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Gauge-Independent Two-Body Amplitudes and Vector Dominance Model of Electromagnetic Form Factors*

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I argue that in a gauge theory of absolute quark confinement two-body amplitudes of mesons are nonvanishing only on an equal-time plane in the rest frame of mesons. The vector dominance model of electromagnetic form factors follows as a consequence. The parton model for inelastic processes would remain valid.

I will argue that a gauge theory of complete quark confinement leads to a two-body description of mesons which is entirely different from the conventional field-theoretical picture. I start from the following postulates: (P1) Color triplet states have infinite masses[1] (absolute confinement). (P2) Physical states are invariant under local gauge transformations. (P3) In spite of P1, it is possible to use a local quark field $q(x)$ to define certain matrix elements, whenever such matrix elements are nonvanishing. P3 is obviously necessary to make any field-theoretical arguments. I believe that P3 could be justified in the sense of the correspondence principle. For simplicity of presentation I will discuss the case of quantum electrodynamics (QED), indicating later how to generalize to quantum chromodynamics (QCD).

In QED, P1 is replaced by P1′: States with quark number ± 1 have infinite masses. An immediate consequence of P1 or P1′ is that the Bethe-Salpeter (B-S) amplitude for a quark-antiquark system

$$\chi^{\mathrm{BS}}(1,2)=(\Psi_0,[q(1)\bar{q}(2)]_+\Psi), \tag{1}$$

where $[\ ]_+$ denotes the time ordering, vanishes. In fact, inserting $\sum_n \Psi_n)(\Psi_n$ between $q(1)$ and $\bar{q}(2)$ for $t_1>t_2$, we obtain the spectral representation of $\chi^{\mathrm{BS}}(1,2)$, which is an integral of $\Delta^{(+)}(x_1-x_2; M_n^2)$ over the intermediate-state mass M_n^2. Such a function vanishes exponentially in the whole spacelike region as $M_n^2\to\infty$. Hence the Wick-rotated amplitude will vanish everywhere.

In order to formulate P2 we need to specify the gauge. We will take the "spacelike" gauge $n_\mu A^\mu=0$, where n_μ is a unit timelike vector. The Coulomb gauge is not appropriate because all components of the electric field, transverse as well as longitudinal, will contribute to the confinement potential. It seems difficult to translate the results that we obtain into the Lorentz or Gupta-Bleuler gauge, which would be possible if there were a unitary transformation linking the gauge that we use with the latter. The formulation in the Lorentz gauge is an unsolved problem. An additional advantage of the spatial gauge is that the canonical quantization can be done in a straightforward way.

For one-hadron states, a natural choice for n_μ is the energy-momentum vector of the hadron, p_μ. Thus, without loss of generality we will hereafter take the rest frame $\vec{p}=0$, and there we have $A_0=0$. In this gauge we will have a limited local gauge transformation

$$\vec{A}'(x)=e^{i\Lambda}\vec{A}(x)e^{-i\Lambda}=\vec{A}(x)-\nabla\lambda(x),$$
$$q'(x)=e^{i\Lambda}q(x)e^{-i\Lambda}=e^{-ie\lambda(x)}q(x), \tag{2}$$

where the generator Λ is given by

$$\Lambda = \int \sigma(x)\lambda(x)\, d^3x, \tag{3}$$

with

$$\sigma(x) = \rho(x) - \nabla\cdot\vec{E}(x). \tag{4}$$

Here $\rho(x)$ is the charge density $\rho(x) = eq^\dagger(x)q(x)$; and $\vec{E}(x) = -\dot{\vec{A}}(x)$ is the electric field. $\lambda(x)$ must be time independent, i.e., $\dot{\lambda} = 0$. The field equation insures that $\dot{\sigma}(x) = 0$, and the postulate P2 states simply that

$$\sigma(x)\Psi = \Lambda\Psi = 0. \tag{5}$$

In other words, P2 requires that Gauss's law holds as a subsidiary condition on any physical state. I stress that Eq. (5) does not select a class of particular states. In fact, it is easy to show that by a certain unitary transformation, Eq. (5) reduces to a single condition on the vacuum of the longitudinal field, $\nabla\cdot\vec{E}\Psi_0 = 0$, which can be trivially satisfied.

