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Determination of the Space-Time Extension of Centrally Produced Hadronic Matter Using the Intensity Interferometer Technique in pp Interactions at 28.5 GeV/c*

C. Ezell, L. J. Gutay, A. T. Laasanen, F. T. Dao, P. Schübelin, and F. Turkot

Purdue University, West Lafayette, Indiana 47907, Tufts University, Medford, Massachusetts 02155, Centre de Recherches Nucléaires, Strasbourg, France, and Fermi National Accelerator Laboratory, Batavia, Illinois 60510

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We have measured in the central region, the ratio of negative pion pairs to that of negative-positive pion pairs as a function of the Kopylov variables. A correlation was observed. From the shape of the correlation function we determined the radius r and lifetime τ of the centrally produced hadronic matter parallel and perpendicular to the collision axis to be $r_{\parallel} = 0.73^{+0.10}_{-0.11}$ fm, $r_{\perp} = 1.65^{+3.56}_{-0.99}$ fm, and $\tau_{\parallel} = (1.97^{+0.57}_{-0.56}) \times 10^{-24}$ sec.

Two decades ago Hanbury-Brown and Twiss expounded the idea of the intensity interferometer.[1] In a subsequent experiment[2] they used their intensity interferometer to measure the angular diameter of α Canis Majoris A. To settle the controversy stimulated by the novel idea of Hanbury-Brown and Twiss, Purcell noted[3] that the idea is sound and is rooted in a fundamental quantum mechanical relationship between spin, statistics, and the symmetry of a wave function. In studying $\bar{p}$ annihilation in nuclei, it was noted by Goldhaber *et al.*[4] that the $\pi\pi$ opening angle distributions are in slight disagreement with the predictions of the statistical model. A possible explanation for this deviation was given by Goldhaber *et al.*[5] which is conceptually a simplified and independently derived version of the Hanbury-Brown–Twiss idea applied to high energy physics. Until recently the effects of Bose-Einstein statistics on multipion production have been discussed in rather general terms.[6] However, Kopylov has recently presented a theoretical description of pion production[7] which incorporates the necessary symmetry constraints on the process; he also shows how it can be used to measure the space and time structure of pion-emitting hadronic matter. Kopylov's analysis is also closely related to that of Ref. 2. A preliminary test of Kopylov's analysis was carried out by Canter *et al.*[8] and subsequently by Deutschmann *et al.*[9] and Grard *et al.*[10] The significance of our result is that not only are we able to demonstrate with superior statistics the effect discussed in Refs. 8–10, but also are able to show that the above effect is truly a manifestation of intensity fluctuations in the Hanbury-Brown–Twiss sense and that is is not due to the reflection of other aspects of multipion production dynamics. Consequently, we were able to determine the two-dimensional space-time extension of the pion-emitting hadronic matter.

Our analysis was made possible by a novel experimental technique[11] which combined the features of multiparticle detection, high data rate, and identification of one or both of the leading particles. The above features are essential in establishing the three conditions which must be satisfied by a reliable analysis: (a) Removal of leading particle effects; (b) showing that the momentum distribution of the centrally produced pions behaves as a Bose-Einstein gas; (c) demonstrating that the $\pi^+\pi^-$ distribution as a function of the Kopylov variables does not exhibit intensity fluctuation and thereby justifying its use as a background distribution.

To satisfy Condition (a), we studied the reaction

$$p_1+p_2\to p_3+N_4+\text{mesons (190 000 events)}, \qquad (1)$$

where we measured the momentum of all the charged particles and identified the fast forward proton[11] p_3. To present evidence for a centrally produced Bose-Einstein meson cloud [Condition (b)], in the inset of Fig. 1(a) we show the Feynman-x distribution of the produced pions. In an earlier Letter we showed[12] that, for centrally produced pions, both scaling in the mean and the Bose-Einstein distribution for $p_\perp$ are valid. In the following analysis, we imposed the constraint[13] of $-0.2<x<0.2$. To demonstrate that the $\pi^+\pi^-$ distribution does not exhibit prominent resonance structure, in Fig. 1(a) we plot the effective-mass distribution for the $\pi^+\pi^-$ ($M_{\pi^+\pi^-}$) di-pion pairs in the central region. From Fig. 1(a) it is clear that the effective-mass distribution is dominated by a low-effective-mass enhancement. It is known that at low effective masses the di-pion system is s-wave dominated.[14]

