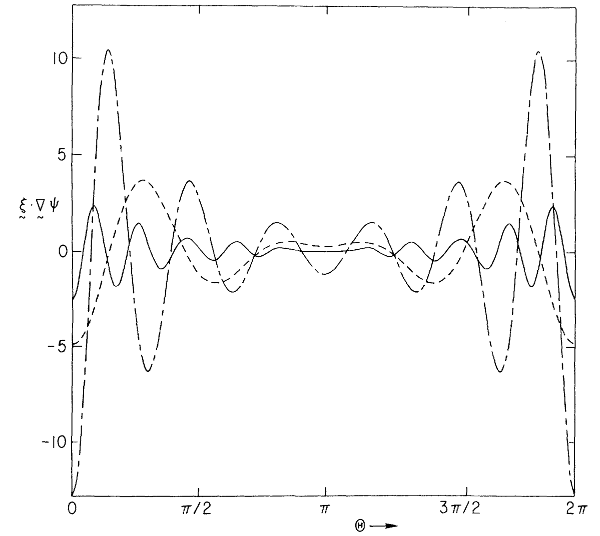

FIG. 6. The displacement $\vec{\xi}\cdot\nabla\Psi$ as a function of the poloidal angle on surfaces with $\Psi/\Psi_b=\frac{1}{12}$ (dashed curve), $\Psi/\Psi_b=\frac{1}{2}$ (broken curve), and $\Psi/\Psi_b=\frac{11}{12}$ (solid curve) for the fixed-wall $n=3$ mode at $\beta^*=3.0\%$.

less than qR. Clearly, a better understanding of these instabilities is essential.

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Magnetohydrodynamic Stability of Flux-Conserving Tokamak Equilibria*

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Large-scale magnetohydrodynamic instabilities of flux-conserving tokamak equilibria are studied computationally. Stable equilibria are found with up to 5% average β. As β is increased, the observed instabilities take on a strong ballooning character, concentrating near the outer edge of the torus with a mix of poloidal harmonics.

For the purpose of controlled thermonuclear fusion, it is desirable to build a tokamak with the highest possible energy density relative to the confining magnetic field—the highest possible average β,

$$\langle\beta\rangle \equiv 2\mu_0\langle p\rangle/B_{\rm tor}^{\,2},$$

where $\langle p\rangle$ is the pressure averaged over the plasma volume and $B_{\rm tor}$ is the vacuum toroidal magnetic field at the geometric center of the plasma. As the plasma pressure in a tokamak is raised, by neutral beam injection, for example, poloidal currents play a progressively larger part in providing the radial force balance to confine the plasma; thus there is an increase in the poloidal β,

$$\beta_{\rm pol} \equiv 2\mu_0\langle p\rangle/(\oint dl\, B_{\rm pol}/\oint dl)^2,$$

where the integral is performed around the plasma boundary. In addition, the vertical magnetic field which restrains the plasma from expanding along the major radius of the torus must be increased.[1] For arbitrarily chosen equilibrium profiles it is theoretically predicted[2] that the vertical field will tend to cancel the poloidal magnetic field at the inner edge of the torus when $\beta_{\rm pol} \sim R/a$, and the resulting magnetic separatrix will reduce the plasma confinement and cause the plasma to shrink. Clarke has suggested[3] that if a perfectly conducting plasma were heated (i.e., if the plasma were heated rapidly on the magnetic skin time scale), the fluxes would be frozen into the plasma and therefore the separatrix could not move in to shrink it. Then arbitrarily high-β tokamak equilibria could be produced with moderately large aspect ratios ($R/a \sim 4$), which is desirable for engineering reasons. The limit on β would then be expected from instability considerations, as will be studied in this paper.

Such sequences of equilibria, called flux-conserving tokamak (FCT) equilibria, have been produced computationally[4] with large β values ($\langle\beta\rangle > 15\%$) and highly peaked current and pressure profiles that are consistent with experimental observations in present-day tokamaks. They are computed by increasing the pressure $p(\psi)$ in an arbitrary way while holding $q(\psi) = d\psi_{\rm tor}/d\psi_{\rm pol}$ fixed.