The B-S amplitude (1) is not invariant under local gauge transformations (2) in the gauge used here and therefore vanishes without invoking P1′. Thus, we have to adopt a gauge-invariant amplitude, an immediate choice being

$$\chi_C(1, 2) = \left(\Psi_0, [q(1)\exp(-ie\int_2^1 \vec{A}(x)\cdot d\vec{x})\bar{q}(2)]_+\Psi\right), \tag{6}$$

where C denotes a path along which the integration is taken. It may look very peculiar that we have to abandon the B-S amplitude in the ordinary QED, but it is nothing but a peculiarity of the present gauge. In fact, if we take as $\vec{A}$ in (6) the longitudinal part of it, $\vec{A}_L$, $\chi_C(1, 2)$ is still gauge invariant. The line integral is then path independent and integrable. We can show easily that the amplitude $\chi_C(1, 2)$ reduces to the B-S amplitude in the Coulomb gauge, which would vanish if we invoke P1′. In the normal phase of the QED, P1′ is not valid and the amplitude (6) with $\vec{A} = \vec{A}_L$ is perfectly legitimate. If, however, there exists a confinement phase in QED, as suggested by some confinement models,[2,3] then we must necessarily include the transverse part of $\vec{A}$ in (6) to avoid the vanishing of the amplitude.

With the full $\vec{A}$ taken, the amplitude (6) depends on the path C in an essential way with Heisenberg operators continuously distributed along the path C. Inserting $\sum_n \Psi_n)(\Psi_n$ at a single point on the path does not produce an infinite time oscillation present in the B-S amplitude. Yet $\chi_C(12)$ vanishes unless $t_1 = t_2$. To see this, we divide the line integral into N infinitesimal sections. For $t_1 > t_2$, we use a parametric representation of the path C, $\vec{x} = \vec{x}(\zeta)$ with $\zeta = t - t_2$. Then by introducing $\zeta_n = (n/N)(t_1 - t_2)$ and $x_n = x(\zeta_n)$, we have

$$\chi_C(1,2) = \lim_{N\to\infty} (\Psi_0, q(1)\prod \exp[-ieA(x_n, \zeta_n)\cdot(x_n - x_{n-1})]\bar{q}(2)\Psi).$$

Now the time dependence can be factored out by using $\vec{A}(x_n, \zeta_n) = \exp(i\zeta_n H)\vec{A}(x_n, 0)\exp(-i\zeta_n H)$ and inserting a complete set of states in between every pair of neighboring operators. The result is

$$\chi_C(1,2) = \sum_{\{\alpha_n\}} F[\{\alpha_n\}, \{x_n\}]\exp(-it\sum_{n=1}^N E_{\alpha n}/N)$$

$$= \int_M^\infty dE\, w(E) e^{-iEt}, \tag{7}$$

where

$$w(E) = \sum_{\{\alpha_n\}} F[\{\alpha_n\}, \{x_n\}]\delta(E - \sum_{n=1}^N E_{\alpha n}/N).$$

In the above, α_n denotes intermediate states inserted at the nth junction. $t = t_1 - t_2$, and I have taken $t_2 = 0$ for simplicity. M is the threshold for the intermediate states and the limit $M\to\infty$ must be taken in accordance with P1′. In this limit $\chi_C(1, 2)$ will vanish for $t_1 - t_2 \neq 0$ by virtue of the Riemann-Lesbegue lemma, provided that $w(E)$ satisfies certain convergence conditions. What happens for $t_1 = t_2$ depends critically on the behavior of the weight function $w(E)$. In the case of the B-S amplitude (1), $w(E)$ has an oscillatory factor $\sin[(E^2 - M^2 r)^{1/2}]$ ($\vec{r} = \vec{x}_1 - \vec{x}_2$), so that it vanishes even for $t_1 = t_2$. I assert that such an oscillation is very unlikely for the amplitude (6). In fact, for a straight-line path C one can extract the corresponding oscillatory factor in F, which is $\exp(i\vec{x}\cdot\sum_n \vec{p}_{\alpha n}/N)$. The average momentum

$$\sum_{n=1}^N \vec{p}_{\alpha n}/N$$

will not grow as fast as the average energy E, contrary to the case of $N = 1$. Thus, barring the oscillation of $w(E)$, I maintain that the amplitude $\chi_C(1, 2)$ is nonvanishing only on the equal-time plane in the rest system. If furthermore $w(E) \to E^p$ as $E\to\infty$, then