Let E_i, p_i denote the energy and momentum (in the overall center-of-mass system) of the ith pion emerging from the centrally produced hadronic matter in Reaction (1). q_0 and q_t (Kopylov's variables) are defined by the equations

$$q_0=|E_i-E_k| , \qquad (2)$$

$$q_t=|\vec{q}-(\vec{q}\cdot\vec{n})\vec{n}| , \qquad (3)$$

where $\vec{q}=\vec{p}_i-\vec{p}_k$ and $\vec{n}=(\vec{p}_i+\vec{p}_k)/|\vec{p}_i+\vec{p}_k|$. The ratio of the number of like and unlike pion pairs in a given q_t and q_0 interval is denoted by $R_{+-}{}^{--}$. Using the above notation, in Figs. 2(a) and 2(b) we plot $R_{+-}{}^{--}$ as a function of q_t for the intervals 0 GeV$<q_0\leq 0.1$ GeV and 0.1 GeV$<q_0\leq 0.3$ GeV. In addition to the q_0 cuts, the $-0.45\leq\cos\theta\leq+0.45$ condition was imposed on the direction on $\vec{n}$, where θ denotes the polar angle of $\vec{n}$ in the overall center-of-mass system. The z axis is chos-

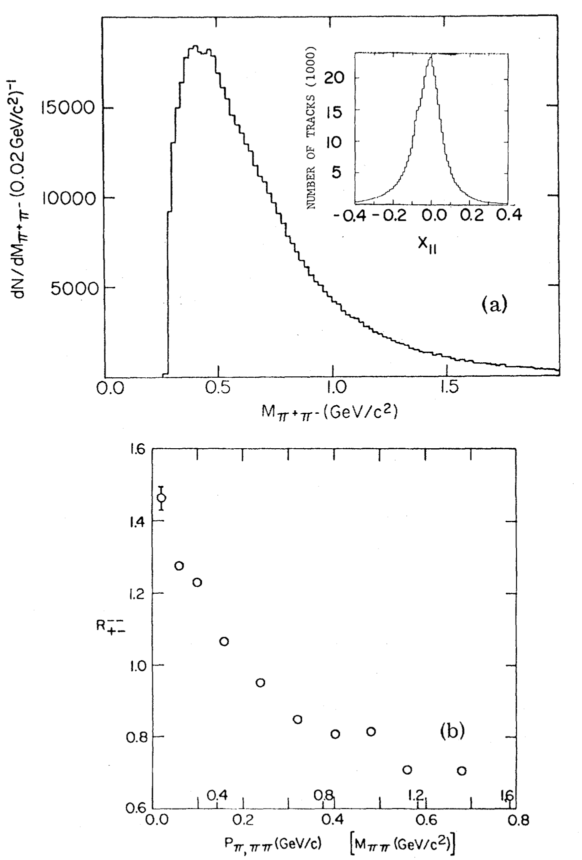

FIG. 1. (a) Effective-mass distribution of ($\pi^+\pi^-$) pairs. The inset is the Feynman-x distribution of produced pions. (b) The correlation ratio, $R_{+-}{}^{--}$, for single-arm events as a function of both the pion momentum in the di-pion rest frame (lower scale) and the dipion effective mass (upper scale).

