F_K. Some observations and speculations on the reason for this will be discussed later.

For convenience we use a field-theoretic notation. A pseudoscalar-meson bound state B of a quark and antiquark is (in the rest frame) expanded as

$$|B(\vec{0})\rangle = \epsilon_{rs}[V/6(2\pi)^3]^{1/2}\int d^3p\, \varphi(\vec{p})a_r^{\dagger}(\vec{p})b_s^{\dagger}(-\vec{p})|0\rangle, \tag{1}$$

where $\epsilon_{12} = -\epsilon_{21} = 1$ and $\epsilon_{11} = \epsilon_{22} = 0$. $\varphi(\vec{p})$ is the momentum-space wave function normalized so that $\int d^3p|\varphi(\vec{p})|^2 = 1$. Also, $a_r^{\dagger}$ and $b_s^{\dagger}$ are, respectively, the appropriate quark and antiquark creation operators normalized in a volume V so that $[a_r(\vec{p}), a_{r'}^{\dagger}(\vec{p}')]_+ = \delta_{r,r'}\delta_{\vec{p},\vec{p}'}$, etc. Note that our meson state is normalized so that $\langle B(\vec{0})|B(\vec{0})\rangle = 1$. Finally summation over the three color degrees of freedom is to be understood. We need to consider the matrix element of the fourth component of the pseudovector current,

$$P_4(x) = iq'^{\dagger}(x)\gamma_5 q(x) \text{ (summed over colors)} \tag{2}$$

$[q(x)$ and $q'(x)$ are the appropriate quark field operators], between the state (1) and the vacuum. Substituting the expansion of $q(x)$ and $q'(x)$ in terms of free-field creation and destruction operators and keeping terms to lowest order in $\vec{p}$ (as appropriate to a nonrelativistic approximation) results in

$$\langle 0|P_4(0)|B(\vec{0})\rangle = i[6/(2\pi)^3V]^{1/2}\int d^3p\,\varphi(\vec{p}). \tag{3}$$

The decay constant for particle B is taken to be

$$F_B = (2V/m_B)^{1/2}|\langle 0|P_4(0)|B(\vec{0})\rangle|, \tag{4}$$

where the magnitude sign corresponds to the fact that the phase of $\varphi(\vec{p})$ is undetermined. Putting (3) into (4) and performing a Fourier transformation to $\vec{x}$ space results in the formula

$$F_B = (2\sqrt{3}/m_B^{1/2})\psi(\vec{0}), \tag{5}$$

$\psi(\vec{x})$ being the s-wave Schrödinger wave function satisfying $\int d^3x|\psi(\vec{x})|^2 = 1$. The factor $\sqrt{3}$ arises because we are using a color quark model rather than the old-fashioned one.

We will now calculate $\psi(\vec{0})$ from the Schrödinger equation using a linear potential acting between a charmed quark and a charmed antiquark. This is roughly consistent with the spectrum of the spin-1 objects[1] and is motivated by any model which effectively produces a stringlike set of excitations.[2,3] The potential is taken to be

$$V(r) = r/2\pi\alpha', \tag{6}$$

where $\alpha' \simeq 1\ \text{GeV}^{-2}$ is the universal Regge slope. The overall constant can either be derived[2] from a stringlike model or observed to agree roughly with the value obtained from fitting the spin-1 spectrum. Note[4] that the addition of a small Coulomb or Yukawa term to $V(r)$ has little effect on $\psi(0)$. The normalized $l = 0$ wave function is then

$$\psi(\vec{x}) = \frac{1}{(4\pi)^{1/2}r}\left(\frac{m}{2\pi\alpha'}\right)^{1/6}\frac{1}{\text{Ai}'[-(m/2\pi\alpha')^{1/3}(2\pi\alpha'E)]}\text{Ai}\left[\left(\frac{m}{2\pi\alpha'}\right)^{1/3}(r-2\pi\alpha'E)\right], \tag{7}$$

where m is the mass of the charmed quark, Ai is the Airy function, $\text{Ai}'(z) \equiv d\text{Ai}(z)/dz$, and E is the bound-state energy eigenvalue. Equation (7) results simply in

$$\psi(\vec{0}) = (m/8\pi^2\alpha')^{1/2}. \tag{8}$$

Substituting (8) into (5) gives the decay constant

$$F(\eta_c) = (2\pi)^{-1}[6m/m(\eta_c)\alpha']^{1/2}. \tag{9}$$

We may simplify this formula further (with about 15% accuracy) by neglecting the binding energy so that we have the easy relation

$$F(\eta_c) \simeq (2\pi)^{-1}(3/\alpha')^{1/2}. \tag{10}$$

From (9) we get the numerical result

$$F(\eta_c) = \begin{cases} 1.78F_\pi & \text{for } \alpha' = 1\ \text{GeV}^{-2}, \\ 1.86F_\pi & \text{for } \alpha' = 0.89\ \text{GeV}^{-2}, \\ 1.97F_\pi & \text{for } \alpha' = 0.76\ \text{GeV}^{-2}. \end{cases} \tag{11}$$

