Mobility of the Electron Bubble in Superfluid ^{3}He†

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(Received 28 February 1977)

The theory of mobility of negative ions in superfluid ^{3}He is presented. The calculated B-phase mobility agrees well with experiment.

According to recent experiments,[1,2] the mobility of negative ions in ^{3}He remains remarkably constant down to the superfluid transition temperature, T_c, and then rises rapidly with decreasing temperature in the superfluid phases. In the B phase the ratio μ_B/μ_N of the superfluid mobility to that in the normal state is, within experimental accuracy, a pressure-independent function of T/T_c only. The B-phase ratio is substantially larger at any given T/T_c than that in the A phase. As we show below by detailed calculation, two important physical effects underlie these data: (i) Because the ion, which is a large bubble containing an electron, undergoes Brownian motion in the fluid, it scatters from the ^{3}He quasiparticles with very small change in energy; and (ii) the ion–^{3}He-quasiparticle momentum-transfer cross section is appreciably reduced in the B phase because of the diverging density of ^{3}He quasiparticle states near the gap edge.

When the typical energy transfer $\bar{\omega}$ in ^{3}He-ion scattering is small compared with k_BT, the mobility in the normal phase is a temperature-independent constant for $T \ll T_F$, the ^{3}He Fermi temperature; should $\bar{\omega}$ become $\gtrsim k_BT$, the mobility would rise with decreasing T because of Pauli-principle effects.[3] If one assumes that the ion recoils as a free particle from a collision with a ^{3}He quasiparticle, then since the typical momentum transfer $\bar{k}$ in the scattering is p_F, the ^{3}He Fermi momentum, we have $\bar{\omega} \ll k_BT$ for $T \gg T_0 = p_F^2/2M^*k_B$. Here $M^* \simeq (2\pi/3)R^3n_3m_3$ is the ion effective mass, R is the ion (bubble) radius, n_3 is the ^{3}He density, and m_3 the ^{3}He bare mass. At $P = 0$, $T_0 \sim 17$ mK and near the melting pressure $T_0 \sim 100$ mK. That μ_N remains constant at temperatures well below T_0 is a consequence of the fact that in a ^{3}He-ion collision the ion is constantly bombarded by other ^{3}He quasiparticles, and thus its spectrum of recoil energies is substantially modified, an effect pointed out by Josephson and Lekner.[4] In particular, in the diffusive limit the ion recoil spectral function $S(k,\omega)$ has the form[5]

$$S(k\omega) \simeq \frac{\beta\omega}{1-e^{-\beta\omega}} \frac{2Dk^2}{\omega^2 + (Dk^2 - \omega^2\tau)^2}, \qquad (1)$$

where k and ω are the changes in ion momentum and energy, $D = k_BT\tau/M^*$ is the ionic diffusion constant, $\beta = 1/k_BT$, and τ is the relaxation time for ion momentum, related to the mobility μ by the Einstein relation $\mu = e\tau/M^*$. The characteristic energy transfer, for given k, is $\sim Dk^2$. Taking experimental values[1,2] of μ_N, and $k \sim p_F$, we have $\bar{\omega} = Dp_F^2 = (\mu p_F^2/e)k_BT \lesssim 0.1k_BT$ at *all* temperatures in the normal state. [If one writes $\tau \simeq M^*/n_3\sigma p_F$, where $\sigma \sim \pi R^2$ is a characteristic ^{3}He-ion cross section, then $Dp_F^2/k_BT \sim 3\pi/(p_FR)^2 \ll 1$.] Because of the self-consistent effects of the scattering process on the recoil spectrum, the ^{3}He-ion scattering is effectively elastic at all T in the normal state [$S(k\omega) \to 2\pi\delta(\omega)$] and the mobility is thus constant for $T \ll T_F$. The observed μ_N is consistent[2] with the ion's acting as a hard sphere of radius R.

In the superfluid phase the mobility is limited by the ion's scattering from ^{3}He quasiparticles as well as creating and annihilating pairs. Just below T_c the scattering may still be treated as elastic, and at low drift velocitites processes changing the number of quasiparticles can be neglected. In the low drift velocity limit the mobility is then given by[6]

$$\frac{e}{\mu} = -\frac{2\pi}{3}\sum_{\vec{p}\vec{p}'}(\vec{p}-\vec{p}')^2|t_{\vec{p}',\vec{p}}|^2\delta(E-E')\frac{\partial n(E)}{\partial E}, \qquad (2)$$

where $t_{\vec{p}',\vec{p}}$ is the amplitude for scattering by an ion at rest of a ^{3}He quasiparticle of momentum $\vec{p}$ and energy E to momentum $\vec{p}'$ and energy E'; and $n(E) = (1+\exp\beta E)^{-1}$ is the equilibrium quasiparticle distribution function (with E measured with respect to the ^{3}He chemical potential). Equation (2) assumes isotropy; in the A phase the mobility has a tensor structure. Carrying out the $\vec{p}'$ sum, and noting that $p \simeq p_F$, we have