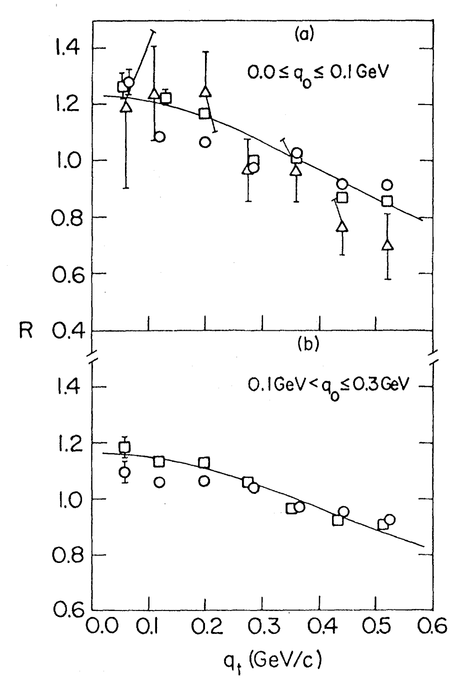

FIG. 2. The correlation ratio as a function of q_t for $\vec{n}$ having $-0.45\leq\cos\theta\leq+0.45$. Circles, $R_{D^{--}}{}^{S^{--}}$ for single-arm events; squares, $R_{+-}{}^{--}$ for single-arm events; triangles $R_{+-}{}^{--,++}$ for double-arm events. The smooth curves represent the fit of $R_{+-}{}^{--}(q_t,q_0)$ [Eq. (4)] to the single-arm data, integrated over the respective ranges of q_0 for Figs. 2(a) and 2(b).

en along the incident proton (p_1) direction. The data points with the square symbols represent events where only p_3 was mass-identified as a proton (single-arm events) and therefore only p_3 was excluded from the $\pi\pi$ correlation analysis. To check that the observed enhancement is not due to a slow proton component of the positive tracks, in Fig. 2(a) we also present our results for events where both protons (p_3 and p_4) were mass-identified and both p_3 and p_4 were excluded from the $\pi\pi$ correlation analyses and are represented by the triangular symbols (double-arm events). Although the errors are large, both curves reveal similar dependence on q_t.

Next we show that our observed peak of R_{+-}^{--} is not a reflection of $\pi^+\pi^-$ correlation [Condition (c)]. We follow Kopylov's suggestion[7] and evaluate the normalized ratio of the number of pairs of negative pions from the same event to the number of pairs of negative pions *with each pion being from a different event,* $R_{D^{--}}^{S^{--}}$, as a function of q_t. Our results for $R_{D^{--}}^{S^{--}}$ are plotted in Figs. 2(a) and 2(b) as circles. The curves so obtained are in good agreement with our results for R_{+-}^{--}, which means that the unlike pion pairs are uncorrelated. In Fig. 1(b) we plot the ratio R_{+-}^{--} as a function of the di-pion effective mass or equivalently the pion momentum in the di-pion rest frame. No restriction is made here on the direction of $\vec{n}$. Figure 1(b) exhibits a strong peak at low effective mass or pion momentum in the di-pion rest frame, in agreement[15] with our results in terms of q_t and q_0. There is very little known of the dependence of R_{+-}^{--} on $\vec{n}$. Thus our procedure presented above is repeated for $-1 \le \cos\theta \le -0.9$. In Fig. 3 we plot the double-arm and the single-arm results for R_{+-}^{--}, as well as $R_{D^{--}}^{S^{--}}$ with the 0 GeV $< q_0 \le 0.1$ GeV cut. We observe a stronger q_t dependence than that for equatorially produced pion pairs.

According to Kopylov, the q_t and q_0 dependence of R_{+-}^{--}, for a uniformly radiating sphere, is

$$R_{+-}^{--}(q_t, q_0) = K\left[1 + \frac{I^2(rq_t)}{1+(\tau q_0)^2}\right], \qquad (4)$$

where K, r, and τ are, respectively, an arbitrary normalization constant, the radius of the radiating sphere, and its lifetime, and $I(x) = 2J_1(x)/x$ (J_1 is a Bessel function). Using the maximum-likelihood method, we fitted the single-arm data of Fig. 2 to Eq. (4). The results are $r_{\|} + 0.73^{+0.10}_{-0.11}$ fm, $\tau_{\|} = (1.97^{+0.57}_{-0.56}) \times 10^{-24}$ sec with χ^2 per degree of freedom of 2.1. The subscript denotes the fact that we determined that dimension which is parallel to the motion of the incident protons. Inspection of Fig. 3 shows that Eq. (4) cannot give an adequate description of the data because of the sharp drop of the correlation ratio as a function of q_t. One possible explanation is that the uniformly radiating sphere model is not adequate when viewing the object from along the incident direction. If the radiating matter has no sharp cutoff, then instead of Eq. (4) we have[7]