Note that the binding energy was taken into account in arriving at (11) [specifying α' gives m since $m(\eta_c) - 2m = 2.338m^{-1/3}(2\pi\alpha')^{-2/3}$]. We have taken $m(\eta_c)$ to be 2750 MeV; by (10) we see that its exact value is not critical for our result. The three values of α' used in (11) correspond, respectively, to rule-of-thumb, ρ-trajectory, and ψ/J determinations. Clearly the most striking feature of this result is that the value of $F(\eta_c)$ is of the same order as F_π and F_K. Since

$$F_K/F_\pi \simeq 1.1 \text{ to } 1.28, \tag{12}$$

(depending on the exact value of the Cabibbo angle) it also seems natural that $F(\eta_c)$ should be larger, rather than smaller, than F_K. If we were to take $F(\eta_c)$ roughly the same as F_π we might then interpret (10) as setting the scale for low-energy pion phenomena in terms of the parameter α' which governs high-energy phenomena. A number of authors[5] have speculated that some such relation should exist.

As a check on our procedure we can apply the above method to calculate

$$(2k_0V)^{1/2}\langle 0|J_\mu^{\rm EM}|\psi(\vec{k})\rangle = F_\psi\epsilon_\mu \tag{13}$$

for orthocharmonium, ψ. This is related to experiment by $\Gamma(\psi\to e^+e^-)=F_\psi^2e^4/12\pi m_\psi^3$. Neglecting the ψ-η_c mass difference we can state the result in the form of a Kawarabarashi-Suzuki-Fayyazuddin-Riazuddin–type relation:

$$|F_\psi|/m_\psi=\tfrac{2}{3}|F(\eta_c)|. \tag{14}$$

Actually F_ψ has been already evaluated[6] with a linear potential model; the result is in reasonable agreement with experiment. Note that, like the $\pi^0\to 2\gamma$ matrix element, the decay constants here provide a test of the color theory. In an old-fashioned quark model without color the predicted $\Gamma(\psi\to e^+e^-)$ would be reduced by a factor $\frac{1}{3}$.

How can we undrstand the physics of the situation where all pseudoscalars have F's of the same order? This is of course what one would expect if they behaved like Nambu-Goldstone bosons. However at first glance the η_c, being a nonrelativistic bound state of two heavy objects, would appear very different from the nearly zero mass, highly relativistic collective objets one considers as Nambu-Goldstone bosons. Nevertheless there are some highly suggestive features of the present calculation which indicate that the η_c may be "trying" to behave in this way. First, consider the usual partially conserved axial-vector current (PCAC) relation[7] for the pseudovector current in (2):

$$\partial_\mu P_\mu = 2im\bar{q}\gamma_5 q. \tag{15}$$

Sandwiching this between the vacuum and (1) and proceeding as before gives

$$m(\eta_c)=2m, \tag{16}$$

which amounts simply to neglecting the binding energy. It is amusing that this consistency with PCAC becomes better as the constituent particles become heavier and more nonrelativistic. Putting (16) into (5) then shows that $F(\eta_c)\propto\psi(0)/m^{1/2}$. The linear potential has the unique feature (see the following) that $\psi(0)\propto m^{1/2}$ so that, with the assumption of (16), $F(\eta_c)$ is independent of m. If the extrapolation to zero mass[8] is allowed, we find that $F(\eta_c)$ of course is nonzero in the limit. This is the usual signature of a Goldstone boson!

In a picture where all sixteen 0^- objects are Nambu-Goldstone bosons it becomes plausible to adopt a sigma model or current-algebra approach for them. The situation is then as follows. In a general version[9] of the linear SU(4) σ model there is enough freedom so that one can choose all F's to have the same order of magnitude. If this model is restricted to be renormalizable,[10] the F's, at tree order, for the particles containing charmed quarks are $\gtrsim 5F_\pi$. Both sigma models predict too large a value for $m(D)$. However in a current-algebra approach which encompasses the sigma-model results as a special case, the value of $F(\eta_c)$ gotten here can be used as an input to find[11] a satisfactory fit to all presently known masses with all F's of the same order.

Finally, it is natural (though not really justified) to try to apply the simple linear potential model also to mesons containing light quarks. If we assume (6) to hold exactly for all mesons, we would immediately get in trouble with the predicted mass spectrum (see Ref. 2, for example). We would also get in trouble with F_K/F_π. This may be seen either from a direct analysis similar to the above or from the following consideration. Suppose that a meson is composed of two constituents with reduced mass μ and can be described by a Schrödinger equation with $V=\text{const}\times r^s$, s being arbitrary. Then by dimensional analysis we can derive[12] the formula

$$\psi(\vec{0})=\mu^{3/2(s+2)}\tilde{\psi}(\vec{0}). \tag{17}$$

where $\tilde{\psi}(\vec{0})$ has no dependence on μ. Applying (5) and (17) to the ratio F_K/F_π gives

$$\frac{F_K}{F_\pi}=\left(\frac{m_\pi}{m_K}\right)^{1/2}\left(\frac{2m_3}{(m_1+m_3)}\right)^{3/2(s+2)}, \tag{18}$$

where m_1 is the mass of the first quark and m_3 is the mass of the third quark.

