Profile Modifications and Plateau Formation Due to Light Pressure in Laser-Irradiated Targets

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A steady-state spherical model has been used in this Letter to study the influence of light pressure on plasma flow and density profile modifications. Whereas in former investigations transitions from subsonic to sonic or supersonic flow and the concomitant formation of density steps have been studied in plane models, this paper treats the transition from supersonic to supersonic flow; and the new phenomenon of density plateau formation just below the critical density is found.

It was stated years ago that light pressure may play a dominant role in the dynamics of laser-produced plasmas.[1] On the basis of the WKB approximation for the wave equation, strong overall acceleration of the plasma was predicted at that time. Subsequent calculations based on the wave equation predicted a much less dramatic increase of the electric field in the critical region where the plasma frequency equals that of the laser, and no appreciable plasma acceleration was found.[2] Because of reflection of the incident wave and the resulting superposition, the light pressure assumes an oscillatory character, on the basis of which local plasma density distortions[2] and striations[3] have been postulated. Density profile modifications at the critical density were first explicitly calculated in 1971: A deep density step was found at the critical point from which the plasma streams out at sound velocity.[4] A simultaneous solution of hydrodynamics and the wave equation showing the interaction and mutual influence of hydrodynamics and light has only very recently been given.[5] The calculation in Ref. 5 is based on a stationary isothermal model in plane geometry. Without the additional light pressure, the only possible solution is a constant density in space and time. However, when a light wave interacts with the plasma flow, a steplike density transition from subsonic to supersonic flow forms at the critical point.

Laser plasmas are divergent and expand into vacuum in most experimental arrangements. Therefore, the assumptions of plane geometry and steady state are somewhat contradictory. Only structures of small extent, e.g., steplike density transitions, can be studied under plane conditions. To be closer to reality, we base our investigations of light-pressure effects on spherical geometry. In this way, the important effect of radial divergence can be introduced and, as we shall see, the new phenomenon of plateau formation just below the critical density will appear when the outflow in the critical region is supersonic. In this geometry, the modulations of density and flow velocity due to the standing-wave pattern are no longer constant and an estimate of their attenuation with distance can be given. Furthermore, a spherical model is much more compatible with the assumption of a steady state.[6]

Steady-state hydrodynamics in spherical geometry is described by conservation of mass,

$$n_e v r^2 = n_0 v_0 r_0^2, \tag{1}$$

and conservation of momentum,

$$v\frac{\partial v}{\partial r} = -\frac{Z}{n_e m_i}\frac{\partial p}{\partial r} - s^2\mu\frac{\partial |E_0|^2}{\partial r}; \qquad \mu = \frac{Ze^2}{4m_e m_i s^2\omega^2}. \tag{2}$$

We replace the energy equation by a polytropic equation of state:

$$p n_e^{-\gamma} = p_0 n_0^{-\gamma}. \tag{3}$$

The symbols have the usual meaning, the subscript "0" referring to an arbitrary but fixed point r_0. The sound velocity s is given by

$$s^2 = \frac{Z}{m_i}\frac{\partial p}{\partial n_e} = \frac{Z}{m_i}\gamma\frac{p}{n_e}, \quad s^2 = s_0^2\left(\frac{n_e}{n_0}\right)^{\gamma-1}. \tag{4}$$

The second term on the right-hand side of Eq. (2) represents the gradient of light pressure per unit mass, E_0 being the local amplitude of the light wave of frequency ω. The light pressure acts on the electrons and is transmitted to the ions by electrostatic fields E_s, which, in principle, must be included in Eq. (2): $\partial|E_0+E_s|^2/\partial r$. However, we treat the case of normal incidence and the most uniform possible irradiation. Then E_s is not resonant and can be neglected because $|E_s/E_0| = v_{osc}/c \ll 1$.[7] For realistic targets the critical radius r_c is much greater than the vacuum wavelength λ_0, and the distribution of E_0 is governed in good approximation by the wave equation for $y = \mu^{1/2} R E_0$ [7,8]

$$\partial^2 y/\partial R^2 + (1 - n_e/n_c) y = 0, \tag{5}$$

where $R = rk$, k is the wave vector, and n_c is the critical electron density. The error due to omission of the angular derivatives in Eq. (5) is of the order of $1/kr \lesssim 10^{-3}$. Introducing the Mach number $M = v/s$ and combining Eqs. (1), (2), and (4), one gets

$$\frac{\partial M}{\partial R} = \frac{(2/R)[1 + \frac{1}{2}(\gamma - 1)M^2] - \mu \partial |E_0|^2/\partial R}{M - M^{-1}}. \tag{6}$$

