ism in π or K reactions would have much less effect than for protons in this x region.

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Strength of Weak Interactions at Very High Energies and the Higgs Boson Mass

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It is shown that if the Higgs boson mass exceeds $M_c=(8\pi\sqrt{2}/3G_F)^{1/2}$ partial-wave unitarity is not respected by the tree diagrams for two-body reactions of gauge bosons, and the weak interactions must become strong.

Modern development in weak interaction theory is based upon the concept of spontaneously broken gauge symmetry.[1] Gauge theories of the weak and electromagnetic interactions contain one or more physical scalar (Higgs) particles, the existence of which is a necessity if the high-energy behavior of the S matrix is to be reasonable.[2] In this Letter we point out that if the Higgs boson mass exceeds about 1 TeV/c^2 new phenomena must appear in weak interactions in the TeV energy regime. Alternatively, if the Higgs boson mass is much less than 1 TeV/c^2, weak interactions may remain weak at all energies.[3]

We derive a quantitative estimate of this critical value of the Higgs boson mass. Our considerations are S-matrix theoretic in nature[4] and depend very little on the formal apparatus of renormalizable field theory. They rely instead on the application of unitarity bounds to tree diagrams. For definiteness we shall consider the minimal scheme of Weinberg and Salam[5] based on the Group SU(2) $\otimes$ U(1), in which there is only one physical Higgs particle. Our results may readily be extended to the case of several neutral Higgs particles.

It has frequently been remarked that a large Higgs boson mass implies a strong interaction among Higgs bosons. Weinberg[6] has emphasized the view that $G_F^{-1/2}$ is a natural mass scale of nature and that, if the Higgs self-coupling is strong, the effective ultraviolet cutoff would be at this energy. More recently Veltman[7] considered Higgs-boson contributions to certain radiative corrections. He concluded that for Higgs boson masses exceeding approximately $G_F^{-1/2}\approx 300$ GeV/c^2 the perturbation expansion of weak interactions may well break down, and speculated on various possible cases in which this might entail. We are in accord with the general view expressed by Veltman. Our demonstration is perhaps more primitive, but direct.

We assume Yang-Mills gauge couplings among vector bosons. Consider first the elastic process $W^+W^-\to W^+W^-$. At the tree-diagram level there are γ and Z^0 exchanges in the s and t channels and a contact (subtraction) term. The high-energy behavior of the amplitude is worst for all W's longitudinally polarized, for which[8]

$$T^{\gamma,Z}\sim(G_F/\sqrt{2})s(1+\cos\theta) \tag{1}$$

as $s\to\infty$. This linear divergence is canceled by the contributions of Higgs boson (H) exchanges in the s and t channels, provided that the HW^+W^- coupling is precisely that of the Weinberg-Salam theory.[9] The resulting amplitude is

$$T\sim(-4G_F/\sqrt{2})M_H^2,\quad s\to\infty. \tag{2}$$

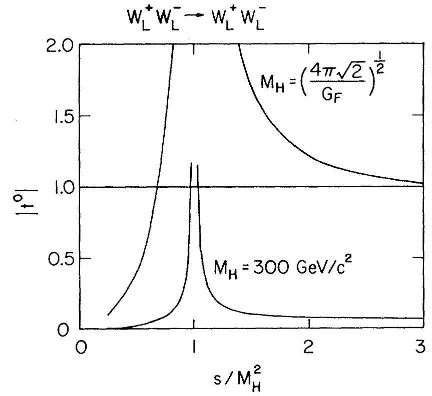

FIG. 1. Sketch of the energy dependence of the $J=0$ partial-wave amplitude for elastic scattering of longitudinally polarized W bosons for two choices of the Higgs boson mass. For $M_H > (4\pi\sqrt{2}/G_F)^{1/2}$ the partial-wave unitarity bound $|t^{J=0}| \leq 1$ is violated for $s > M_H^2$.

Application of the partial-wave unitarity bound, $|t^J| \leq 1$, to the $J=0$ amplitude defined through $T = 16\pi\sum_J(2J+1)t^J P_J(\cos\theta)$ then yields

$$M_H^2 \leq 4\pi\sqrt{2}/G_F. \tag{3}$$

The behavior of t^0 is shown in Fig. 1 for values of M_H below and at this critical value. The conclusion to be drawn from this argument is that if M_H is substantially less than the critical value, the Born amplitude is well within the unitarity bounds, except near resonances such as Z^0 and H, where finite-width corrections suffice to tame the amplitude. On the other hand, if M_H is comparable to or greater than the critical value the weak interaction must become strong for $s \approx 4\pi\sqrt{2}/G_F$, in the sense the lowest-order perturbation theory fails utterly to represent physics.