$$\frac{e}{\mu} = -n_3p_F\int_{-\infty}^{\infty}d\xi\left|\frac{dE}{d\xi}\right|\frac{\partial n}{\partial E}\sigma_{tr}(\xi), \qquad (3)$$

where $\xi \simeq (p-p_f)v_f$ is the quasiparticle energy in the normal state, and $\sigma_{tr}(\xi) = \int d\Omega(1-\cos\theta)(d\sigma/d\Omega)$ is the momentum transfer cross section; $d\sigma/d\Omega = \sum(p\partial p/\partial E)^2|t_{\vec{p}',\vec{p}}|^2/4\pi^2$ is the differential cross

section for scattering of a ^{3}He quasiparticle, the sum being over all final states with energy E in solid angle $d\Omega$. In the normal state (where in the limit $p_F R \gg 1$, $\sigma_{tr} = \pi R^2$ for hard-sphere scattering), $e/\mu_N = n_3 p_F \sigma_{tr}$.

In the superfluid phases the mobility is increased by the reduction in the thermally excited quasiparticle density as well as by a decrease in σ_{tr}. In the B phase $E = (\xi^2 + \Delta^2)^{1/2}$. Then if one assumes σ_{tr} in the superfluid phase to be the same as in the normal phase, one finds from (3) that $\mu/\mu_N = \frac{1}{2}(e^{\beta\Delta} + 1)$, a result derived by Bowley.[7]

Calculations based on this formula, which are shown as the dashed line in Fig. 2, account for about half of the observed increase in the mobility.

As a first approximation to the modification of σ_{tr}, one might be tempted to assume that the scattering amplitude t^S in the superfluid phase is simply found by carrying out a Bogoliubov transformation on that in the normal phase t^N, i.e., $t_{\vec{p}',\vec{p}}{}^S = uu' t_{\vec{p}',\vec{p}}{}^N - vv' t_{\vec{p}',\vec{p}}{}^{N*}$ where u and v are the usual coherence factors, and spin indices are suppressed. Squaring and carrying out the spin average, one has[8]

$$|t_{\vec{p}',\vec{p}}{}^S|^2 = \frac{1}{2}\left(1 + \frac{\xi\xi'}{EE'}\right)|t_{\vec{p}',\vec{p}}{}^N|^2 - \frac{1}{2}\frac{\vec{\Delta}\cdot\vec{\Delta}'}{EE'}\,\mathrm{Re}[(t_{\vec{p}',\vec{p}}{}^N)^2], \tag{4}$$

with $\vec{\Delta}(\hat{p})$ the vector describing the p-wave pairing gap. In the B phase $\vec{\Delta}\cdot\vec{\Delta}' = \Delta^2\cos\theta$, and one finds on summing over $\xi' = \pm(E^2 - \Delta^2)^{1/2}$ that $d\sigma^S/d\Omega \sim (E/\xi)^2$. This result diverges as $\xi \to 0$ (except for forward scattering if t^N is real), a violation of unitarity, and leads as well to a logarithmically divergent integral in (3). The source of the difficulty is the divergent density of quasiparticle states $\propto |E/\xi|$.

The correct calculation of t^S requires that one include, in addition, the modification by the superfluidity of intermediate states in the scattering. Space does not permit a detailed description of the calculation here; we indicate only the crucial steps. In the presence of pairing the full t matrix, for quasiparticle-ion scattering, which is 4×4 in the usual matrix Green's-function formalism, obeys the equation $T = V + VG(\xi)T$, where V is the bare interaction matrix and $G(\xi)$ is the quasiparticle Green's-function matrix. The real part of G may be replaced by its normal-state value, but the full effects of the pairing on ImG, which is proportional to the superfluid density of states, must be included. Introducing the normal-state K matrix, $\sim\tan\delta_l$, via $K_N = V + V(\mathrm{Re}G)_N K_N$, we may write $T = K_N + iK_N(\mathrm{Im}G)T$. In the B phase the gap matrix is invariant under simultaneous rotations in real and spin space; thus as in usual spin-orbit problems, the T matrix is diagonal in quasiparticle states of the form

$$|\Psi(\xi, j, \pm, m)\rangle = \begin{pmatrix} u(\xi)|\, j, \pm, m\rangle \\ iv(\xi)\sigma_2\vec{\sigma}\cdot\hat{p}\,|\, j, \pm, m\rangle \end{pmatrix}. \tag{5}$$

where $|\, j, \pm, m\rangle$ is an eigenstate with total angular momentum j, orbital angular momentum $l = j \pm \frac{1}{2}$, and $J_z = m$. One finds by lengthy calculation that for $\xi^2 = \xi'^2 = E^2 - \Delta^2$,