Among the cases with various γ values the simplest one to treat is that of $\gamma = 1$, which describes a steady-state isothermal rarefaction wave, i.e., the temperature and sound velocity are constant everywhere [see Eq. (4)]. There are also physical arguments for this assumption: electron temperature measurements by x-ray emission[9] and the presence of a hot-electron component with large mean free path at high laser intensities.[10] In addition, numerical calculations show that isothermal conditions are an acceptable assumption if heat conduction is not drastically reduced in the critical region.[6,11] Then Eq. (6) reduces to

$$\frac{\partial M}{\partial R} = \frac{(2/R) - \mu \partial |E_0|^2/\partial R}{M - M^{-1}}, \tag{6'}$$

or, by integration, to

$$M^2 - \ln M^2 = M_0^2 - \ln M_0^2 + 2\mu(|E_0|_0^2 - |E_0|^2) + 4\ln(R/R_0). \tag{7}$$

This generalized Bernoulli equation relates the Mach number at each position r to that at the fixed point r_0. The right-hand side expression $M^2 - \ln M^2$ of Eq. (7) tends to infinity when M tends to both infinity and zero (Fig. 1). At the sonic point ($M = 1$) it reaches its minimum. Without light pressure there are two distinct solutions, one with increasing Mach number over the radius and the other with Mach number tending to zero as r tends to infinity. Since no transitions between the two solutions are possible, the latter with $M \leq 1$ everywhere must be excluded for physical reasons. Figure 2 shows a particular result of an isothermal rarefaction wave without light pressure for light of $\lambda_0 = 1.315$ μm (iodine laser) which impinges from the right-hand side onto a pellet of critical radius r_c slightly larger than $r_0 = 50$ μm. The Mach number M_0 chosen was $M_0 = 1.5$ at r_0. The density and velocity distributions n_e/n_c and M are smooth and the electric field amplitude (solid line) has the characteristic pattern of a standing wave due to reflection at the critical point r_c.

Figure 3 shows for the same Mach number $M_0 = 1.5$ at r_0 how hydrodynamic changes when light pressure is taken into account. The dashed lines again represent the undisturbed density and Mach-number distributions. The most characteristic

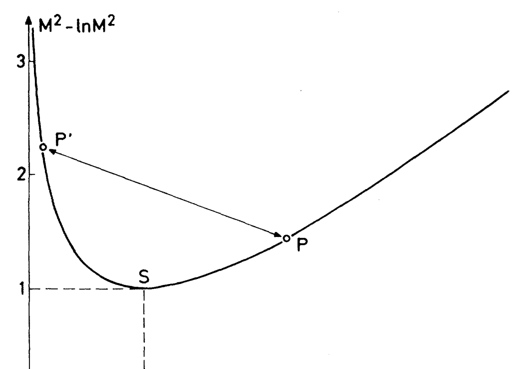

FIG. 1. Curve of $M^2 - \ln M^2$ as a function of M^2 shows, in connection with Bernoulli's equation (7), that without light pressure a steady-state isothermal rarefaction wave can exist only for $M \leq 1$, the $M < 1$ branch being forbidden. When light pressure is present transition from one to the other branch is possible (in the special case of Fig. 3 from P to P').

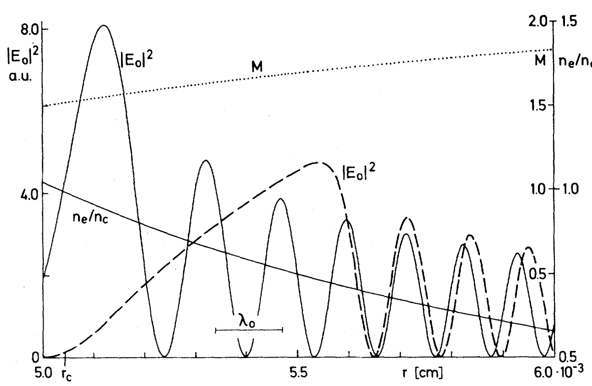

FIG. 2. Distributions of density n_e/n_c, Mach number $M = v/s$ and electric field amplitude $|E_0|^2$ (solid line) over the radius r when radiation pressure is omitted. The iodine laser ($\lambda_0 = 1.315$ μm) impinges from the right. The dashed line is the amplitude distribution of Fig. 3 when light pressure is taken into account. Laser intensity at the critical radius r_c is 2.2×10^{24} W/cm^2, $T_e = T_i = T = 1$ keV.