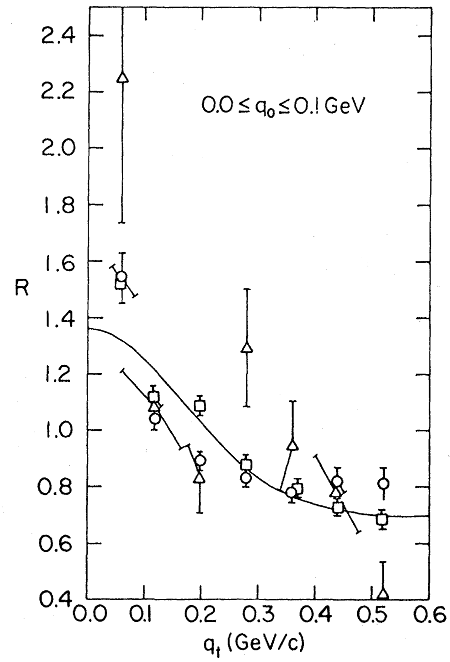

FIG. 3. The correlation ratio as a function of q_t for $\vec{n}$ having $-1.0 \le \cos\theta \le -0.9$. Circles, $R_{D^{--}}^{S^{--}}$ for single events; squares, R_{+-}^{--} for single-arm events; triangles, $R_{+-}^{--,++}$ for double-arm events. The smooth curve represents the fit of $R_{+-}^{--}(q_t)$ [Eq. (5)] to the single-arm data.

$$R_{+-}^{--}(q_t) = K_1\{1 + K_2(q_0\tau)\exp[-(rq_t)^2]\}, \qquad (5)$$

where $K_2(q_0\tau) \approx 1$ for small values of q_0. Fitting the data in Fig. 3 to Eq. (5), we obtain $r_\perp = 1.65^{+3.56}_{-0.99}$ fm with χ^2 per degree of freedom of 3.1, indicating that the transverse dimension of the radiator is larger than the longitudinal dimension.

In conclusion, we have obtained a value for the longitudinal size and lifetime of the centrally produced radiating matter. Our values are in good agreement with the recent estimate of E. Calligarich *et al.*[16] who studied lower-energy π^--induced reactions. There is a qualitative indication that the transverse dimension is larger than the longitudinal.

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[15]Note that the variables q_0 and q_t are not Lorentz scalars. However, foı a *parallel emission* of a di-pion pair ($\vec{p}_i+\vec{p}_k \neq 0$), small values of q_0 and q_t are related to small values of di-pion effective mass. Since the di-pion effective mass, $M_{\pi\pi}$, is a Lorentz scalar, the plot of $R_{+-}{}^{--}$ as a function of $M_{\pi\pi}$ in Fig. 1(b) suggests that the correlation observed in Figs. 2 and 3 is not a result of our choice of coordinate system.

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Proton-Proton Bremsstrahlung at 730 MeV*

B. M. K. Nefkens, O. R. Sander,† and D. I. Sober‡
University of California, Los Angeles, California 90024
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We have measured the differential cross section for proton-proton bremsstrahlung at sixteen photon angles for 730-MeV incident protons. At all angles, the photon spectra fall smoothly with increasing photon energy up to $E_\gamma \sim 80$ MeV and are in quantitative agreement with the prediction of external-emission dominance (EED). Above $E_\gamma \sim 80$ MeV the spectra are consistently above the EED calculations.

We report on new measurements of proton-proton bremsstrahlung (PPB) at an incident energy of 730 MeV. This energy is substantially higher than that of previous PPB measurements,[1] making our experiment different from earlier ones in some important ways. First, the maximum photon energy at many angles is so large that the photon wavelength is smaller than the hadronic interaction radius. At such photon energies, PPB can be sensitive to the internal structure of the nucleon-nucleon interaction. Second, the incident proton energy is well above the pion production threshold and approximately half of the pp total cross section at 730 MeV is inelastic. All previous PPB experiments have been below pion production threshold. There is no potential model which describes the elastic scattering at 730 MeV and there are no theoretical calculations of PPB in the inelastic region. Even the soft-photon approximation (SPA), which is based on the theorem of Low[2] and is expected to be valid for low-energy photons, cannot be evaluated at our