$$\frac{m^* p_F}{8\pi^2}\langle\Psi(\xi, j, \pm, m)|T|\Psi(\xi', j, \pm, m)\rangle = \frac{-uu'\alpha_+ + vv'\alpha_- - i(E/|\xi|)\alpha_+\alpha_-[uu' + vv' - (\Delta/E)(uv' + vu')]}{1 - i(E/|\xi|)(\alpha_+ - \alpha_-) + \alpha_+\alpha_-}; \tag{6}$$

here $\alpha_\pm = \tan\delta_{l=j\pm1/2}$, the δ_l are normal-state phase shifts, and m^* is the normal-state ^{3}He quasiparticle effective mass. [Setting the factors $\Delta/E = 0$ and $E/|\xi| = 1$ in (6), one readily recovers (4).]

For $l \lesssim p_f R \equiv l_0$ the $\alpha_\pm$ are of order unity; thus we find for these l the important result that for $|\xi| \ll \Delta$ the right-hand side of (6) becomes $|\xi|/2iE$. This means that the scattering amplitudes are equal in all partial waves, up to $l \sim l_0$. Consequently the quasiparticle-ion scattering cross section is strongly peaked in the forward direction, as in diffraction, with an angular width $\sim l_0^{-1/2}$; since forward-scattering processes transfer small momentum, the transport cross section, for $|\xi| \ll \Delta$, is drastically reduced from its normal-state value. This forward-scattering effect is illustrated in Fig. 1 which shows the B-phase spin-averaged transport and total cross sections, compared to their normal-state values, computed with δ_l describing hard-sphere scattering for $l_0 = 10.3$, appropriate to a pressure of 18.0 bar. (In this calculation the small effects due to distortion of the order parameter near the ion are neglected.) The transport cross section

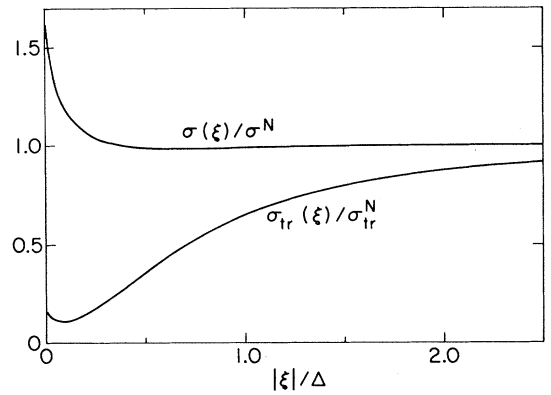

FIG. 1. Spin-average transport and total cross sections for ^{3}He quasiparticle-ion scattering, in the B phase, as a function of $|\xi|/\Delta$, and normalized to their normal state values. A bubble radius $R = 10.3/p_F$ was used.

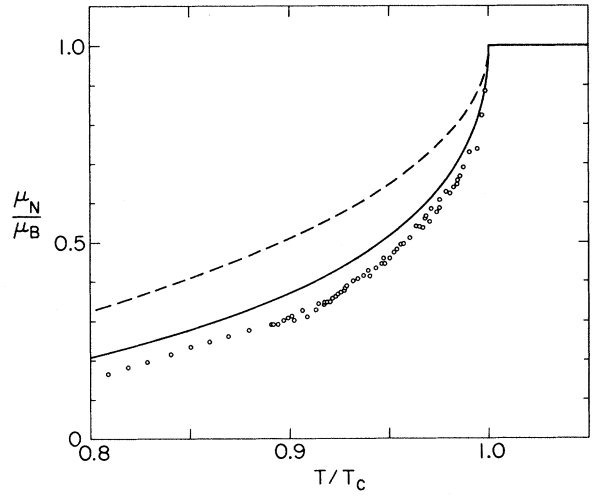

FIG. 2. Ratio of normal to B-phase mobility at $P = 18.0$ bar, as a function of T/T_c. The data points are from Ref. 2, and the solid curve is the present calculation. The dashed curve results from taking $\sigma_{tr}/\sigma_{tr}^N = 1$.

has a minimum of order σ_{tr}^N/l_0, while the total cross section remains remarkably constant. The rise in the cross sections at very small ξ is due to coherent contributions from partial waves with $l > l_0$.