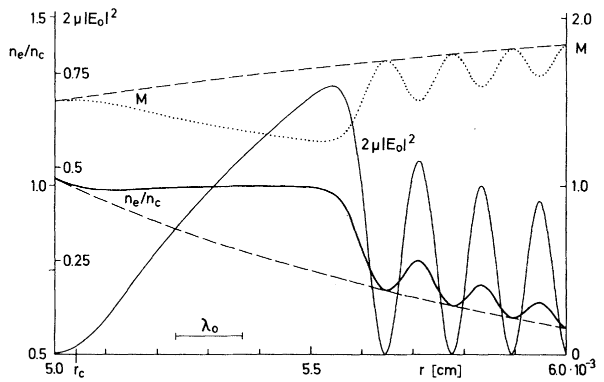

FIG. 3. Plateau formation and striations in the density n_e/n_c as a consequence of radiation pressure. The laser beam impinges from the right. The laser intensity at the critical radius is 2.2×10^{14} W/cm^2, $T=1$ keV. The density maxima coincide with the maxima of the electromagnetic energy density; in a subsonic flow they would coincide with the minima of $|E_0|^2$. The dashed lines show the density and velocity distributions without radiation pressure.

feature of Fig. 3 is the formation of a density plateau just below the critical density ($n_e/n_c = 0.998$) and the broadening of the first $|E_0|^2$ maximum. From Bernoulli's Eq. (7) it follows that striations of density appear at the right-hand side because of the modulated amplitude E_0. At a given intensity the more pronounced they are, the less the flow velocity is and we have

$$\frac{\Delta n_e}{n_e} \simeq \frac{2\mu|E_0|^2}{M^2}. \qquad (8)$$

At the points with $E_0 = 0$ the density and Mach number become identical with the undisturbed values of Fig. 2. In order to show by direct comparison how the electric field distribution changes when light pressure is taken into account, in Fig. 2 the $|E_0|^2$ curve of Fig. 3 is drawn as a dashed line for the same incident laser intensity. Both correspond to a laser intensity of 2.2×10^{14} W/cm^2 at r_c if $T_e = T_i = 1$ keV is assumed. Note also the reduction of field strength with respect to the undisturbed one.

The minimum Mach number is reached at the end of the plateau, where $2/R = \mu\partial|E_0|^2/\partial R$ is satisfied, i.e., very near the maxima of $|E_0|^2$. The extension of the plateau Δr is determined from Eqs. (1) and (7) by setting $n_e = n_0$: $r^2M_{\rm min} = r_0^2M_0$ and $M_0^2 - M_{\rm min}^2 = 2\mu|E_0|_{\rm max}^2$, which immediately yield

$$\frac{\Delta r}{r_0}\left(1 - \frac{3}{2}\frac{\Delta r}{r_0}\right) \frac{\mu|E_0|_{\rm max}^2}{2M_0^2}. \qquad (9)$$

As the laser intensity increases the minimum Mach number approaches unity, the plateau reaches its maximum length $\Delta r/r_0 = M_0^{1/2} - 1$ and the corresponding intensity is given by $|E_0|_s^2 = (M_0^2 - 1)/2\mu$.