$$\lim_{M\to\infty} M^{-p}\chi_C(1, 2) \propto (it)^{-1} e^{iMt},$$

so that with an appropriate normalization we may write $\chi_C(1,2) \propto \delta(t_1 - t_2)$. I will assume in the following that in fact this is the case. This behavior of the amplitude does not violate the relativistic invariance. If we move to another Lorentz frame $\vec{p} \neq 0$, the equal-time plane is no longer equal-time, but A_0 no longer vanishes either. The line integral in (6) must be replaced by $\int_2^1 A_\mu(x)dx^\mu$ and the simple factorization of the time dependence such as (7) is no longer possible. Thus we arrive at a covariant version of the above statement,

$$\chi_C(1;2) = \delta(p \circ (x_1 - x_2)) \exp(ip \circ x_1)\tilde{\chi}_C(x_1 - x_2). \tag{8}$$

Our picture of the two-body amplitude (8) may be related to the string-model picture[4] where a string sweeps a two-dimensional strip in space-time. However it is not obvious how to relate them.

The vector dominance model of the electromagnetic form factors[5] is a consequence of the four-dimensionally flat structure of hadrons as expressed in Eq. (8). The quark and the antiquark, restricted to an equal-time plane in the rest frame of the initial meson, cannot adjust themselves instantaneously with emission of a photon to an equal-time plane in the rest frame of the recoil particle. Thus, bare electromagnetic vertices for elastic and inelastic one-particle transitions should vanish. On the other hand, a three-meson vertex is finite since the overlap integral

$$\int d^4x_1 d^4x_2 d^4x_3\, \delta(p_1 \cdot (x_1 - x_2))\, \delta(p_2 \cdot (x_2 - x_3))\, \delta(p_3 \circ (x_3 - x_1)) \tag{9}$$

is nonvanishing. Therefore, a photon can couple to two mesons only through vector mesons. This suppression of the bare electromagnetic vertex obviously fails for inelastic processes involving more than two hadrons in initial and final states. The amplitude for such a process would involve a nonvanishing overlap integral like (9) with a photon emitted at one of the points x_i. Hence we may expect that the parton model for deep inelastic processes would remain to be valid.

A gauge-invariant two-particle amplitude for mesons in QCD, corresponding to (6), is given by

$$\chi_C(1,2) = \left(\Psi_0, \mathrm{Tr}^c[\exp(ig\int_1^2 \tilde{A}(x)\cdot dx) q(1)\overline{q}(2)]_+ \Psi\right), \tag{10}$$

where

$$\tilde{\mathrm{A}}(x) = \sum_{a=1}^{8} \tfrac{1}{2}\lambda_a \vec{\mathrm{A}}^a(x).$$

$\{\lambda_a\}$ is a set of eight color-spin matrices. $q(1)\overline{q}(2)$ is regarded as a matrix in spinor indices, flavor SU(4) indices, and color-spin indices. Tr^c represents a trace with respect to the color-spin, projecting out the color-singlet operator. The ordering is now along the path C, and is essential because of the presence of the color-spin matrices λ_a. The previous argument for $\chi_C(1,2)$ can be repeated without any change.

A gauge-invariant amplitude for baryons in QCD is given by

$$\chi_{C_1,C_2,C_3}(1,2,3;\vec{Z}) = \epsilon_{ijk}(\Psi_0, U_{C_1}{}^i(Z,1) U_{C_2}{}^j(Z,2) U_{C_3}{}^k(Z,3)\Psi), \tag{11}$$

where

$$U_{C_1}(Z,1) = [\exp(ig\int_1^z \tilde{A}(x)\cdot dx)]_+ q(1). \tag{12}$$

C_1, C_2, and C_3 are the paths connecting the point Z with 1, 2, and 3. U^i represents ith color-spin component of U. It is not difficult to prove the gauge invariance of the amplitude (11). It will again vanish unless $t_1 = t_2 = t_3$. How to obtain the two-body or three-body wave equation to determine the energy eigenvalue of hadrons starting from the amplitudes (10) and (11) will be discussed elsewhere.

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[1]I will make a minimum assumption here. It could be that all states except for color-singlet states have infinite energies.

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