Figure 2 shows (as the solid line) the B-phase mobility ratio at 18.0 bar computed from Eq. (3) with the above $\sigma_{tr}/\sigma_{tr}^N$, together with the data of Ref. 2, and for comparison Bowley's result [the dashed line]. These calculations used $\Delta(T) = 1.07\Delta_{BCS}(T)$, where Δ_{BCS} is the weak-coupling gap [$\sim 3.06 k_B T_c(1 - T/T_c)^{1/2}$ near T_c]; the factor 1.07 includes strong effects as inferred from ultrasonic attenuation experiments.[9] We see how inclusion of the reduction in the transport cross section brings theory into essential agreement with experiment. The deviations from experiment at lower T are consistent with the fact that our no-recoil assumption becomes less valid as μ_B increases; for example, at $T/T_c \simeq 0.9$ and $P = 18.0$ bar, Dp_F^2 is $\sim \frac{1}{3} k_B T$. Possible pressure dependence of μ_N/μ_B can arise from the rather weak dependence of $\sigma_{tr}/\sigma_{tr}^N$ on R, and from strong-coupling effects on $\Delta(T)$; the latter appear, from both theory and experiment, to be small.

In the A phase the calculation of the mobility is more complicated because of the anisotropy of the gap. We note, though, that the singularity in the averaged density of states is only logarithmic and therefore the reduction in the energy-averaged transport cross section is much less than in the B phase. Hence, even though strong-coupling effects are more important in the A phase, one does not expect as sharp a rise in the mobility below T_c.[10]

We are grateful to the Otaniemi low temperature group for discussions of the experiments, and to Aa. Bohr, A. Fetter, J. Kurkijärvi, B. Mottelson, D. Rainer, D. G. Ravenhall, and J. Serene for theoretical discussions.

†Research supported in part by National Science Foundation Grants No. DMR75-22241 and No. DMR76-24011.

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Electroclinic Effect at the A–C Phase Change in a Chiral Smectic Liquid Crystal*

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When a smetic-A phase is composed of chiral molecules, it exhibits an electroclinic effect, i.e., a direct coupling of molecular tilt to applied field. The pretransitional behavior of the electroclinic effect in the A phase is used to study the critical behavior near the second order, smectic-A–smectic-C phase transition. This behavior is measured by monitoring the change in birefringence of a sample as the electroclinic effect causes a tilt of the molecules. A large pretransitional effect is measured, and constants describing the critical behavior are determined.

Meyer *et al.*[1] established the existence and behavior of ferroelectric liquid crystals. Since then many of the properties of these liquid crystals have been investigated: synthesis and properties of several compounds which have enhanced ferroelectric properties[2-4]; optics and bulk properties[1]; polarization and helical pitch[5]; shear induced polarization[6]; pyroelectricity[7]; and fluctuations and domain walls in free smectic films.[8] Our experiment is a measurement of a new phenomenon—the electroclinic effect (ECE)—and the critical behavior on the smectic-A side of the smectic-A–smectic-C phase transition. Although this effect is very closely related to piezoelectricity in crystalline phases, we use a distinctive name because the fluid nature of the liquid crystalline phase does not allow the static shear strains associated with piezoelectricity. This work is one of the first detailed studies of the critical properties of this phase transition. Our results raise some question about the exact nature of the phase change.

A symmetry argument, similar to that predicting ferroelectricity in a chiral smectic-C,[1] describes the origin of the ECE in a smectic-A phase composed of chiral molecules. The application of an electric field parallel to the smectic layers of such a smectic-A biases the free rotation of the molecules and therefore produces a nonzero average of the transverse component of the molecular polarization. When such a dipole moment is present, a tilt of the long molecular axis (the director) is induced in a plane perpendicular to the dipole moment.

In an aligned smectic-A sample, a tilt of the director is directly related to a tilt of the optic axis; therefore, the ECE results in a linear electro-optic response. The electro-optic effect manifests itself as a modulation of the birefringence.

The ferroelectric smectic-A–smectic-C phase transition is second order and exhibits a divergence of the electric polarization susceptibility characteristic of a Curie point.[9] On the smectic-A side of the transition, a soft mode is observed for which the susceptibility A controlling tilt of the director goes to zero at the critical temperature T_c according to a power law:

$$A = a[(T - T_c)/T_c]^{\gamma}. \quad (1)$$

Despite the striking resemblances to the crystalline ferroelectric Curie point, the phase change is driven by the intermolecular forces inducing the smectic-C phase and not by the spontaneous polarization which is a minor perturbation on these forces. In this sense, we consider the pretransitional behavior of the ECE to be a probe of the critical behavior near the smectic-A–smectic-C phase transition rather than a measure of the effects of dipole-dipole interactions.

Because of the overall symmetry properties of the nonchiral smectic-A–smectic-C phase change, de Gennes has drawn an analogy between this transition and the normal-to-superfluid transition in ^{4}He where γ cannot be directly measured.[10] In the high-temperature-series expansion analysis, the asymptotic values for γ are 1.0 in the mean-field limit and 1.315 near the transition.[11]

The spatially homogeneous, static free-energy density of a chiral smectic-A in the presence of an electric field E parallel to the smectic layers