Taking $s=1$ (the linear potential) in (18) and either a "constituent" determination of quark masses, $m_3/m_1\simeq\frac{5}{3}$, or a "current" determination, $m_3/m_1\simeq 25$, would give $F_K\simeq F_\pi/2$ in contradiction with (12). To get F_K/F_π around unity in (18) requires $s\simeq -1.5$ with $m_3/m_1=\frac{5}{3}$. This result crudely suggests that the binding potential for light quarks contains an important Coulomb (or perhaps Yukawa) piece in addition[13] to the linear one. Such a model has been suggested by a number of authors[1-3] for a variety of reasons. The overall picture would be one in which the heavy charmed quarks and high excitations of the light ones would mainly feel the linear "confining" potential. For light quarks, the Coulomb-like forces may be important and reliable calculation may involve field-theoretic techniques.

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[1]T. Appelquist and H. D. Politzer, Phys. Rev. Lett. 34, 43 (1975); E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, Phys. Rev. Lett. 34, 369 (1975); B. J. Harrington, S. Y. Park, and A. Yildiz, Phys. Rev. Lett. 34, 168 (1975); J. F. Gunion and R. S. Willey, Phys. Rev. D 12, 174 (1975); J. S. Kang and H. J. Schnitzer, Phys. Rev. D 12, 841 (1975).

[2]A. P. Balachandran, R. Ramachandran, J. Schechter, K. C. Wali, and H. Rupertsberger, Phys. Rev. D 13, 361 (1976).

[3]See Y. Nambu, Phys. Rev. D 10, 4262 (1974).

[4]See, for example, Kang and Schnitzer, Ref. 1.

[5]L. S. Brown and R. L. Goble, Phys. Rev. D 4, 723 (1971); H. Pagels, Nuovo Cimento 17A, 1 (1973); M. R. Pennington and C. Rosenzweig, Nucl. Phys. B57, 305 (1973); L. Susskind, A. Casher, and J. Kogut, Phys. Rev. D 8, 4448 (1973).

[6]See Harrington, Park, and Yildiz, Ref. 1.

[7]To solve the U(1) problem it may be necessary to modify the coefficient on the right-hand side of (15). See V. Mirelli and J. Schecter, Phys. Rev. D 15, 1361 (1977). Also m in (15) should be the "current" quark rather than the "constituent" quark; for the heavy charmed quark we are considering these two to be close [see H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976)].

[8]Note that if we did not use the PCAC relation (16) we would find $F(\eta_c) \rightarrow 0$ as $m \rightarrow 0$. This follows from (9) and the fact that $m(\eta_c) - 2m \propto m^{-1/3}$. Of course, our extrapolation is taken without regard for the validity of the nonrelativistic approximation as $m \rightarrow 0$.

[9]J. Schechter and M. Singer, Phys. Rev. D 12, 2781 (1975); Y. Ueda, "New Resonances in the Linear Chiral SU(4)×SU(4) Model" (to be published).

[10]J. Kandaswamy, J. Schechter, and M. Singer, Phys. Rev. D 13, 3151 (1976); M. Vaughan, Phys. Rev. D 13, 2621 (1976). See also S. S. Gershtein and M. Yu. Khlopov, Pis'ma Zh. Eksp. Teor. Fiz. 23, 374 (1976) [JETP Lett. 23, 338 (1976)].

[11]J. Kandaswamy, J. Schechter, and M. Singer, to be published. With an exact fit to the U(4) pseudoscalar masses we get a typical solution $F_K/F_\pi = 1.19$, $F_D/F_\pi = 1.48$, $F_F/F_\pi = 1.61$, and (input) $F(\eta_c)/F_\pi = 1.78$.

[12]For the potential $V = -cr^s$ we may write the Schrödinger equation (with $\hbar = 1$) as $(\nabla^2 + 2\mu c r^s + \epsilon)\psi = 0$ with $\epsilon = 2\mu E$. We regard this as an equation for both ψ and ϵ. Equation (17) follows when we make use of the fact that the only dimensional quantity on which $\psi(\vec{0})$ and $\epsilon = 2\mu E$ can depend is the combination μc.

[13]This may also provide an explanation of the Van Royen–Weisskopf puzzle. B. Van Royen and V. F. Weisskopf, Nuovo Cimento 50, 617 (1967); C. H. Llewellyn Smith, Phys. Lett. 28B, 335 (1968).