At the $M = 1$ point, one deduces for the slope of M from Eq. (6′)

$$\frac{\partial M}{\partial R} = \pm\left(-\frac{1}{R^2} - \frac{\mu}{2}\frac{\partial^2}{\partial R^2}|E_0|^2\right)^{1/2}.$$

For the slope at the right-hand side of the sonic point the + sign must be chosen because according to Eq. (7) the flow has to be supersonic in all cases for which the maximum of $|E_0|^2$ near the critical point is highest. However, at the left-hand side of the sonic point, both the + and the − signs are possible, i.e., bifurcation of the solution occurs. For the case with $M_0 > 1$ treated here the − sign is valid. If the + sign is taken, the flow becomes subsonic in the whole region at the left of the sonic point, and the distribution of density, velocity, and $|E_0|^2$ behave very differently: Instead of a plateau, a steep density step forms as calculated in Ref. (5), which causes the electromagnetic wave to become evanescent over a very short distance (skin depth). Here the Mach number reaches its lowest value M_0 ($E_0 \simeq 0$) and then increases to the left up to $M = 1$ and the density decreases, i.e., when light pressure is present transition from subsonic to supersonic flow can also occur (see Fig. 1). M_0 is now related to the maximum of $|E_0|^2$ according to Eq. (7) by $M_0^2 - \ln M_0^2 = 1 + 2\mu|E_0|_s^2$ [e.g., the subsonic M_0 value associated with the supersonic one of Fig. 3 ($M_0 = 1.5$) is $M_0 = 0.345$]. At a given $M_0 > 1$ a plateau forms just below the critical density for laser intensities not exceeding the limit $|E_0|_s^2$, whereas for the steplike solution all quantities are uniquely determined by the value of $M_0 < 1$.

In a steady state we now have the following picture: As the light intensity increases, a more and more pronounced density plateau forms until the electric field reaches the limiting value $|E_0|_s$ depending on M_0. At this point, two different situations are possible: (i) the plateau is destroyed and a steplike profile builds up with a subsonic M_0; (ii) by heating the plasma further and/or by increasing the overdense region due to energy transport, the supersonic M_0 increases and hence the limit $|E_0|_s$ for the existence of a plateau. However, the authors cannot exclude the possibility that above a certain intensity no steady-state solution, either plateau or step, will exist. For a steplike solution, an overdense subsonic flow region is needed. On the other hand, it has been shown in Refs. 6, 11, and, in particular Ref. 12, that if heat conduction is not drastically reduced M_0 (taken at $E_0 \simeq 0$) is supersonic. We conclude, therefore, that for steplike solutions strong reduction of heat conduction is needed.

The importance of radiation-pressure profile modifications lies in the fact that light absorption will be modified. Furthermore, it appears as evident that the thresholds for instabilities, calculated in a WKB-like manner (see, for example, Rosenbluth[13]) for inhomogeneous plasmas, cannot be expected to be correct. As a matter of fact, none of the many instabilities predicted analytically and localized at the critical point has been identified in a convincing manner in a laser plasma experiment. The calculations presented in this paper were performed without absorption. As far as absorption follows Beer's exponential low (e.g., inverse bremsstrahlung), no modifications have been seen because in this case local absorption is very low.

It should be pointed out here that the steplike solution in spherical geometry which is obtained for $M_0 < 1$ has practically the same structure as in the plane case (see Ref. 5) as long as only a narrow region around the critical point is considered. However, no bounded, i.e., physically correct, solution for the electric field is obtained in plane geometry if one starts with $M_0 > 1$, because in this case maintains its initial curvature with growing R, as can be seen from Eqs. (5) and (6′) with $2/R \to 0$. For the formation of the plateau it is essential that (i) M_0 be supersonic and (ii) the flow be divergent.

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Interfacial Surface Energy between the Superfluid Phases of He^3

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We report the first measurements of σ_{AB}, the surface energy associated with the interface between the A and B phases of superfluid ^{3}He as they coexist at T_{AB}. At melting pressure we find $\sigma_{AB} = 6\times 10^{-6}$ erg cm^{-2} in no magnetic field ($T_{AB}/T_C = 0.79$) rising to $\sigma_{AB} = 1.6\times 10^{-5}$ erg cm^{-2} in a magnetic field of 4 kOe ($T_{AB}/T_C = 0.5$). Theoretical calculations that we present give an estimate about 50% larger.

At low temperatures liquid He^3 forms two very different superfluid phases.[1-3] In magnetic fields the A phase is always stable near T_c, but below a transition temperature T_{AB} depending both upon field and density, the B phase becomes the stable phase. The phase transition at T_{AB} is first order, and normally is observed to supercool and/or superheat. As a result, experimen-