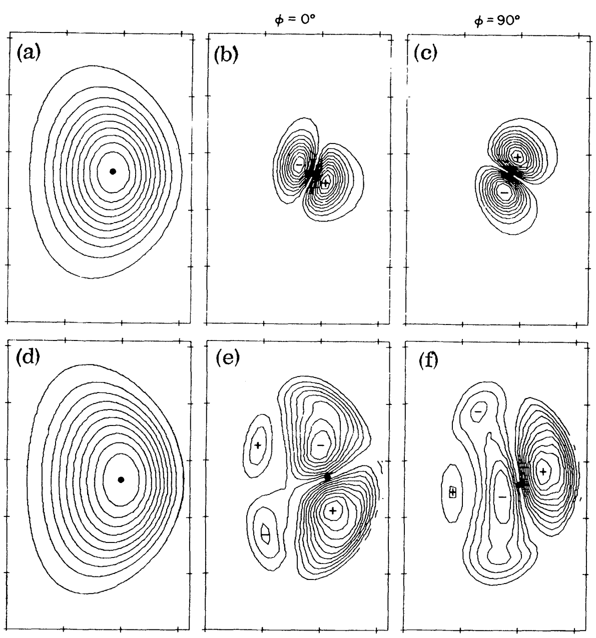

FIG. 1. Eigenfunctions for low-β and high-β $n=1$ instabilities near their marginal points. (a), (d) Equilibrium pressure contours. (b)–(e) Perturbed pressure contours on poloidal planes a quarter wavelength apart, the signs indicating positive and negative perturbations. (a)–(c) $\langle\beta\rangle = 1.56\%$, $\beta_{\rm pol} = 1.02$, $q_{\rm axis} = 0.9$, and $\gamma = 0.0087$. (d)–(f) $\langle\beta\rangle = 4.85\%$, $\beta_{\rm pol} = 3.6$, $q_{\rm axis} = 1.1$, and $\gamma = 0.006$. The toroidal center line is to the left.

The Grad-Shafranov equation

$$-\left[R\,\frac{\partial}{\partial R}\left(\frac{1}{R}\,\frac{\partial\psi}{\partial R}\right)+\frac{\partial^2\psi}{\partial Y^2}\right]=R^2p'(\psi)+II'(\psi),$$

$$I(\psi)\equiv RB_{\varphi},\quad \vec{B}=\nabla\times(\psi\nabla\varphi)+B_{\varphi}\hat{\varphi}, \qquad (1)$$

is solved while iterating $I(\psi)$ until the desired $q(\psi)$ is found. D-shaped plasmas with mild elongation ($b/a = 1.6$) have been shown[5] to enhance stability with respect to the Mercier criterion.[6] Such equilibria with $R/a = 4$ and $q_{\rm edge}/q_{\rm axis} = 3.3$ are illustrated in Figs. 1(a) and 1(d). The stability of these equilibria with respect to large-scale instabilities is studied here using the Oak Ridge National Laboratory magnetohydrodynamic (MHD) instability code.[7] In addition to being simple and flexible, this initial-value code has the advantage that it is almost identical to its nonlinear version.

In order to expedite a comparative study of these instabilities, the equilibria are scaled in the following way:

$$\psi \to s\psi. \qquad (2)$$

It follows from Eq. (1) that

$$p \to s^2 p, \qquad (3)$$

$$I_{\rm edge}{}^2 - I^2 \to s^2(I_{\rm edge}{}^2 - I^2). \qquad (4)$$

This scaling method fixes $\beta_{\rm pol}$, the geometry, and the shape of the profiles while changing the magnetic fields, the q value, and $\langle\beta\rangle$ according to simple algebraic relations.

Growth rates for the instabilities with $n = 1$ (n = toroidal mode number) are shown in Fig. 2, plotted against the q value at the magnetic axis. Instabilities with $n \geq 2$ (not shown) are found only at lower q values but with considerably larger

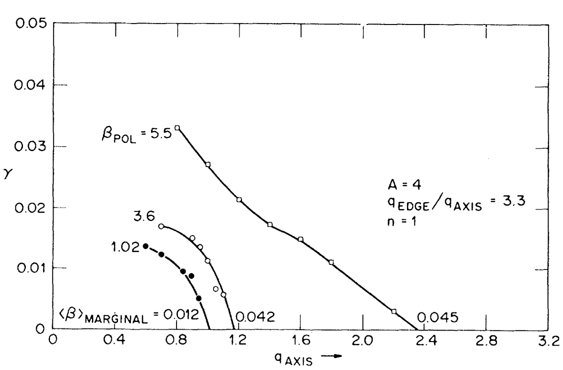

FIG. 2. Growth rates as a function of q value for $n=1$ instabilities. On each curve, corresponding to a different value of $\beta_{\rm pol}$, the equilibrium is scaled according to Eqs. (1)–(3) and the $\langle\beta\rangle$ is noted for the marginal point $\gamma = 0$.

growth rates. All growth rates are measured in units of the Alfvên transit time across the plasma minor radius. The different curves in Fig. 1 correspond to equilibria with different values of β_{pol}. Moving through a flux-conserving sequence corresponds to moving along a vertical line of this graph. As β_{pol} is raised, the q value of the extrapolated marginal stability point increases; this increase must be accompanied by a decrease in the toroidal current and consequently in a maximum stable β. For this one particular set of equilibria, the maximum stable β was found to be nearly 5%. It should be emphasized that for FCT equilibria, the Mercier stability criterion permits marginal β values that increase indefinitely with β_{pol}.[4,5] It is the large-scale ballooning instabilities that appear to limit β.[8]