We may refine the bound (3) somewhat by considering the three-channel system consisting of $W_L^+W_L^-$, $(1/\sqrt{2})Z_LZ_L$, and $(1/\sqrt{2})HH$. (The subscript L denotes longitudinal polarization.) Other neutral two-body channels decouple from this system as $s \to \infty$. The resulting 3×3 t matrix for $J=0$ is

$$t^0 = \frac{-G_F M_H^2}{4\pi\sqrt{2}}\begin{bmatrix} 1 & \frac{1}{\sqrt{8}} & \frac{1}{\sqrt{8}} \\ \frac{1}{\sqrt{8}} & \frac{3}{4} & \frac{1}{4} \\ \frac{1}{\sqrt{8}} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}. \tag{4}$$

Applying the partial-wave unitarity bound to the eigenchannel $(2W^+W^- + ZZ + HH)$ with the largest eigenvalue, we obtain

$$M_H^2 \leq \frac{8\pi\sqrt{2}}{3G_F} \equiv M_c^2 \approx 1\ (\mathrm{TeV}/c^2)^2. \tag{5}$$

Let us assess the meaning of this result.

(1) If M_H lies between 4.5 GeV/c^2, the lower bound given by Linde and Weinberg,[10] and $2M_W$, we presume the Higgs boson closely follows the expectations set forth by Ellis, Gaillard, and Nanopoulos.[11]

(2) For $2M_W < M_H \lesssim 600$ GeV/c^2, the Higgs scalar will decay preferentially into a pair of intermediate vector bosons. In this range, a perturbative estimate of the decay rates should be reliable. We find

$$\frac{\Gamma(H\to W^+W^-)}{M_H} = \frac{G_F M_W^2}{8\pi\sqrt{2}}\frac{(1-x)^{1/2}}{x}(3x^2-4x+4), \tag{6}$$

$$\frac{\Gamma(H\to Z^0Z^0)}{M_H} = \frac{G_F M_W^2}{16\pi\sqrt{2}}\frac{(1-x')^{1/2}}{x}(3x'^2-4x'+4), \tag{7}$$

where $x = 4M_W^2/M_H^2$ and $x' = 4M_Z^2/M_H^2 = x/\cos^2\theta_W$. The resulting partial decay widths are shown in Fig. 2. A Higgs boson in this mass range may be produced in colliding-beam facilities now being contemplated. At the resonance peak the production cross section is

$$\sigma(e^+e^-\to H) \approx \frac{4\pi}{M_H^2}\frac{\Gamma(H\to e\bar{e})}{\Gamma(H\to \mathrm{all})}, \tag{8}$$

with

$$\frac{\Gamma(H\to e\bar{e})}{M_H} = \frac{G_F m^2}{4\pi\sqrt{2}}, \tag{9}$$

where m is the electron mass. The value of m to be chosen for $p^\pm p$ collisions (quark mass?) requires further study. If $M_H = 200$ GeV/c^2, we have

$$\sigma \approx 10^{-33}\left(\frac{m}{1\ \mathrm{GeV}/c^2}\right)^2\ \mathrm{cm}^2. \tag{10}$$

(3) For $M_H \gtrsim 600$ GeV/c^2, the total width given by (6) and (7) exceeds 100 GeV. However, for $M_H \gtrsim M_c$ it is possible that strong interactions among the gauge bosons create a scalar bound state which serves as a low-mass Higgs boson for the purposes of low-energy phenomenology. Qualitative considerations suggest that this is indeed a conceivable outcome, but we have not demonstrated its inevitability.

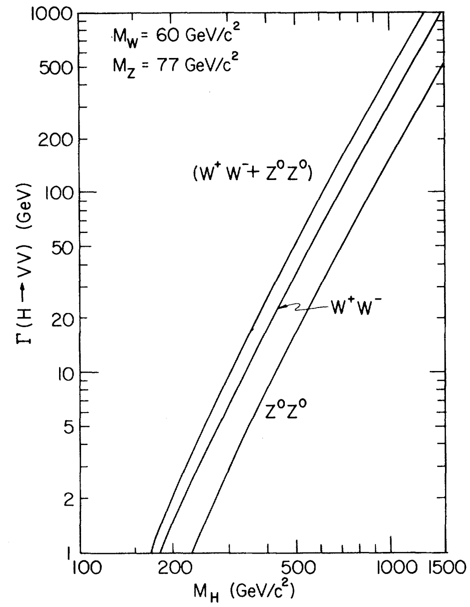

FIG. 2. Partial decay widths of the Higgs boson into intermediate boson pairs vs the Higgs boson mass. For this illustration we have taken $M_W = 60$ GeV/c^2 and $M_Z = 77$ GeV/c^2.

(4) It is perhaps less daring to speculate that if $M_H \gtrsim M_c$ (so that the unitarity bound is saturated or surpassed), weak interactions do become strong and begin to display the attributes exhibited in the GeV energy regime by strong interactions: resonances of intermediate vector bosons, multiple production of intermediate bosons, and so forth.

In view of the unitarity bound (5) we find it appealing to believe that new phenomena are to be found in the weak interactions at energies not much larger than 1 TeV, in addition to the anticipated discovery of the intermediate bosons. Either a light Higgs boson will exist or weak interactions will approach the richness of low-energy strong interactions. Details and further exploration of the consequences of this observation will be presented elsewhere.

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