As β_{pol} is raised, we find that the character of the instabilities near the marginal point changes substantially. For $\beta_{pol} \sim 1$, corresponding to present-day tokamaks, the spatial domain of the instability shrinks into the magnetic axis as the q value is raised to the marginal point, as shown in Figs. 1(b) and 1(c) for a typical $m=1$, $n=1$ instability. However, as β_{pol} is raised, the structure of the instability near the marginal point moves toward the periphery of the plasma and the instability takes on a strong ballooning character. The perturbation is strongest near the outer edge of the toroid where the gradients are steepest and the curvature has the most destabilizing effect,[8] as shown in Figs. 1(e) and 1(f). The perturbed pressure shown in these contour plots results from convection around vortex cells in the poloidal plane.[7] It is observed that several harmonics, corresponding to poloidal harmonics $e^{in\theta}$ in the straight circular cylinder analogy, exist simultaneously at different distances from the magnetic axis, the number of harmonics increasing with n The harmonics always reinforce each other at the outer edge of the toroid, resulting in the ballooning character. Also, as β_{pol} is raised, the higher harmonics near the periphery of the plasma become stronger than the ones near the magnetic axis.

These differences between low-β and high-β instabilities are expected to have a profound effect on the nonlinear development and consequences of the instabilities. The resistive form of the low-β instability[9] is believed to result in the experimentally observed sawtooth oscillations[10] which flatten the temperature and current profiles within the $q=1$ mode rational surface. These instabilities are relatively benign when q_{edge} is high and β is low. Also it is found that a moderate amount of shear in low β will cause MHD instabilities near the periphery of the plasma to saturate nonlinearly at small amplitudes.[11] It is not necessary or even desirable to operate present-day tokamaks in a completely stable mode, for it is experimentally observed that the best tokamak operating conditions are achieved when the enhanced energy loss due to unstable activity is balanced by the incremental power input.[10] However, because these ballooning instabilities at high β appear first near the periphery of the plasma where the pressure gradients are steepest, they are potentially very dangerous unless they saturate at small amplitudes due to shear or nonideal effects.

The results presented in this paper were obtained from an initial-value computer code using the ideal MHD equations on a Cartesian grid with 54 by 35 points over the cross section. Similar results were also obtained with 38 by 25 grid points. Instabilities with $n \geq 2$ are found to be insensitive to the boundary conditions. A highly resistive region is used outside the plasma to simulate a vacuum.

Work is in progress to study a wider class of equilibria in order to determine the effect of plasma cross section, pressure profile, and shear on large-scale MHD instabilities. Also, non-FCT sequences of equilibria (with reversed toroidal current densities near the inner edge of the toroid and with steeper q profiles indicative of a separatrix near the plasma) are being studied. The nonlinear Oak Ridge National Laboratory instability code[12] will be used to look for nonlinear saturation of the ideal MHD ballooning modes. Further work is needed on the nonideal form of these instabilities.

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High-Pressure Brillouin Scattering Study of the Order-Disorder Transition in KCN†

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Pressure and temperature dependences of the Brillouin spectra for KCN are shown for pressures of 0 to 7 kbar and temperatures of 178 to 295 K. It is shown that multiphonon interactions are the dominant anharmonic effect, and it is suggested that the phase transition at 168 K comes from a ferroelastic ordering of CN^- dipoles strongly coupled anharmonically to the phonons.

Crystalline KCN displays many interesting properties which arise from the molecular character of the CN^- ion. Of interest in this Letter is the phase transition[1] occurring at 168 K at pressures of 1 bar, in which KCN transforms from a high-temperature fcc structure with CN^- directional disorder to a lower-temperature orthorhombic structure with directional order but head-to-tail randomness. The transition temperature, T_c, is strongly pressure dependent, increasing 2 K per kbar of hydrostatic pressure.[2]

The phase transition is seen clearly in the elastic constants, which have been studied by two different techniques. Haussühl[3] used 15-MHz ultrasonic measurements to show that C_{44} softens very markedly, tending to zero at a temperature T_0 about 14 K below T_c, and therefore concluded that C_{44} was directly involved in the phase transition. Brillouin scattering studies were doen by Krasser, Buchenau, and Haussühl[4] in order to study the temperature dependence of C_{44} at GHz frequencies, where CN^- orientational motion might be reflected in C_{44} and therefore involved in the phase transition.

In this Letter we report both the pressure[5] and temperature dependences of C_{44} obtained by Brillouin spectroscopy. It will be shown that C_{44} and T_0 are independent of pressure and that the softening of C_{44} is *not* the driving mechanism for the phase transition, as suggested by Rowe *et al.*[6] Instead, the TA mode associated with C_{44} will be seen to be strongly anharmonic, with a large multiphonon contribution to the self-energy near T_c. It will be suggested that the phase transition